Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering)
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
www.jzus.zju.edu.cn; www.springerlink.com
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Regenerative Bayesian detection of foundation constant with variable scale gradient theory*

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https://doi.org/10.1631/jzus.A1800467

Before the geotechnical structures are designed, it is often required to determine the mathematical physical model of the corresponding geotechnical medium to properly describe and predict the behavior of the rock and soil during construction and operation in deformation and stability (Bouderba et al., 2013). With the deepening of our understanding of physical phenomena, the geotechnical mechanics, which is evolved from the limit equilibrium theory on basis of rigid body mechanics to the elastic, elastoplastic or viscoelastic theory based on continuum mechanics, is now progressed to various non-continuous media mechanics methods (Al-Hammoud et al., 2011). In this process, various mathematical models have been established for the purpose of describing the system response of the geotechnical medium system even when external environmental conditions are involved.

In the process of numerical simulation analysis of actual geotechnical engineering, the value of geotechnical parameter is often an important factor affecting the accuracy, objectivity and practicability of geotechnical engineering simulation analysis results (Bousahla et al., 2014; Meziane et al., 2014; Zidi et al., 2014). Because the current geotechnical parameters obtained by the test method often have large deviations from the actual parameters, the geotechnical parameters obtained through parameter inversion have always been a research hotspot in geotechnical engineering (Hamdia et al., 2018). In recent decades, the research on geotechnical engineering problem in calculation theory has made great progress. Not only have many new mechanical models and elastic-plastic-viscous calculation methods been established, but also indoor simulation tests, field measurements, and prototypes have been adopted (Nanthakumar et al., 2013, 2016; Khader et al., 2016). Observations and other numerical calculations are further used to establish new techniques to guide geotechnical engineering design and construction with using field monitoring and measurement information, which leads to the fact that the geotechnical engineering calculation theory is gradually becoming more and more complete. However, since the value of the calculative load or the parameter of the rock mass is still subjectively random and the parameters of the quantitative deformation of the material by the laboratory test are often inconsistent with the actual conditions of the project, the practical value of these calculation theories still appear quite limited, which is prompting people to study the theory of determining the value of mechanical parameters with the inversion of on-site measurement information in order that the established calculation theory can be adopted in engineering practice. Compared with direct searching methods, the spacial matrix scale is continually

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* Project supported by the National Natural Science Foundation of China (No. 41272325), the Natural Science Foundation of Jiangsu Province (No. BK20130787), the Fundamental Research Funds for the Central Universities (No. NS2014003), the Research Fund of Graduate Education and Teaching Reform of Nanjing University of Aeronautics and Astronautics (NUAA) (No. 2017-2), and the Research Fund of Education and Teaching Reform of College of Aerospace Engineering, NUAA (No. 2017-5), China
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changed to create new search directions during the iterations in variable scale gradient methods, which hastens the detection of the displacement constants and has satisfactory iterative efficiency. In this paper, a regenerative Bayesian detection model of stochastic foundation constants based on variable scale gradient theory is derived and analyzed.

From the Bayesian theory (Bennoun et al., 2016; Vu-Bac, 2016), the regenerative Bayesian objective function J can be derived as

$$J=\sum_{i=1}^{n}(\boldsymbol{W}_i^*-\boldsymbol{W}_i)^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{W}_i^*}^{-1}(\boldsymbol{W}_i^*-\boldsymbol{W}_i)+(\boldsymbol{X}-\boldsymbol{X}_0)^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{X}}^{-1}(\boldsymbol{X}-\boldsymbol{X}_0), \tag{1}$$

where the stochastic vector $\boldsymbol{X}=[x_1x_2\ldots x_m]^{\mathrm{T}}$ (m is the dimension of the vector $\boldsymbol{X}$). $\boldsymbol{X}_0$ is the expectation vector of the stochastic foundation constants $\boldsymbol{X}$. The measured systematic response data $\boldsymbol{W}_i^*$ of each time are extracted from the total measured data $\boldsymbol{W}^*$. $\boldsymbol{W}_i$ is the systematic response vector of the computational results. $\boldsymbol{C}_{\boldsymbol{X}}^{-1}$ is the inverse matrix of covariance matrix of the stochastic foundation constants $\boldsymbol{X}$. $\boldsymbol{C}_{\boldsymbol{W}_i^*}^{-1}$ is the inverse matrix of covariance matrix of the measured systematic response data $\boldsymbol{W}_i^*$.

