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Upper bound solution of supporting pressure for a shallow square tunnel based on the Hoek-Brown failure criterion*

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Abstract: To analyze the stability of a shallow square tunnel, a new curved failure mechanism, representing the mechanical characteristics and collapsing form of this type of tunnel, is constructed. Based on the upper bound theorem of limit analysis and the Hoek-Brown nonlinear failure criterion, the supporting pressure derived from the virtual work rate equation is regarded as an objective function to achieve optimal calculation. By employing variational calculation to optimize the objective function, an upper bound solution for the supporting pressure and the collapsing block shape of a shallow square tunnel are obtained. To evaluate the validity of the failure mechanism proposed in this paper, the solutions computed by the curved failure mechanism are compared with the results calculated by the linear multiple blocks failure mechanism when the Hoek-Brown nonlinear failure criterion is converted into the Mohr-Coulomb linear criterion. The influences of rock mass parameters on the supporting pressure and collapsing block shape are discussed.

Key words: Shallow square tunnel, Variational calculation, Curved failure mechanism, Supporting pressure, Shape of collapsing block, Upper bound theorem

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1 Introduction

Since arch structures evenly distribute stress, the cross-sectional shape of a majority of railway and highway tunnels are multi-circular. However, there are numerous square and rectangular tunnels widely used in subway stations, underground cavities and mine tunnels. The mechanical characteristics of square tunnels are not as good as those of circular tunnels, especially at the junction of the tunnel roof and the tunnel wall. Furthermore, to make traveling convenient, subway stations and underground parks are all excavated in shallow soil strata. So how to keep the stability of shallow tunnels with a reasonable supporting pressure is an issue of great engineering significance. The aim of this paper is to find a minimal supporting pressure which prevents the possible collapse of the tunnel roof during the construction process.

The upper bound theorem has been widely used in slope stability analysis since it was first introduced into geotechnical engineering by Chen (1975). Later, some studies found this theory to be a valid method for analyzing the stability problem of tunnels. Davis *et al*. (1980) constructed a linear multi-block collapse mechanism to develop the upper bound solution of the stability ratio. By comparing the lower bound solution and experimental result, they claimed that their result was valid for a shallow tunnel under undrained condition when the depth ratio was less then 3. On the basis of collapse and blow-out failure mechanisms, Leca and Dormieux (1990) calculated upper bound solutions of supporting pressure for a tunnel face. As the validity of the 3D failure mechanisms has been proved by Chambon and Corte (1994) using a

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centrifuge model test, these failure mechanisms have been cited frequently by others. Based on these mechanisms and taking seepage force into account, Lee and Nam (2001) computed the supporting pressure of an underwater tunnel face in the framework of the upper bound theorem. The stability of the front of a slurry shield-driven tunnel was studied by Li *et al.* (2008) employing the upper bound theorem of limit analysis. With the rapid development of computer technology in recent years, large optimization problems can be solved efficiently by using complex algorithms to decrease the computing time. As a result, some authors employed the finite element method in conjunction with the limit analysis theorem to study the stability problem of tunnels. To investigate the stability of a square tunnel in undrained soil, Sloan and Assadi (1991) derived the rigorous bounds of supporting pressure by combining the finite element method and limit analysis theory. Similarly, using the same approach, Assadi and Sloan (1991) used two numerical techniques to calculate the rigorous bounds on the loads required to prevent shallow tunnels from active and passive failures.

