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Condition-based scheduled maintenance optimization of structures based on reliability requirements under continuous degradation and random shocks*

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Abstract: In this paper, a condition-based scheduled maintenance model with aperiodic inspections of structures is developed. The structures are experiencing both a gradual degradation process and a random shock process. The former is characterized by a stationary gamma process (SGP), and the latter is assumed to be a homogeneous Poisson process (HPP). Two typical common failure modes are considered in the reliability and the condition-based maintenance model, namely: (1) soft failures caused by the continuous degradation process, together with sudden damage increments due to shocks with moderate impacts, and (2) hard failures caused by the same shock process when a severe shock occurs. A remaining useful lifetime-based (RUL-based) inspection policy is utilized to determine the inspection schedule. Thereafter, at each inspection point, different maintenance actions are to be determined to minimize the average cost rate for either an infinite or a finite time span. The developed models are demonstrated by a numerical example. Sensitivity analyses of the optimal solution with various model parameters are also performed. It is illustrated that, as compared with the pure continuous degradation process, the additional shock loads exert notable impacts on the optimal maintenance strategies.

Key words: Soft failure; Hard failure; Remaining useful lifetime (RUL); Reliability; Maintenance; Cost rate; Finite horizon
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1 Introduction

Engineering structures, such as civil infrastructural facilities, are often exposed to severe operating conditions during their service life. Failures of a structure or a component may occur due to internal degradation or to external shocks under such circumstances. Failure mechanisms including wear degradation, corrosion, erosion, fatigue, fracture, and overload are often observed for engineering structures. A system is considered to experience multiple dependent competing failure processes (MDCFP) when two or more dependent failure mechanisms are involved. More generally, two dependent failure processes are often identified: (1) soft failures caused by a continuous degradation and additional sudden damage increments due to a shock process, and (2) hard failures caused by the same shock process when a severe shock occurs. These two failure processes are dependent and competing, making reliability modeling and analysis a challenge.

Several researchers have investigated the reliability of a system experiencing MDCFP. Peng et al. (2010) proposed a reliability model for systems involving two dependent failure processes: (1) soft failures caused by a linear degradation process

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together with additional sudden damage increments due to a shock process, and (2) hard failures caused by fatal shock loads from the same shock process. Rafiee et al. (2014) developed a reliability model for a system subjected to random shocks and a linear degradation path with a changing degradation rate. Huang and Chen (2015) proposed a time-dependent reliability model of deteriorating structures subjected to aging modeled as a gamma process and random shocks. To incorporate the effect of model uncertainty, Bayesian inference methods are applied to update the model parameters. However, in the literature mentioned above, an implicit assumption exists that the arrival of each shock will cause a sudden degradation regardless of the shock level. Generally, however, structures with high reliability possess the ability to resist small shocks, and thus, shock loads below a certain magnitude have no impacts on the degradation process. In (Jiang et al., 2015), only those shocks beyond a certain level influence the degradation process, which makes the deterioration model more realistic. An and Sun (2017) proposed a reliability model for a system subject to a shock load process and multiple linear degradation processes, by considering the immunity of the system to small shocks and the dependence between these processes. Without loss of generality, degradation of a system constitutes a monotone increasing process. The gamma process is well suited for modeling this temporal variability of the degradation process of structures or systems (van Noortwijk, 2009; Ponchet et al., 2011; Huynh et al., 2012; Huang and Chen, 2015), and will be utilized in the current paper. van Noortwijk (2009) summarized the application of gamma processes in describing the degradation process of systems in detail. Methods for estimation, approximation, and simulation of gamma processes are also reviewed.

Failures of a structure can have catastrophic consequences, especially for structures of great significance to human society, such as bridges, power plants, or pipeline systems. In order to avoid severe environmental and/or economic consequences and to prolong the useful lifetime of a system, preventive maintenance (PM) actions are usually executed throughout its operating period. In the extant literature, perfect preventive maintenance (PPM), which can restore a system condition to an "as good as new" state but at large expense, has been studied extensively. By contrast, imperfect preventive maintenance (IPM), with a lower cost, will restore the system condition to a state between "as bad as old" and "as good as new" after intervention (Nakagawa, 1988; Lin et al., 2001; Zequeira and Bérenguer, 2006; Castro, 2009; Wu and Zuo, 2010). These two kinds of PM will be considered in this paper.

