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Rate-dependent constitutive model of poly(ethylene terephthalate) for dynamic analysis*

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Abstract: Uniaxial tensile testing at strain rates ranging from 10^{-3} to 10^{-1} s^{-1} was carried out to study the rate-dependent mechanical behavior for poly(ethylene terephthalate) (PET) used in the packaging industry. The experimental results show that a rate-dependent plastic behavior exists for PET material. The value of the yield strength was found to increase with the increasing strain rate. A new constitutive model based on the improved Cowper-Symonds rate-dependent constitutive model is proposed to describe the mechanical behavior of PET material in the strain rate ranging from 10^{-3} to 10^{-1} s^{-1}, providing more accurate material data for the subsequent simulation analysis of drop test and dynamic buckling. The predictions obtained using the proposed model are compared with experimental results of the improved Cowper-Symonds model. The simulating results of the proposed model agree well with the experimental data. For a low strain rate, the predictions of this model are more precise than those obtained using the improved Cowper-Symonds model. This confirms that the new constitutive model is suitable for describing the mechanical behavior of PET material at a low strain rate and modeling impact problem.

Key words: Rate-dependent, Tensile testing, Constitutive model, Strain rate, Poly(ethylene terephthalate) (PET)
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1 Introduction

Plastic containers are extensively used for transportation and storage of a vast variety of fluids, such as oils, fuel, water, etc. (Karac, 2003). It is commonly accepted that plastic packs have a number of advantages over glass packaging: lighter, less easily broken, and less expensive (van Dijk *et al*., 1998). Poly(ethylene terephthalate) (PET) has become the common choice of material for bottled beverages because it is an amorphous engineering thermoplastic with an excellent balance of toughness, weight, and clarity. The PET bottles are manufactured by an injection stretch blow molding process in which the preform (shaped like a test tube) is heated and expanded into conformity with the inside surface of a injection cavity, thus forming a thin-walled bottle (Yang *et al*., 2004).

The important durability standards for PET bottles are drop and top-load tests (QB/T 1868-2004), which are dynamic problems. The conventional method satisfies the performance requirement using repeated tests, which are costly and time-consuming. Consequently, using the finite element method (FEM), the dynamic performance of PET bottle can be simulated without numerous models and tests. In this case, strain rate-dependent stress-strain curves are required to make accurate crashworthiness predictions based on dynamic finite element analyses with a view to improve the impact resistance of PET bottle.

Studies of the mechanical behavior of PET bottles have been conducted in recent years. Karalekas *et al*. (2001) compared the numerically obtained strains

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with those experimentally measured ones. Finite element analysis can calculate stresses and deformations at various critical points of the PET bottles. Chittepu *et al*. (2009) accurately simulated the deformation behavior of filled PET bottles in the labeling process using the smoothed particle hydrodynamics (SPH) approach for accounting inertia of the liquid in combination with machine kinematics. The results can help to reduce bottle losses in the machine carrousel and avoid bad placement of the label. Morrison *et al*. (2010) investigated the stress cracking behavior of PET bottles, and proposed a new method for the prevention of stress cracking based on water hardness. However, the analyses above do not consider the complex behavior of PET material in reality. Suvanjumrat *et al*. (2007) proposed a mathematical model of the PET thermoplastic material for drop test analysis, which was implemented in the FEM software MSC Dytran. The proposed equation, however, map well up to only a certain strain for all strain rates in its paper.

Various types of constitutive models that incorporate strain hardening and strain rate effects have been presented (Cowper and Symonds, 1957; Bodner and Partom, 1975; Johnson and Cook, 1983; Zerilli and Armstrong, 1987; Dean and Read, 2001; Rusinek and Klepaczko, 2001; Yu *et al*., 2008). In the commercial finite element programs, the phenomenological model is widely used to analyze the rate-dependent mechanical behaviors for common materials, mainly because of the few material constants and simplicity in applications (Li *et al*., 2009). More physicals models were reported in Durrenberger *et al*. (2007) and Voyiadjis and Almasri (2008). They are normally quite complicated and rarely used in FEM code. A critical review on this topic is provided by Liang and Khan (1999) and Rusinek *et al*. (2007).