All studies mentioned above adopted a linear failure criterion in conjunction with limit analysis theorem and finite element method to study the stability problem of tunnels. However, with the development of geotechnical experimental techniques, numerous experimental results have proved that the strength envelopes of geomaterials are nonlinear. Therefore, the nonlinear failure criterion is now widely used in various geotechnical stability analyses (Yang *et al.*, 2004; Yang and Yin, 2006; Yang, 2007; Zhang *et al.*, 2010). As there was no suitable method to estimate the mechanical characteristics and the joint rock mass strength, Hoek and Brown (1980) proposed the original Hoek-Brown failure criterion. Due to its simplicity and accuracy, numerous scholars employed this criterion to analyze various geotechnical stability problems. Using the limit analysis method, Li *et al.* (2008) calculated stability charts for the rock slope using the Hoek-Brown failure criterion, and Merifield *et al.* (2006) estimated the ultimate bearing capacity of a foundation resting on a rock mass which is subjected to the Hoek-Brown failure criterion, and presented rigorous bounds of the ultimate bearing capacity for the foundation. To study the collapsing block shape of a deep tunnel with arbitrary cross-sections, Fraldi and Guarracino (2010) proposed a 2D failure mechanism to compute the external power and the internal dissipation power in the framework of the limit analysis method and the Hoek-Brown failure criterion. Though their upper bound solutions of the collapsing surface derived from variational calculation are effective, their mechanism is not suitable for shallow tunnels. Thus, it is necessary to construct a new mechanism which describes the collapsing features of surrounding rock over a shallow tunnel roof.

In this paper, a new collapse mechanism which represents the collapsing block of surrounding rock over a shallow tunnel roof is proposed to construct a virtual work equation on the basis of the upper bound theorem combined with the Hoek-Brown failure criterion. What is new in this paper is that the supporting pressure is regarded as an objective function to achieve optimal calculation. Taking advantage of the variational calculation, the optimal upper bound solutions for the supporting pressure and an analytical expression of the collapsing block shape over the tunnel roof are obtained. Thus, with support pressure being considered, we extend the study of the collapsing block shape of a deep cavity conducted by Fraldi and Guarracino (2009) to shallow tunnels.

2 Hoek-Brown failure criterion

The generalised Hoek-Brown failure criterion is always expressed by the maximum effective stress σ_1 and minimum effective stress σ_2 (Hoek and Brown, 1980; 1988; Hoek *et al.*, 2002). However, the energy dissipation power along the collapsing surface is composed of two parts, i.e., the energy dissipation caused by normal stress and shear stress. Consequently, the Hoek-Brown criterion represented by the normal and shear stresses (Hoek and Brown, 1997) applied in this study can be written as

$$\tau = A\sigma_{\mathrm{ci}}\left(\frac{\sigma_{\mathrm{n}}+\sigma_{\mathrm{tm}}}{\sigma_{\mathrm{ci}}}\right)^{B}, \tag{1}$$

where σ_{n} is the normal stress, τ is the shear stress, A

and B are material parameters, and σ_{ci} and σ_{tm} are the uniaxial compressive strength and the tensile strength of the geomaterials, respectively. Due to the lack of a suitable failure criterion to evaluate the strengths of a poor rock mass, Hoek and Brown (1997) stated that the Hoek-Brown failure criterion can be applied to projects in very poor quality rock mass, which is classified as engineering soil. So the application of the Hoek-Brown failure criterion to the study of the failure block of shallow square tunnels is sensible and feasible.

3 Upper bound theorem of limit analysis

According to Chen (1975), the upper bound theorem can be described as follows: in any kinematically admissible velocity field, the load determined by the equation of virtual work is larger than the actual collapsing load when the velocity boundary condition is satisfied by

$$\int_v \sigma_{ij}\dot{\varepsilon}_{ij}\mathrm{d}v \geq \int_s T_i V_i \mathrm{d}s + \int_v X_i V_i \mathrm{d}v, \tag{2}$$

where σ_{ij} is the stress tensor, ε_{ij} is the strain rate in the failure mechanism, T_i is a surcharge load on the velocity field boundary s, X_i is the body force, v is the volume of the mechanism, and V_i is the velocity along the velocity discontinuity surface. As noted by Chen (1975), the limit analysis theorem can be applied to the geotechnical material if the material has the following ideal properties: (1) it is perfectly plastic material which ignores the strain hardening and strain softening features of stress-strain; (2) the failure surface of the material is convex and the associated flow rule is applicable; (3) the geometric deformation of the failure mechanism induced by limit load is insignificant.