The partial differentiation of regenerative objective function J to the stochastic foundation constants $\boldsymbol{X}$ is expressed as

$$\frac{\partial J}{\partial \boldsymbol{X}}=\nabla J=\sum_{i=1}^{n}2\left(\frac{\partial \boldsymbol{W}_i}{\partial \boldsymbol{X}}\right)^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{W}_i^*}^{-1}(\boldsymbol{W}_i-\boldsymbol{W}_i^*)+2\boldsymbol{C}_{\boldsymbol{X}}^{-1}(\boldsymbol{X}-\boldsymbol{X}_0). \tag{2}$$

The achieved value $\hat{\boldsymbol{X}}$ of the foundation constants $\boldsymbol{X}$ can be written as

$$\hat{\boldsymbol{X}}=(\boldsymbol{I}-\boldsymbol{MS})\boldsymbol{X}_0+\boldsymbol{MW}^*-\boldsymbol{M}(\overline{\boldsymbol{W}}-\boldsymbol{S}\overline{\boldsymbol{X}}), \tag{3}$$

where

$$\boldsymbol{M}=\left(\sum_{i=1}^{n}\boldsymbol{S}_i^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{W}_i^*}^{-1}\boldsymbol{S}_i+\boldsymbol{C}_{\boldsymbol{X}}^{-1}\right)^{-1}\left[\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{W}_1^*}^{-1},\boldsymbol{S}_2^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{W}_2^*}^{-1},\cdots,\boldsymbol{S}_n^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{W}_n^*}^{-1}\right]. \tag{4}$$

$\boldsymbol{I}$ is a unit matrix, and $\boldsymbol{W}^*=\left[\boldsymbol{W}_1^*,\boldsymbol{W}_2^*,\cdots,\boldsymbol{W}_n^*\right]^{\mathrm{T}}$, where $\boldsymbol{W}_i^*$ is the measured systematic response vector of the ith time. $\overline{\boldsymbol{W}}=\left[\overline{\boldsymbol{W}}_1,\overline{\boldsymbol{W}}_2,\cdots,\overline{\boldsymbol{W}}_n\right]^{\mathrm{T}}$, where $\overline{\boldsymbol{W}}_i$ is the computational systematic response vector of the ith time at the expectation point $\overline{\boldsymbol{X}}$. $\boldsymbol{S}=[\boldsymbol{S}_1, \boldsymbol{S}_2, \ldots, \boldsymbol{S}_n]^{\mathrm{T}}$, where $\boldsymbol{S}_i(\overline{\boldsymbol{X}})=\left.\frac{\partial \boldsymbol{W}_i}{\partial \boldsymbol{X}}\right|_{\boldsymbol{X}=\overline{\boldsymbol{X}}}$ is the sensitivity matrix of measured systematic responses of the ith time. Using the non-singularity property of $\boldsymbol{C}_{\boldsymbol{X}}$ and $\boldsymbol{C}_{\boldsymbol{W}^*}$, the variance of $\hat{\boldsymbol{X}}$ can be obtained as

$$\boldsymbol{C}_{\hat{\boldsymbol{X}}}=\left[\boldsymbol{C}_{\boldsymbol{X}}^{-1}+\sum_{i=1}^{n}\boldsymbol{S}_i^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{W}_i^*}^{-1}\boldsymbol{S}_i\right]^{-1}, \tag{5}$$

where $\boldsymbol{C}_{\boldsymbol{W}^*}=\mathrm{diag}\left(\boldsymbol{C}_{\boldsymbol{W}_1^*},\boldsymbol{C}_{\boldsymbol{W}_2^*},\cdots,\boldsymbol{C}_{\boldsymbol{W}_n^*}\right)$ and $\boldsymbol{C}_{\boldsymbol{W}_i^*}$ is the covariance matrix of the measured systematic response data of the ith time.

In the previous studies (Bourada et al., 2015; Hamidi et al., 2015, 2017), the governing differential equations for the Winkler foundation model is obtained with elastic Mindlin plate theory, and the Fourier closed solution of the foundation model is achieved with Fourier transform method. The systematic response vector of the computational results $\boldsymbol{W}_i$ can be obtained.

With variable scale gradient method, the corresponding steps for the detection model of foundation constants with regenerative Bayesian theory are as follows:

(1) Select the initial values $\boldsymbol{X}^0$ of foundation constants $\boldsymbol{X}$ and assign the convergence criterion ε_1 and ε_2.