In maintenance optimization problems, determining the optimal maintenance times at which PMs are performed is a critical issue. Time-based maintenance (TBM) and condition-based maintenance (CBM) are two basic strategies to cope with this issue. In TBM, it is assumed that the failure behavior of a system is predictable, and the intervention is executed based on a failure intensity function or reliability function (Doyen and Gaudoin, 2004; El-Ferik and Ben-Daya, 2006; Castro, 2009; Chen et al., 2018). In CBM, which is another popular maintenance technique, maintenance decisions are made based on system degradation information at each inspection instant. Currently, CBM is receiving increased attentions and is more appropriate for application in cases where the on-line system information is taken into account during the maintenance decision-making process (Tan et al., 2010; Ahmad and Kamaruddin, 2012; Do Van and Bérenguer, 2012; Saydam and Frangopol, 2015). It is worth noting that maintenance optimization of systems experiencing MDCFP is of practical importance. Peng et al. (2010) proposed a periodic time-based maintenance model to identify the optimal inspection time interval to minimize cost rate. Guo et al. (2013) studied two periodic time-based inspection/replacement models for a non-repairable system to minimize cost rate. Wang and Pham (2011) presented a periodic CBM model ignoring the potential hard failures caused by shocks. Li and Pham (2005) developed a CBM model with a predefined inspection time schedule. In the above studies, optimal maintenance policies were determined for systems over an infinite time span. However, since a system or a structure usually has a finite service life, determining the optimal maintenance strategy over a finite time span would be substantially more valuable.

This paper first proposes a time-dependent reliability model for systems involving MDCFP that takes into consideration the gamma degradation process together with different shock level effects. This model is generalized from those in the literature (Peng et al., 2010; Rafiee et al., 2014; Huang and Chen, 2015), in which any shocks are assumed to induce damage in a structure or component regardless of their intensions. Thereafter, the proposed model is incorporated with a maintenance strategy optimization as described below. For the CBM modeling, a remaining useful lifetime-based (RUL-based) inspection policy proposed by Do et al. (2015) is utilized to identify the inspection schedule according to a reliability criterion, and a condition-based maintenance optimization model is developed for minimizing the average cost rate. The cost includes inspection costs, maintenance costs, and downtime losses. The proposed CBM model in this study differs from those in other studies (Wang and Pham, 2011; Do et al., 2015), in which both soft failure and hard failure modes are taken into account when applying the RUL criterion. Finally, considering that managers usually pay more attentions to the profits and losses of structures in a certain time period, optimal maintenance policies for both infinite and finite time horizons are investigated and compared.

In this study, we first present the description of a system and its reliability analysis. Then, we introduce the RUL-based inspection policy and CBM modeling. The impacts of imperfect preventive maintenance on the deterioration level and on the mean deterioration rate are also addressed. Optimal maintenance policies for infinite and finite time horizons are discussed. At last, to illustrate the proposed reliability and maintenance models, a numerical example is studied.

2 Reliability analysis for a system subject to degradation and shocks

Fatigue, corrosion, and erosion are examples of component degradation in engineering. Several stochastic processes have been investigated for describing the continuous degradation process, such as Brownian motion with drift (Doksum and Hóyland, 1992), the compound Poisson process (Esary et al., 1973), and the gamma process (van Noortwijk et al., 2007; van Noortwijk, 2009). When a system or a component suffers not only the gradual deterioration process, but also random shock loads, the degradation process will become increasingly complicated. Specially, the system will experience two dependent competing failure processes: (1) soft failure caused by gradual deterioration and moderate shocks, and (2) hard failure caused by some extreme random shock loads. These two failure processes are dependent because they share the same random shock loads. The system will fail once the total degradation exceeds a threshold H or the shock load exceeds the upper threshold (An and Sun, 2017). These competing failure modes may be observed, for example, in an electric device, in which the wear-out process corresponds to a soft failure, while overloading stresses caused by voltage spikes result in a hard failure (Wang and Pham, 2011). Another example is a micro-engine application, in which the dominant failure mode is identified as the wear on the rubbing surface between the gear and the pin joint. The wear volume is primarily caused by the aging degradation process. In addition, shock tests on micro-engines demonstrate that shock loads may cause substantial wear debris between the gear and the pin joint (Peng et al., 2010; Rafiee et al., 2014; Jiang et al., 2015; An and Sun, 2017). For engineering structures, various overloads produce shock impacts.