4 Curved failure mechanism of a shallow square tunnel

Fraldi and Guarracino (2009) proposed a 2D curved failure mechanism to investigate the shape of a collapsing block for a deep cavity. The curved failure mechanism is constructed by an arched curve which confines the possible collapse of the surrounding rock over the tunnel roof to a certain region. However, as the overburden layer of shallow tunnels is thin, the collapsing surface of tunnels extends to the ground surface. So, in this study, a new failure mechanism (Fig. 1) is used to describe the collapsing block of the surrounding rock of shallow tunnels. This failure mechanism is composed of two symmetrical curves which extend from the junction of the tunnel roof and wall to the ground. As velocity discontinuity occurs along the collapse surface in the velocity field, the failure surface can also be called a detaching surface. L is half of the width of the failure surface, σ_s is the surcharge at ground surface, q is the supporting pressure, H is the buried depth of the shallow square tunnel, b is half of the width of the square tunnel and $f(x)$ is the analytical expression of the detaching surface (Fig. 1).

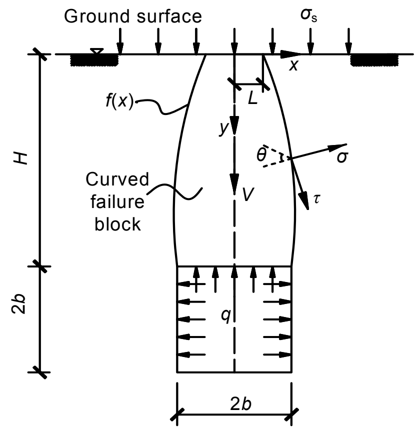

Fig. 1 Curved failure mechanism of a shallow square tunnel

5 Upper bound solution of supporting pressure q for a shallow tunnel

According to the associated flow rule, the plastic strain rate is proportional to the stress gradient of the plastic potential, and the plastic potential is coincidental with the Hoek-Brown failure curve. Therefore, the normal and shear strains are calculated, and the energy dissipation of a random point on the detaching surface was calculated by Fraldi and Guarracino (2009) as

$$D=\sigma_{\rm n}\dot{\varepsilon}_{\rm n}+\tau\dot{\gamma}_{\rm n}$$
$$=\left\{\sigma_{\rm tm}-\sigma_{\rm ci}\left[ABf'(x)\right]^{\frac{1}{1-B}}(1-B^{-1})\right\}\frac{v}{t}\frac{1}{\sqrt{1+f'(x)^2}}, \quad (3)$$

where $\varepsilon_{\rm n}$ is the normal strain rate, $\gamma_{\rm n}$ is the shear strain rate, $f'(x)$ is the first derivative of $f(x)$, and t is the thickness of the failure surface. By integrating D over the interval $[L, b]$, the internal energy dissipation power along the detaching surface is obtained:

$$W_D=\int_L^b\left\{\sigma_{\rm tm}-\sigma_{\rm ci}\left[ABf'(x)\right]^{\frac{1}{1-B}}(1-B^{-1})\right\}\frac{v}{t}{\rm d}x. \quad (4)$$

The power of the collapse block caused by weight can be determined by

$$W_\gamma=\left[H\gamma b-\int_L^b\gamma f(x){\rm d}x\right]v, \quad (5)$$

where γ is the unit weight of the geomaterials. The power of the supporting pressure q can be written as

$$W_q=-bqv. \quad (6)$$

The surcharge $\sigma_{\rm s}$ is applied at ground surface and its power can be written as

$$W_{\sigma_{\rm s}}=\sigma_{\rm s}Lv. \quad (7)$$

Based on the upper bound theorem of limit analysis, the virtual work rate equation composed of the external power and the internal energy dissipation power can be expressed as

$$W_D=W_q+W_\gamma+W_{\sigma_{\rm s}}. \quad (8)$$

By substituting Eqs. (4)–(7) into Eq. (8), the supporting pressure q is obtained as