(2) From Eqs. (1) and (2), $J(\boldsymbol{X}^0)$ and $\nabla J(\boldsymbol{X}^0)$ are respectively computed. Set the initial matrix $\boldsymbol{H}^0$ equal to the unit matrix $\boldsymbol{I}$ and the iterative variable k equal to 0.

(3) The searching direction vector is determined by $\boldsymbol{d}^k=-\boldsymbol{H}^k\nabla J(\boldsymbol{X}^k)$.

(4) With Eq. (4), the optimal step length λ is confirmed to make $J(\boldsymbol{X}^{k+1})=\min_{\lambda} J(\boldsymbol{X}^k+\lambda\boldsymbol{d}^k)$, where λ can be determined with the 1D quadratic parabola interpolation search method.

(5) When λ is achieved, compute $J(\boldsymbol{X}^{k+1})$ and $\nabla J(\boldsymbol{X}^{k+1})$.

(6) If one of the below criteria is satisfied, the iteration is convergent, the iteration stops and the procedure proceeds to the last step.

$$\left\| \frac{\boldsymbol{X}^{k+1} - \boldsymbol{X}^k}{\boldsymbol{X}^{k+1}} \right\|_2 < \varepsilon_2, \tag{6}$$

$$\left| \frac{J(\boldsymbol{X}^{k+1}) - J(\boldsymbol{X}^k)}{J(\boldsymbol{X}^k)} \right| < \varepsilon_1. \tag{7}$$

Otherwise, proceed with the following step.

(7) If $J(\boldsymbol{X}^{k+1}) > J(\boldsymbol{X}^k)$, then let $\boldsymbol{X}^0=\boldsymbol{X}^{k+1}$ and go back to step (2) to reiterate. Otherwise, continue iteration.

(8) Compute the iterative variable scale matrix $\boldsymbol{H}^{k+1}$:

$$\boldsymbol{H}^{k+1} = \boldsymbol{H}^k + \Delta \boldsymbol{H}^k, \tag{8}$$

$$\Delta \boldsymbol{H}^k = \frac{\boldsymbol{S}^k (\boldsymbol{S}^k)^{\mathrm{T}}}{(\boldsymbol{S}^k)^{\mathrm{T}} \boldsymbol{Y}^k} - \frac{\boldsymbol{H}^k \boldsymbol{Y}^k (\boldsymbol{Y}^k)^{\mathrm{T}} \boldsymbol{H}^k}{(\boldsymbol{Y}^k)^{\mathrm{T}} \boldsymbol{H}^k \boldsymbol{Y}^k}, \tag{9}$$

$$\boldsymbol{S}^k = \boldsymbol{X}^{k+1} - \boldsymbol{X}^k, \tag{10}$$

$$\boldsymbol{Y}^k = \nabla J(\boldsymbol{X}^{k+1}) - \nabla J(\boldsymbol{X}^k), \tag{11}$$

where $\boldsymbol{S}^k$ is the variable vector difference and $\boldsymbol{Y}^k$ is the gradient vector difference. Then, let k=k+1 and return to step (3) for reiteration.

(9) The detection results of foundation constants $\boldsymbol{X}$ are $\hat{\boldsymbol{X}} = \boldsymbol{X}^{k+1}$ and the covariance $\boldsymbol{C}_{\hat{X}}$ of the foundation constants $\boldsymbol{X}$ is obtained from Eq. (5).

To put the regenerative Bayesian detection of the stochastic foundation constants with variable scale gradient theory into action, the detection procedure named VSGFND.for was compiled and three different kinds of Winkler foundations were considered, on which three different concrete bearing plates were set, each with a simply supported boundary around the four edges as shown in Fig. 1. The foundation mechanical model and the dimensions and other parameters of the plates are presented in (Zhang et al., 2017, 2018). The uniform load q_1 along the z coordinate direction was equal to 180.0 N/cm^2 and applied on the first Winkler foundation. The same kind of uniform load q_2 equal to 300.0 N/cm^2 was applied on the second Winkler foundation. A concentrated load P equal to 2500 kN was applied to the concrete bearing plate on the third Winkler foundation. The displacement measurements and the displacement standard deviations of the five selected points on the concrete bearing plate are based on the reference (Zhang et al., 2018). The displacement of each selected point was measured five times with consideration of measurement fluctuation.