Since both continuous degradation and shocks are responsible for failures, they are both considered in the reliability analysis. In particular, continuous degradation is characterized by a stationary gamma process, and the shock process is modeled by a homogeneous Poisson process (HPP). By dividing the shocks into three ranges and using a relationship between the moderate shocks and the induced sudden damage increments, a reliability equation is derived. This result is used in the RUL-based inspection policy for determining the next scheduled inspection time, as described in Section 3.1.

2.1 Problem definition

1. Considering a single component system and focusing on the most important or critical component, its deterioration level at time t can be observable and summarized by a random scalar variable $X_S(t)$. Soft failure occurs if the cumulative deterioration level $X_S(t)$ reaches a predefined threshold H, i.e. if $X_S(t) \geq H$.

2. A hard failure takes place if a random shock load W_i greater than the upper threshold W_U occurs, i.e. if $W_i \geq W_U$. Those that are smaller than a predefined

threshold W_L, i.e. $W_i<W_L$, have no effect on the system. For moderate shocks, i.e. $W_L\le W_i<W_U$, damage will be induced to the system, and it can be added to $X_S(t)$.

3. Inspections are assumed to be instantaneous, perfect, and non-destructive, and failures can only be detected by inspections. When the system fails, it is not immediately repaired. Hence, the system is unavailable from the time at which the failure occurs until the next scheduled inspection instant (Peng et al., 2010; Ponchet et al., 2011).

2.2 Continuous degradation modeling using a gamma process

The gamma process is a monotone increasing process suitable for describing most physical degradation phenomena, such as wear, fatigue, corrosion, erosion, crack growth, and creep.

In mathematics, a continuous random quantity X follows a gamma distribution if its probability density function (PDF) is given by

$$f_X(x;\alpha,\beta)=\frac{1}{\Gamma(\alpha)}\frac{1}{\beta^{\alpha}}x^{\alpha-1}\mathrm{e}^{-x/\beta}I_{(0,\infty)}(x), \quad (1)$$

where $\alpha>0$ is the shape parameter, $\beta>0$ denotes the scale parameter, $I_A(x)$ is an indicator function with $I_A(x)=1$ if $x\in A$ and $I_A(x)=0$ otherwise, and $\Gamma(x)=\int_0^{\infty}t^{x-1}\mathrm{e}^{-t}\mathrm{d}t$ is the gamma function for $x>0$. The expectation and variance are $E[X]=\alpha\beta$ and $V[X]=\alpha\beta^2$, respectively. A gamma process with shape function $\alpha(t)>0$ and scale parameter $\beta>0$ is a continuous-time stochastic process $\{X(t), t\ge0\}$ with the following probability density function $f_X(x;\alpha(t),\beta)$, in which $\alpha(t)$ is a non-decreasing, right-continuous, and real-value function for $t\ge0$ and $\alpha(0)\equiv0$.

A component/system is considered to fail when its cumulative degradation amount exceeds a predefined threshold H. Let the time at which failure occurs be T_f, as shown in Fig. 1. The lifetime distribution and the reliability function can be written as:

$$\begin{aligned}F(t)&=\Pr\{T_f\le t\}=\Pr\{X(t)\ge H\}\\&=\int_H^{\infty}f_X(x;\alpha(t),\beta)\mathrm{d}x=\frac{\Gamma(\alpha(t),H/\beta)}{\Gamma(\alpha(t))},\end{aligned} \quad (2)$$

and

$$\begin{aligned}R(t)&=1-F(t)=\Pr\{X(t)<H\}\\&=\int_0^{H}f_X(x;\alpha(t),\beta)\mathrm{d}x=\frac{\gamma(\alpha(t),H/\beta)}{\Gamma(\alpha(t))},\end{aligned} \quad (3)$$

where $\Gamma(\alpha,x)=\int_x^{+\infty}t^{\alpha-1}\mathrm{e}^{-t}\mathrm{d}t$ is the upper incomplete gamma function, and $\gamma(\alpha,x)=\int_0^{x}t^{\alpha-1}\mathrm{e}^{-t}\mathrm{d}t$ is the lower incomplete gamma function. The following relation holds:

$$R(t)=\frac{\gamma(\alpha(t),H/\beta)}{\Gamma(\alpha(t))}=1-\frac{\Gamma(\alpha(t),H/\beta)}{\Gamma(\alpha(t))}. \quad (4)$$