$$q=\frac{1}{b}\int_L^b\left\{\sigma_{\rm ci}\left[ABf'(x)\right]^{\frac{1}{1-B}}(1-B^{-1})-\sigma_{\rm tm}-\gamma f(x)\right\}{\rm d}x+\frac{\sigma_{\rm s}L}{b}+H\gamma$$
$$=\frac{1}{b}\int_L^b\psi\left[f(x),f'(x),x\right]{\rm d}x+H\gamma+\frac{\sigma_{\rm s}L}{b}, \quad (9)$$

where $\psi\left[f(x),f'(x),x\right]$ is a functional which can be expressed as

$$\psi\left[f(x),f'(x),x\right]=\sigma_{\rm ci}\left[ABf'(x)\right]^{\frac{1}{1-B}}(1-B^{-1})-\sigma_{\rm tm}-\gamma f(x). \quad (10)$$

The upper bound theorem of limit analysis states that the loads obtained by the equation of virtual work are larger than the actual loads for any kinematical failure mechanism. So the optimalizing upper bound solution close to the real solution is the extremum of the upper bound expression q represented by Eq. (9). Furthermore, the supporting pressure q is determined only by the function ψ (Eq. (10)). Consequently, the calculation of the optimalizing upper solution of q can be regarded as searching for the extremum value of the objective function ψ. Since the objective function ψ is a functional whose extremum is difficult to derive, variational calculation is employed here to solve this problem. Based on the variational principle, the calculation of the extremum of ψ can be converted into solving a Euler's equation which is used to derive the equation of a detaching surface in the failure mechanism. Based on the principle of the stationary value of variational calculation, we can obtain the Euler equation of ψ from

$$\gamma-\sigma_{\rm ci}(AB)^{\frac{1}{1-B}}\left[f'(x)\right]^{\frac{2B-1}{1-B}}f''(x)\frac{1}{1-B}=0. \quad (11)$$

It is evident that Eq. (11) is a second-order homogeneous differential equation. Thus, the expression of $f(x)$ is obtained by conducting two integrations:

$$f(x)=A^{-1/B}\left(\frac{\gamma}{\sigma_{\rm ci}}\right)^{(1-B)/B}\left(\frac{c_0}{\gamma}+x\right)^{1/B}+c_1, \quad (12)$$

where C_0 and C_1 are integration constants which can be calculated by the boundary condition. By substituting Eq. (12) into Eq. (10), the function ψ is determined:

$$\psi\left[f(x),f'(x),x\right] = -A^{-1/B}\sigma_{\mathrm{ci}}^{(B-1)/B}\gamma^{1/B}B^{-1}\left(x+\frac{c_0}{\gamma}\right)^{1/B} - \sigma_{\mathrm{tm}} - c_1\gamma. \tag{13}$$

By substituting Eq. (12) into Eq. (9), the upper solution for the supporting pressure q is obtained:

$$q = H\gamma + \frac{\sigma_{\mathrm{s}}L}{b} - \frac{1}{b}(b-L)(\sigma_{\mathrm{tm}}+c_1) - \frac{1}{b(1+B)}A^{-1/B}\sigma_{\mathrm{ci}}^{(1+B)/B}\gamma^{1/B}\left[\left(b+\frac{c_0}{\gamma}\right)^{(1+B)/B} - \left(L+\frac{c_0}{\gamma}\right)^{(1+B)/B}\right]. \tag{14}$$

To achieve the optimal calculation for the supporting pressure, the unknown constants C_0, C_1 and L in Eq. (14) should be determined. Since there is no distribution of shear stress on the ground surface, the shear stress at the junction of the failure surface and the ground surface is zero. In line with the mechanical equilibrium equation, the expression of shear stress at the location (x=L, y=0) is derived:

$$\tau_{xy}(x=L, y=0) = 0. \tag{15}$$

Furthermore, there are two geometric equations (Fig. 1) which can be used to determine these integration constants:

$$f(x=L) = 0, \tag{16}$$

$$f(x=b) = H. \tag{17}$$

In accordance with Eqs. (15)–(17), the constants C_0, C_1 and L are obtained. After substituting these constants into Eqs. (12) and (14), the final forms of detaching curve $f(x)$ and supporting pressure q can be expressed as follows:

$$f(x) = A^{-1/B}\left(\frac{\gamma}{\sigma_{\mathrm{ci}}}\right)^{(1-B)/B}\left[x + AH^B\left(\frac{\gamma}{\sigma_{\mathrm{ci}}}\right)^{B-1} - b\right]^{1/B}, \tag{18}$$

$$q = H\gamma + \frac{\sigma_{\mathrm{s}}}{b}\left[b - AH^B\left(\frac{\gamma}{\sigma_{\mathrm{ci}}}\right)^{B-1}\right] - \frac{1}{b(1+B)}A\sigma_{\mathrm{ci}}^{1-B}\gamma^B H^{1+B} - \frac{1}{b}\sigma_{\mathrm{tm}}AH^B\left(\frac{\gamma}{\sigma_{\mathrm{ci}}}\right)^{B-1}. \tag{19}$$

6 Comparisons with existing mechanism

To analyze the stability of shallow tunnels in cohesive material, Davis *et al*. (1980) proposed a linear multiple blocks failure mechanism to calculate the stability ratio of tunnels in conjunction with the Mohr-Coulomb failure criterion. This mechanism is composed of numerous rigid failure blocks (Fig. 2). The results of Chambon and Corte (1994) derived from a centrifuge model test show that the linear multiple blocks failure mechanism is an effective failure mechanism in the upper bound analysis of shallow tunnels. Consequently, to estimate the validity of the method applied in this study, the results derived from the curved failure mechanism are compared with the solutions calculated by the linear multiple blocks failure mechanism. However, how to convert the parameters of the Hoek-Brown failure criterion into the parameters of the Mohr-Coulomb failure criterion is a problem in this comparison. It is obvious that the Hoek-Brown failure criterion converts into the Mohr-Coulomb linear criterion when parameter B=1. Thus, in this case, Eq. (1) can be expressed as

$$\tau = A\sigma_{\mathrm{n}} + A\sigma_{\mathrm{tm}}. \tag{20}$$

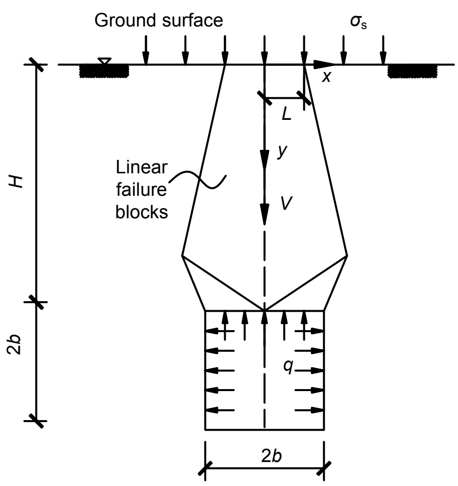

Fig. 2 Linear multiple blocks failure mechanism for a shallow square tunnel

Therefore, the parameters in the Hoek-Brown criterion are equal to those in the Mohr-Coulomb when B=1:

$$A = \tan\phi, \quad \sigma_{\mathrm{tm}} = C / \tan\phi, \tag{21}$$

where ϕ is the friction angle and C is the cohesion.

The supporting pressure values of the curved failure mechanism are plotted in Fig. 3 as functions of the tunnel depth ratio H/b with the equivalent parameters of the Mohr-Coulomb criterion calculated by Eq. (21), while the results of the linear multiple blocks failure mechanism are calculated by a nonlinear sequential quadratic programming algorithm. The values of q for both failure mechanisms are nearly the same in most cases except for a tunnel depth ratio larger than 1.6. As a result, the slight difference in the supporting pressure between these two mechanisms proves that the curved failure mechanism for shallow tunnels used in this study is effective.