In this paper, the experimental studies for the samples of PET material have been performed at strain rates of 0.00333, 0.0333, 0.1333, and 0.333 s^{-1}. A new plastic constitutive equation based on the improved Cowper-Symonds constitutive model is proposed to describe the mechanical behavior of PET material at various strain rates. The parameters in this model can be determined by the experimental data. A comparison is reported between the proposed rate-dependent constitutive model and the improved Cowper-Symonds model fitted to experimental data.

2 Experiments

2.1 Materials and specimens

The material tested in this study was PET homopolymer, manufactured and delivered by Wahaha Company (Hangzhou, China) as a blow-molded mineral water bottle. Its mass density is 1.37×10^3 kg/m^3. Based on the GB/T 1040.3-2006, the final specimen dimensions used in the tension test was determined. The width and thickness of specimens were all measured with micrometer to obtain the right cross-section area for all specimens. The gauge length, fillet radius, and total length of the specimen were 25, 14, and 115 mm, respectively. All the samples (Fig. 1) were machined directly from PET bottles using a punch machine and then kept at room temperature and a relative humidity of 50% for at least 2 d prior to testing in order to relax residual stress.

Fig. 1 Poly(ethylene terephthalate) specimens for tensile testing

2.2 Testing and apparatus

Tension tests at strain rates ranging from 10^{-3} to 10^{-1} s^{-1} were performed with a Zwick/Roell Z020 testing machine (Zwick GmbH & Co. KG, Germany) (Fig. 2) according to the GB/T 1040.1-2006 at room temperature and a relative humidity of 50%. According to the manufacture's specifications, this machine can be used to perform tests at a maximum velocity of 1000 mm/min and a maximum load of 20 kN. The maximum stretching distance is 1000 mm. The strain is directly measured in the parallel range of the tensile specimen with a sensor arm extensometer to avoid strain of the specimen in the non-parallel region and yielding of the grips. The force applied to the specimen is measured using a standard load cell, and the engineering stress σ is determined as the ratio of the stretch force to the cross-sectional area of specimens in the stress-free state. According to the standards, the grip separation speeds were 5, 50, 200, and 500 mm/min, then the strain rates were determined from the responding tensile velocities and the gauge length of the specimens.

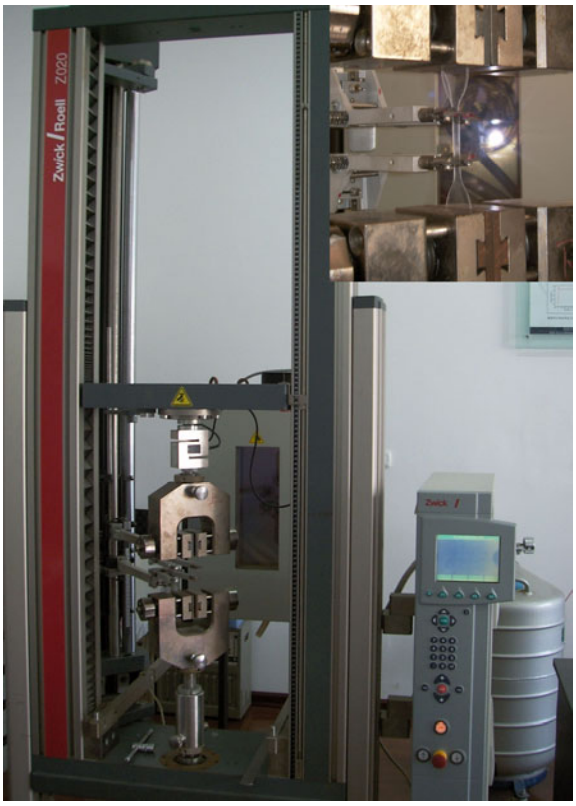

Fig. 2 Zwick/Roell Z020 testing machine (Zwick GmbH & Co. KG, Germany)

2.3 Experimental results

The experimental results of the true stress-strain curves at 0.00333, 0.0333, 0.1333, and 0.333 s^{-1} are shown in Fig. 3, in which the data reported are averaged from at least five samples. This obtained stress-strain experiment data is used to form the strain rate-dependent constitutive model of the PET material.