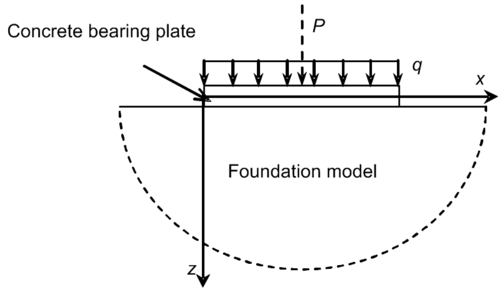

Fig. 1 Concrete bearing plate on a Winkler foundation

To validate the reliability of the variable scale searching model of the stochastic foundation constant and the correctness of the completed analytical procedure, the priori information of the stochastic foundation constant was firstly presumed to satisfy the precise condition. The priori data of the three Winkler foundation constants were respectively 80 N/cm^3, 110 N/cm^3, and 60 N/cm^3. The two groups of initial constant data k_{10} and k_{20} were given as 20 N/cm^3 and 180 N/cm^3, respectively. The convergence tolerances of ε_1 and ε_2 were both equal to 0.001. Combined with the displacement measurement and the displacement standard deviation, the variable scale searching results are shown in Fig. 2 and Table 1.

Fig. 2 and Table 1 show that although different initial foundation constant data were set, the relative errors of the foundation constants were far lower than 5%, and the variable scale searching iterative processes steadily converged to the true values. This indicates that the regenerative Bayesian detection of the foundation constant was correct and the completed procedure was reliable. The coefficient of variation was about 0.06, which was improved with the given data. From the analysis of the mechanical detection model, the variable scale gradient method ceaselessly alters the spacial matrix scale to engender new search directions during the iterative processes and optimizes the objective function efficiently, leading to high computational efficiency. In contrast, the Powell direct searching method has to call many more runs of the computation subroutine of the regenerative Bayesian objective function because of its dependence on the revision of the objective function (Zhang et al., 2018).

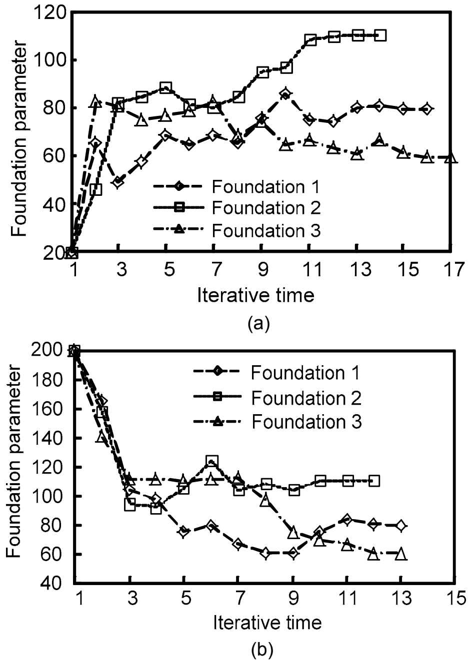

Fig. 2 Variable scale iterative process in the detection of the stochastic foundation constant
(a) k_{10} is selected; (b) k_{20} is selected

Table 1 Variable scale searching results in convergence verification for different foundations

Parameter	Foundation No.	Final constant (N/cm^3)	Iterative time	Relative error (%)
k_{10}	1	79.460	16	0.675
	2	110.151	14	0.137
	3	59.331	17	1.114
k_{20}	1	80.165	14	0.207
	2	110.261	12	0.262
	3	60.411	14	0.685

In brief, we have derived a regenerative Bayesian detection model of the stochastic Winkler foundation constant based on variable scale gradient theory. The variable scale iterative processes steadily converge to the true values, indicating that the derived detection model is correct and reliable. Unlike the Powell direct searching method, which simply depends on the revision of the objective function, the variable scale gradient method always improves the spacial matrix scale to engender new search directions during the iterative processes and optimizes the objective function efficiently. Thus, the variable scale theory has higher computational efficiency.

Contributors

Jian ZHANG wrote the first draft of the manuscript. Jian ZHANG, Chao JIA, and Feng WANG performed the experiments. Jian ZHANG and Wei SUN revised and edited the final version.

Conflict of interest

Jian ZHANG, Wei SUN, Chao JIA, and Feng WANG declare that they have no conflict of interest.