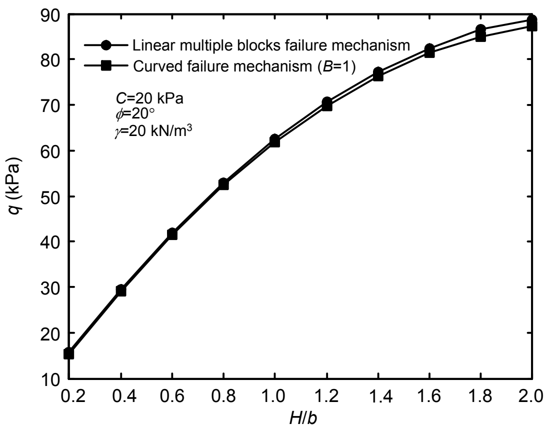

Fig. 3 Comparison of q between curved and linear failure mechanisms

7 Parameter study for supporting pressure q

Hoek and Brown (1997) claimed that different parameters in the Hoek-Brown criterion represent different qualities of the surrounding rock. To discuss the effect of a single parameter on the supporting pressure of shallow tunnels, reference can be made to Fig. 4, where the values of q are plotted as the function of a tunnel depth ratio H/b for different B, A, σ_{tm}, and γ respectively, when other parameters are fixed. The values of q increase nonlinearly with the increase in the tunnel depth ratio, which means that tunnel depth is the key factor influencing the supporting pressure. Furthermore, the values of q tend to increase directly with the values of B and γ, and inversely with the values of A and σ_{tm} when other parameters are constant. Therefore, it can be concluded that the stability of shallow square tunnels in surrounding rock with a low value of B and a high value of σ_{tm} can be achieved with a relatively small supporting pressure.

The majority of subway stations and underground cavities are located in urban centers where there are numerous existing structures. Due to the thin overburden, the surrounding rock of a shallow tunnel will not form a collapsing arch to bear the ground load. Thus, to analyze the influence of surcharge σ_{s} on the supporting pressure q is an issue of great research value. Fig. 5 shows the change law of the supporting pressure for varying the tunnel depth ratio and surcharge with soil parameters corresponding to A=0.5, σ_{ci}=2.5 MPa, $\sigma_{\mathrm{tm}}=\sigma_{\mathrm{ci}}/100$, b=10 m, B=0.7, and γ= 15 kN/m^3. q increases directly with the tunnel depth ratio and surcharge. Thus, the surcharge is an important factor which should not be neglected when the tunnel is excavated in shallow soil strata.

Moreover, Hoek and Brown (1997) proposed that the parameters of the Hoek-Brown criterion represented by the normal and shear stresses can be converted into the parameters in the generalised Hoek-Brown criterion using the linear regression technique. Based on this method, the correspondence between the generalised Hoek-Brown criterion parameters (a, m_{b}, s, σ_{ci}, GSI) and those represented by the normal and shear stresses in Eq. (1) is demonstrated in Table 1.

8 Parameter study for the shape of the collapse surface of a shallow square tunnel

To study the influence of different parameters on the shape of the collapse surface of shallow tunnels, the collapse surfaces of soil parameters corresponding to A=0.1–0.3, σ_{ci}=1.5–3.5 MPa, b=10 m, B=0.5–0.9, and γ=15–25 kN/m^3 are illustrated in Fig. 6. To analyze the influence of a single factor on the shape of the