It is concluded from Fig. 3 that the true stress value at the same strain becomes larger with the increase of the strain rate up to a certain strain. The initial yield stresses at high strain rates are larger than those at lower strain rates, indicating that the strain rate has a significant effect on the strain hardening behavior.

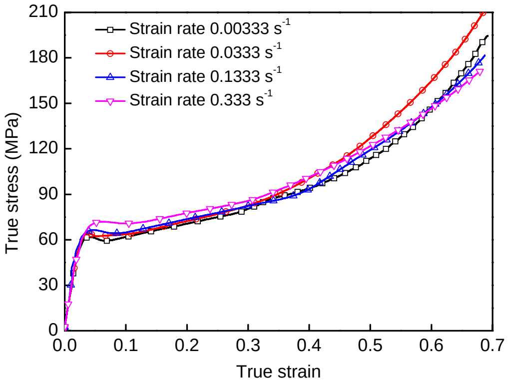

Fig. 3 True stress-strain curves of PET at four strain rates

3 Constitutive models

3.1 Improved Cowper-Symonds model

The PET material exhibits the elastic-plastic behavior, the stress-strain relation of this isotropic material is linear elastic and the nonlinear plastic deformation associates with the effects of strain hardening for large strains. Since the hardening curve of PET depends on the strain rate associated with the applied tensile load (Zaroulis and Boyce, 1997), the material model of PET must consider the effect of the strain rate.

To establish a constitutive model suitable for FEM code, the relationship between representative stress and strain after onset of yielding is assumed to be the Cowper-Symonds model, which is widely used to predict the yield strength and describe the rate-dependent yield behavior of various materials. This model, which combines the effects of strain hardening and strain-rate effects in a multiplicative manner, can be expressed as (Cowper and Symonds, 1957)

$$\sigma = f(\varepsilon_{\mathrm{p}}, \dot{\varepsilon}_{\mathrm{p}}) = \sigma_0(\varepsilon_{\mathrm{p}}) f(\dot{\varepsilon}_{\mathrm{p}}), \tag{1}$$

$$\sigma_0(\varepsilon_{\mathrm{p}}) = \sigma_0 + K\varepsilon_{\mathrm{p}}^{n}, \tag{2}$$

$$f(\dot{\varepsilon}_{\mathrm{p}}) = 1 + (\dot{\varepsilon}_{\mathrm{p}} / D)^{1/P}, \tag{3}$$

where σ is the equivalent yield stress, $\sigma_0(\varepsilon_{\mathrm{p}})$ is the quasi-static yield stress, ε_{p} and $\dot{\varepsilon}_{\mathrm{p}}$ are true plastic strain and strain rate, respectively, $f(\dot{\varepsilon}_{\mathrm{p}})$ is a function of strain rate that represents the strain rate-dependent behavior, K, P, D, and n are material parameters.

The Ludwik equation (Eq. (2)) (Ludwik, 1909) is used to model strain hardening behavior, and the testing at 0.00333 s^{-1} is assumed to be quasi-static tensile testing. Then the strain rate, 0.00333 s^{-1}, is selected as the reference curve to determine the material parameters in Eq. (2). Using the curve-fit method, the stress-strain relation in plastic region for PET material at the reference strain rate can be expressed as

$$\sigma_0(\varepsilon_\mathrm{p}) = 59.65 + 352.13\varepsilon_\mathrm{p}^{2.11}. \quad (4)$$

The curve of Eq. (4) compared with the experimental data at the reference strain rate is illustrated in Fig. 4a. It is seen that the model prediction does not agree well with the experimental data. To better quantify the strain hardening response, based on the Khan-Huang model (Khan and Huang, 1992), an exponential term is added to the Ludwik equation to describe the strain hardening behavior of PET material as (Yu *et al.*, 2008)

$$\sigma_0(\varepsilon_\mathrm{p}) = \sigma_0 + K\varepsilon_\mathrm{p}^n - a\exp(-\alpha\varepsilon_\mathrm{p}), \quad (5)$$

where a and α are material constants determined from experimental data.