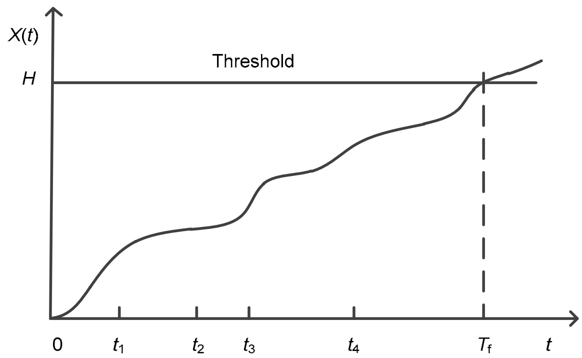

Fig. 1 Illustration of gradual degradation

Empirical studies showed that the expected deterioration at time t often follows a power law, namely,

$$E[X(t)]=\alpha(t)\cdot\beta=ct^b\cdot\beta=at^b\propto t^b, \quad (5)$$

where parameters $a>0$ (or $c>0$), and $b>0$. Some engineering knowledge is available about the parameter b, e.g. degradation of concrete due to corrosion of reinforcement (linear: $b=1$), sulfate attack (parabolic: $b=2$), diffusion-controlled aging (square root: $b=0.5$), creep ($b=1/8$), and the expected scour-hole depth ($b=0.4$) (van Noortwijk, 2009). In this study, the continuous degradation process is supposed to be stationary, i.e. $b=1$.

2.3 Effects of shock processes

Random shocks may frequently occur when devices/systems are exposed to external shock environments, such as unexpected usage and overloads. In the literature, there are four categories of random shock models (Peng et al., 2010; Rafiee et al., 2014). (1) Extreme shock model: failure occurs when the magnitude of any shock exceeds a specified threshold; (2) Cumulative shock model: failure occurs when the cumulative damage from shocks exceeds a critical value; (3) Run shock model: failure occurs when there is a run of k shocks exceeding a critical magnitude; (4) δ-shock model: failure occurs when the time lag between two successive shocks is shorter than a threshold δ (Wang and Zhang, 2005). Both the extreme shock model and the cumulative shock model are utilized in this research.

Let W_i be independent and identically distributed (i.i.d) random variables with density function $f_W(w)$, and cumulative distribution function $F_W(w)$. It is assumed that the random shock arrival time follows an HPP $\{N(t), t\geq0\}$ with intensity λ. Let $N(t)$ be the total number of shock loads occurring in (0, t]. The probability that the shock occurrence number equals n becomes:

$$\Pr\{N(t)=n\}=\frac{(\lambda t)^n}{n!}\cdot \mathrm{e}^{-\lambda t}, \quad n=0,1,2,\cdots. \tag{6}$$

According to the properties of the HPP, the interval between the ith and the (i−1)th shocks, i.e. $\Delta\tau_i=\tau_i-\tau_{i-1}$ (i=1, 2, …), follows an exponential distribution with parameter λ. Thus, the expected value of $\Delta\tau_i$ (i=1, 2, …) is $1/\lambda$, i.e. $E[\Delta\tau_i]=1/\lambda$ (i=1, 2, …) as shown in Fig. 2.

Shock loads can be classified into three levels: (1) small shock loads (also called first class loads), which fall below a certain level W_L, i.e. $W_i<W_L$, and have no impact on the system; (2) moderate shock loads (also called second class loads), which fall in the extent of $W_L\leq W_i<W_U$, and influence the system degradation process and induce a sudden degradation increment; (3) fatal shock loads (also called third class loads), which will cause sudden failure of the system with a magnitude larger than the shock threshold W_U, i.e. $W_i\geq W_U$.

Let p_1, p_2, and p_3 represent, respectively, the occurrence probability of the three kinds of shock loads ($p_1+p_2+p_3=1$), and let $N_1(t)$, $N_2(t)$, and $N_3(t)$ be the occurrence number of these three kinds of loads. According to the decomposition theorem of a Poisson process, the arrival time of these three shock loads is also a Poisson process with intensities λp_1, λp_2, and λp_3.