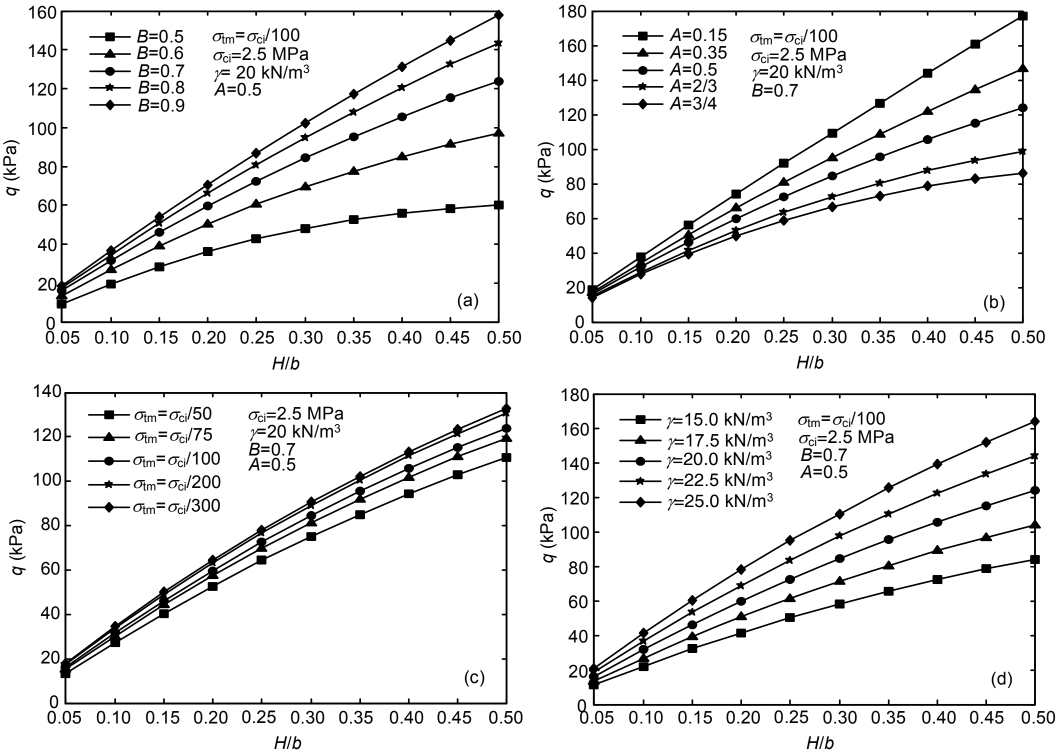

Fig. 4 Upper bound values of q for different B (a), A (b), σ_{tm} (c), and γ (d)

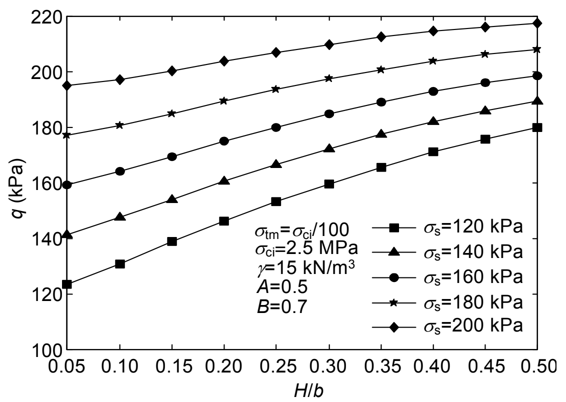

Fig. 5 Upper bound values of q for different surcharge σ_s

Table 1 Correspondence between the generalised Hoek-Brown criterion parameters and those represented by the normal and shear stresses

Parameter	Value	Parameter	Value
a	0.7	A	0.5
m_b	1.9	B	0.7
s	0.02	σ_{ci} (MPa)	2.5
σ_{ci} (MPa)	2.5	σ_{tm} (MPa)	0.025
GSI	25		

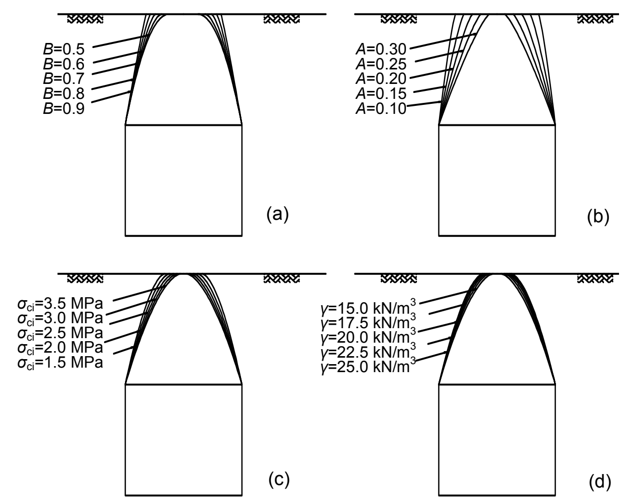

Fig. 6 Shapes of failure surface of a shallow square tunnel with supporting pressure for different B (a), A (b), σ_{ci} (c), and γ (d)