References

Al-Hammoud R, Soudki K, Topper TH, 2011. Fatigue flexural behavior of corroded reinforced concrete beams repaired with CFRP sheets. *Journal of Composites for Construction*, 15(1):42-51.
https://doi.org/10.1061/(ASCE)CC.1943-5614.0000144

Bennoun M, Houari MSA, Tounsi A, 2016. A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates. *Mechanics of Advanced Materials and Structures*, 23(4):423-431.
https://doi.org/10.1080/15376494.2014.984088

Bouderba B, Houari MSA, Tounsi A, 2013. Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations. *Steel and Composite Structures*, 14(1):85-104.
https://doi.org/10.12989/scs.2013.14.1.085

Bourada M, Kaci A, Houari MSA, et al., 2015. A new simple shear and normal deformations theory for functionally graded beams. *Steel and Composite Structures*, 18(2):409-423.
https://doi.org/10.12989/scs.2015.18.2.409

Bousahla AA, Houari MSA, Tounsi A, et al., 2014. A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates. *International Journal of Computational Methods*, 11(6):19-30.
https://doi.org/10.1142/S0219876213500825

Hamdia KM, Silani M, Zhuang XY, et al., 2017. Stochastic analysis of the fracture toughness of polymericnanoparticle composites using polynomial chaos expansions. *International Journal of Fracture*, 206:215-227.
https://doi.org/10.1007/s10704-017-0210-6

Hamdia KM, Ghasemi H, Zhuang XY, et al., 2018. Sensitivity and uncertainty analysis for flexoelectric nanostructures. *Computer Methods in Applied Mechanics and Engineering*, 337:95-109.
https://doi.org/10.1016/j.cma.2018.03.016

Hamidi A, Houari MSA, Mahmoud SR, et al., 2015. A sinusoidal plate theory with 5-unknowns and stretching effect

for thermomechanical bending of functionally graded sandwich plates. *Steel and Composite Structures*, 18(1): 235-253.
https://doi.org/10.12989/scs.2015.18.1.235

Khader MH, Zhuang XY, He PF, et al., 2016. Fracture toughness of polymeric particle nanocomposites: evaluation of models performance using Bayesian method. *Composite Science and Technology*, 126(4):122-129.
https://doi.org/10.1016/j.compscitech.2016.02.012

Meziane MAA, Abdelaziz HH, Tounsi A, 2014. An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. *Journal of Sandwich Structures and Materials*, 16(3):293-318.
https://doi.org/10.1177/1099636214526852

Nanthakumar SS, Lahmer T, Rabczuk T, 2013. Detection of flaws in piezoelectric structures using XFEM. *International Journal of Numerical Methods in Engineering*, 96(6):373-389.
https://doi.org/10.1002/nme.4565

Nanthakumar SS, Lahmer T, Zhuang X, et al., 2016. Detection of material interfaces using a regularized level set method in piezoelectric structures. *Inverse Problems in Science and Engineering*, 24(1):153-176.
https://doi.org/10.1080/17415977.2015.1017485

Vu-Bac N, Lahmer T, Zhuang X, et al., 2016. A software framework for probabilistic sensitivity analysis for computationally expensive models. *Advances in Engineering Software*, 100:19-31.
https://doi.org/10.1016/j.advengsoft.2016.06.005

Zhang J, Zhou CW, Jia C, et al., 2017. Powell inversion mechanical model of foundation parameters with generalized Bayesian theory. *Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering)*, 18(7): 567-578.
https://doi.org/10.1631/jzus.A1600440

Zhang J, Jia C, Zhou CW, 2018. Updated Bayesian detection of foundation parameter with Jeeves pattern search theory. *Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering)*, 19(9):691-703.
https://doi.org/10.1631/jzus.A1700573

Zidi M, Tounsi A, Houari MSA, et al., 2014. Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory. *Aerospace Science and Technology*, 34:24-34.
https://doi.org/10.1016/j.ast.2014.02.001

中文概要

题　目：基于变尺度梯度理论地基参数的修正 Bayes 探索

目　的：建立弹性地基参数的修正 Bayes 分析模型，并获得地基参数的变尺度优化结果。

创新点：推求地基参数的变尺度梯度优化方法，建立地基参数的修正 Bayes 探索分析模型。

方　法：建立修正 Bayes 目标函数及弹性地基参数的修正 Bayes 探索分析模型，并利用变尺度梯度搜索方法进行参数的优化迭代计算。

结　论：地基参数的变尺度梯度搜索分析模型在优化过程中能够稳定地收敛于地基参数的真值（图 2）。变尺度梯度优化理论能够适时地修正空间矩阵尺度以产生新的搜索方向，并有效地优化修正 Bayes 目标函数。

关键词：变尺度法；Bayes 目标函数；地基参数；优化