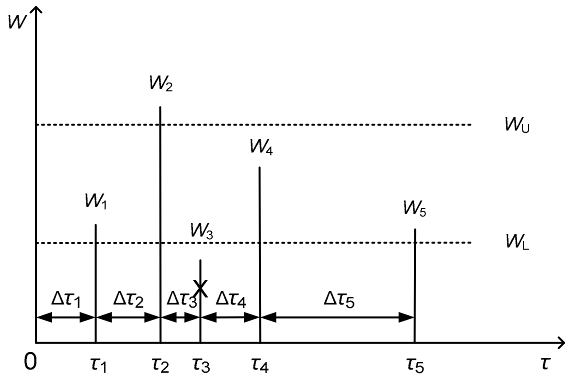

Fig. 2 Illustration of a possible random shock process

For the extreme shock load model, as shown in Fig. 2, a system failure occurs when the shock load exceeds the shock threshold W_U. Then, the survival probability for the ith shock is

$$\Pr\{W_i<W_U\}=F_W(W_U), \quad i=1,2,\cdots. \tag{7}$$

The cumulative shock load model is used to evaluate the effects of moderate shock loads. The instantaneous increases in the total degradation are assumed to be i.i.d random variables, and take the following form:

$$Y_i=\kappa\cdot(W_i-W_L), \quad W_L\leq W_i<W_U, \tag{8}$$

where κ represents the sudden degradation increment caused by a unit change in the shock load. The cumulative degradation due to random shock loads until time t then reads

$$S(t)=\sum_{i=1}^{N_2(t)}Y_i, \quad N_2(t)>0. \tag{9}$$

It can be seen that the degradation $S(t)$ is a compound Poisson process, and the probability of survival becomes

$$\Pr\{S(t)<H\}=\sum_{i=0}^{\infty}\Pr\left\{\left(\sum_{j=0}^{N_2(t)}Y_j\right)<H\,\middle|\,N_2(t)=i\right\}\cdot\Pr\{N_2(t)=i\}. \tag{10}$$

2.4 Reliability analysis considering both a continuous degradation and shocks

For a system experiencing gradual degradation and shocks, the total damage for the system deterioration is the cumulative effect of both continuous degradation and those moderate shocks, i.e.

$$X_S(t) = X(t) + S(t) = X(t) + \sum_{j=1}^{N_2(t)} Y_j. \tag{11}$$

The distribution of $X_S(t)$ is

$$\begin{aligned} F_{X_S}(t) &= \Pr\{X_S(t) \le x\} \\ &= \sum_{i=0}^{\infty} \Pr\{[X(t)+S(t)] < x \mid N_2(t)=i\} \cdot \Pr\{N_2(t)=i\}. \end{aligned} \tag{12}$$

To keep the system in good condition without failure, it is required that no fatal shocks occur and that the total deterioration is lower than the predefined failure threshold H. Reliability of the system over time t can be derived as in Eq. (13):

$$\begin{aligned} R(t) &= 1 - P_f = 1 - [\underbrace{\Pr\{N_3(t) \ne 0\}}_{\text{Hard failure}} \\ &\quad + \underbrace{\Pr\{\text{Failure} \mid N_3(t)=0\} \cdot \Pr\{N_3(t)=0\}}_{\text{Soft failure}}] \\ &= \Pr\{N_3(t)=0\} \\ &\quad - \Pr\{\text{Failure} \mid N_3(t)=0\} \cdot \Pr\{N_3(t)=0\} \\ &= \Pr\{N_3(t)=0\} \cdot (1 - \Pr\{\text{Failure} \mid N_3(t)=0\}) \\ &= \Pr\{N_3(t)=0\} \cdot \Pr\{\text{Safe} \mid N_3(t)=0\} \\ &= \Pr\{N_3(t)=0\} \cdot \Pr\{X_S(t) < H\} \\ &= \Pr\{N_3(t)=0\} \\ &\quad \cdot [\Pr\{X_S(t) < H \mid N_2(t)=0\} \cdot \Pr\{N_2(t)=0\} \\ &\quad + \Pr\{X_S(t) < H \mid N_2(t) \ne 0\} \cdot \Pr\{N_2(t) \ne 0\}] \\ &= \Pr\{N_3(t)=0\} \cdot \Bigg[\Pr\{X(t) < H\} \cdot \Pr\{N_2(t)=0\} \\ &\quad + \sum_{i=1}^{\infty} \Pr\left\{ \left[X(t) + \sum_{j=1}^{i} Y_j \right] < H \right\} \cdot \Pr\{N_2(t)=i\} \Bigg] \\ &= \frac{\gamma(\alpha(t), H/\beta)}{\Gamma(\alpha(t))} \cdot e^{-\lambda(p_2+p_3)t} \\ &\quad + \sum_{i=1}^{\infty} \Bigg\{ \Bigg[\int_0^H F_{S_i}(H-x) \cdot f_X(x;\alpha(t),\beta)\,dx \\ &\quad - F_{S_i}(0) \cdot \int_0^H f_X(x;\alpha(t),\beta)\,dx \Bigg] \\ &\quad \cdot e^{-\lambda(p_2+p_3)t} \cdot \frac{(\lambda p_2 t)^i}{i!} \Bigg\}, \end{aligned} \tag{13}$$