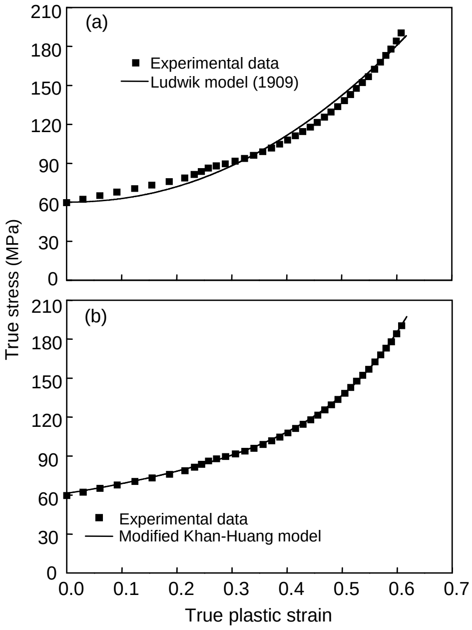

Fig. 4 Comparison of the Ludwik model (a) and modified Khan-Huang model (b) with the experimental data ($\dot{\varepsilon}_\mathrm{p} = 0.00333\ \mathrm{s}^{-1}$)

The comparison curve between Eq. (5) and the experimental data is shown in Fig. 4b. The result shows that the Khan-Huang model prediction agrees very well with the experimental data. Then the stress-strain relation in plastic region for PET material at strain rate 0.00333 s^{-1} can be written as

$$\sigma_0(\varepsilon_\mathrm{p}) = 59.65 + 74.76\varepsilon_\mathrm{p}^{1.09} + 1.35\exp(6.86\varepsilon_\mathrm{p}). \quad (6)$$

3.2 Precision of the mathematical model

According to the experimental observation shown in Fig. 3 and the method to modify Johnson-Cook model (Yu *et al.*, 2008), a new plastic constitutive model based on the improved Cowper-Symonds model is proposed to describe the strain rate-dependent behavior of PET material as

$$\sigma = f(\varepsilon_\mathrm{p}, \dot{\varepsilon}_\mathrm{p}) = \sigma_0 f(\dot{\varepsilon}_\mathrm{p}) + K\varepsilon_\mathrm{p}^n - a\exp(-\alpha\varepsilon_\mathrm{p}). \quad (7)$$

In this model, the strain rate effect is only characterized by exponent form function $f(\dot{\varepsilon}_\mathrm{p})$ (Eq. (3)). Using the least square method, the correlation between yield stress and the four strain rates is illustrated in Fig. 5. Then Eq. (3) can be written as

$$f(\dot{\varepsilon}_\mathrm{p}) = 1 + (\dot{\varepsilon}_\mathrm{p} / 3.42)^{1/1.41}. \quad (8)$$

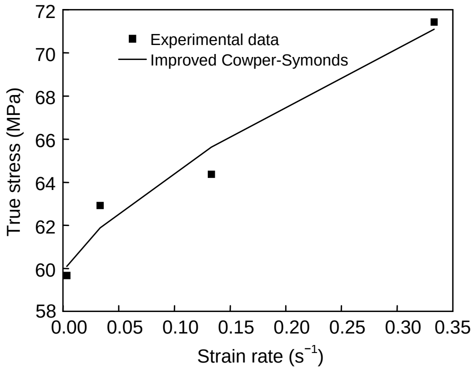

Fig. 5 Correlation between yield stress and the four strain rates

Therefore, the strain rate-dependent constitutive model of PET material for dynamic analysis can be written as

$$\sigma(\varepsilon_\mathrm{p}, \dot{\varepsilon}_\mathrm{p}) = \sigma_0\left(1 + (\dot{\varepsilon}_\mathrm{p} / D)^{1/P}\right) + K\varepsilon_\mathrm{p}^n - a\exp(-\alpha\varepsilon_\mathrm{p}), \quad (9)$$

where the coefficients σ_0=59.65 MPa, K=74.76 MPa, a=−1.35 MPa, D=3.42, P=1.41, n=1.09, and α=−6.86.