collapse surface, each diagram in Fig. 6 is plotted for one varying parameter while the other parameters remain constant. Note that all the collapse surfaces extend from the tunnel roof to the ground along two symmetrical curved lines for different rock mass parameters. Moreover, the top width of the failure block L increases directly with B and γ, and inversely with A and σ_{ci}.

As illustrated in Eq. (20), the Hoek-Brown criterion converts into the Mohr-Coulomb criterion when B=1 and the parameters in the Hoek-Brown failure criterion are equal to those in the Mohr-Coulomb failure criterion. To study the shape of the collapse surface of shallow tunnels which satisfies the Mohr-Coulomb failure criterion, the collapse surfaces for the surrounding rock characterized by A=0.1–0.3, σ_{ci}=2.5 MPa, b=10 m, B=1, and γ=20 kN/m^3 are plotted in Fig. 7. Unlike the general situation, in the limit case B=1, the collapse surface extends from the tunnel roof to the ground along a straight line rather than a curved line (Fig. 6). Furthermore, similar to the changing law mentioned above, the top width of the failure block L still decreases with the increase of A. According to Eq. (21), A is equivalent to tanϕ when the Hoek-Brown failure criterion converts into the Mohr-Coulomb failure criterion. Also, in this limit case the expression of the detaching curve $f(x)$ becomes

$$f(x) = A^{-1}(x + AH - b). \tag{22}$$

It is obvious that Eq. (22) is relevant only to rock mass parameter A. Moreover, A is equal to tanϕ. Therefore, the shape of the collapse surface of shallow tunnels which satisfies the Mohr-Coulomb criterion is determined by only the friction angle ϕ, and is not affected by the cohesion.

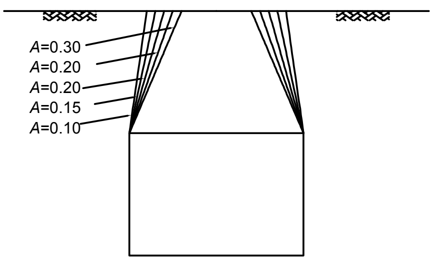

Fig. 7 Shapes of failure surface of a shallow square tunnel with supporting pressure for different parameters when B=1

9 Conclusions

1. Based on the mechanical mechanism of shallow square tunnels, a new 2D curved failure mechanism which describes the collapsing block of this type of tunnel is proposed. According to the Hoek-Brown nonlinear failure criterion and upper bound theorem, the optimal upper solution for the supporting pressure and the collapsing block shape of shallow tunnels are obtained with the help of variational calculation. By using equivalent parameters of the Mohr-Coulomb criterion, the upper solutions for the supporting pressure calculated by the curved failure mechanism were compared with the results using the linear multiple blocks failure mechanism when B=1. A slight difference between the results from the two mechanisms indicates that the new failure mechanism is a valid failure mechanism for computing the upper solutions for the supporting pressure of shallow tunnels.

2. The influence of a single parameter on the supporting pressure of shallow square tunnels was studied when other parameters were fixed. The change laws of the supporting pressure which varies with the rock mass parameter showed that the stability of a shallow tunnel in the surrounding rock with a low value of B and a high value of tensile strength σ_{tm} can be achieved with a relatively small supporting pressure.

3. The shapes of the failure surface drawn by the analytical solution of a velocity discontinuity surface $f(x)$ showed that the range of the collapse surface increases directly with B and γ and inversely with A and σ_{ci}. Furthermore, the shapes of the failure surface, with the Hoek-Brown criterion reducing to the Mohr-Coulomb criterion, were also investigated, and were determined by only the friction angle ϕ in this limit case.

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