3.3 Comparison of constitutive models

Experimental data and simulation results of the improved Cowper-Symonds model and the proposed model at different strain rates for PET material are shown in Fig. 6. It is seen that the proposed model provided a significant improvement over the improved Cowper-Symonds model, and it can capture strain hardening behavior and strain rate sensitivity perfectly at strain rates ranging from of 10^{-3} to 10^{-1} s^{-1}. Moreover, this model is as simple as other phenomenological constitutive models used in numerical simulation, and can be coded with commercially FEM software easily such as ABAQUS and LS-DYNA.

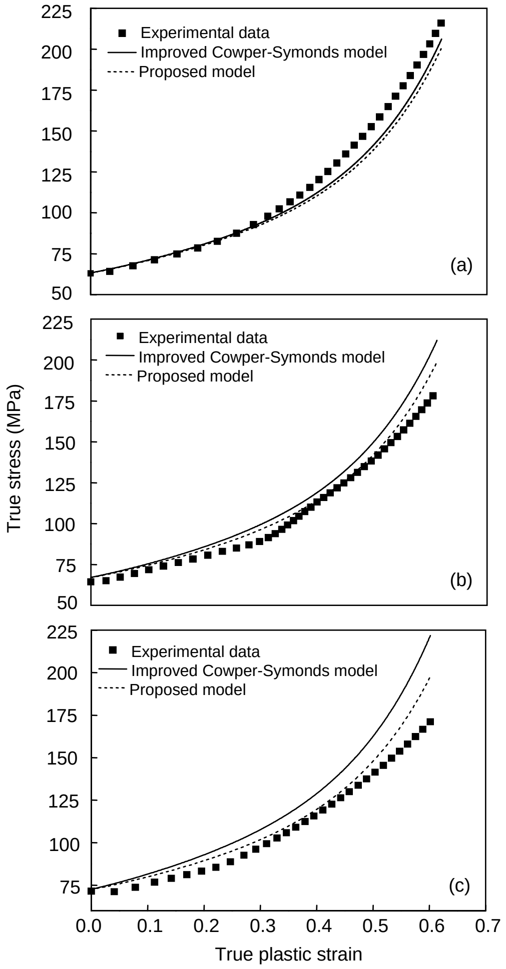

Fig. 6 Comparison of the proposed model with the improved Cowper-Symonds model for the experimental data at different strain rates
(a) 0.0333 s^{-1}; (b) 0.1333 s^{-1}; (c) 0.333 s^{-1}

4 Conclusions

Uniaxial tensile experiments for PET material at strain rates 0.00333, 0.0333, 0.1333, and 0.333 s^{-1} have been performed with Zwick/Roell Z020 tensile testing apparatus. The stress-strain curves of PET in the strain rates ranging from 10^{-3} to 10^{-1} s^{-1} have been obtained. The experimental results show that the tensile behavior of PET is sensitive to strain rate. Yield strength increases significantly with increasing strain rate.

A new strain rate-dependent constitutive equation based on the improved Cowper-Symonds model is proposed to describe the mechanical behavior of PET material for dynamic analysis. Comparisons between the constitutive model proposed in this paper and the improved Cowper-Symonds model fitted to experimental data are illustrated, indicating that the proposed model provides better fitting to the experimental data than the improved Cowper-Symonds model. The prediction with this model is more accurate than the improved Cowper-Symonds model for the mechanical behavior of PET material at different strain rates. Therefore, this model can be used in the simulation of dynamic analysis.

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