where P_f is the failure probability, $f_X(x; \alpha(t), \beta)$ is the probability density function of the gamma distributed random variable as shown in Eq. (1), and $F_{S_k}(\cdot)$ denotes the cumulative distribution function of the sum of k i.i.d Y_i variables. Let W_i follow a normal distribution (An and Sun, 2017), i.e. $W_i \sim N(\mu_W, \sigma_W^2)$, and consequently the sudden damage sizes Y_i also follow a normal distribution. A numerical integration approach can be utilized in calculating the reliability (Eq. (13)). The reliability model constitutes the basis of the RUL-based inspection policy application in formulating the CBM optimization in Section 3.

For illustration purposes, parameters for a system experiencing a gamma process and a random shock process are provided in Table 1. Based on Eq. (13) and Table 1, the survival probability of the system is calculated and plotted in Fig. 3. A sensitivity analysis can be performed to investigate the effects of model parameters on $R(t)$. The results are shown in Fig. 4.

Table 1 Parameters for a system experiencing a gamma degradation process and a random shock process

Parameter	Value	Parameter	Value
H (mm)	20	W_U (kN)	4.0
c^*	1	μ_W (kN)	3.0
b^*	1	σ_W (kN)	0.5
β (kN)	1.0	λ	0.5
W_L (kN)	1.0	κ (mm/kN)	0.5

* c and b are the parameters contained in Eq. (5)

Fig. 3 shows that the random shock process greatly influences the reliability. Systems experiencing a gamma process, as well as a random shock process, are much more vulnerable to failure as compared to the case involving only continuous degradation.

Fig. 4 shows that all of the studied model parameters have significant effects on the reliability function. For example, when the intensity of shock loads increases, which implies that shock loads occur more frequently, the reliability of the system decreases, as shown in Fig. 4d.

3 Condition-based maintenance modeling

In order to avoid severe environmental and/or economic consequences and prolong the useful lifetime of a system, preventive maintenance actions are usually performed throughout its operating period. In the framework of CBM optimization, system information is usually employed to make decisions about both the inspection time and maintenance action.

In this study, both soft and hard failure modes are taken into account in the CBM model. In addition, a remaining useful lifetime-based (RUL-based) inspection policy is adopted to determine the inspection schedule based on a reliability criterion. Thereafter, at each inspection time, different maintenances are to be determined.

3.1 RUL-based inspection

Different inspection policies, which aim to optimize the time interval between two successive inspection points, have been widely studied in the extant literature. The RUL-based inspection seems to be

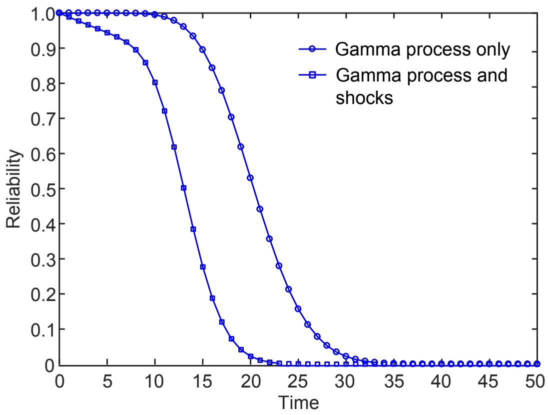

Fig. 3 Plots of reliability function $R(t)$

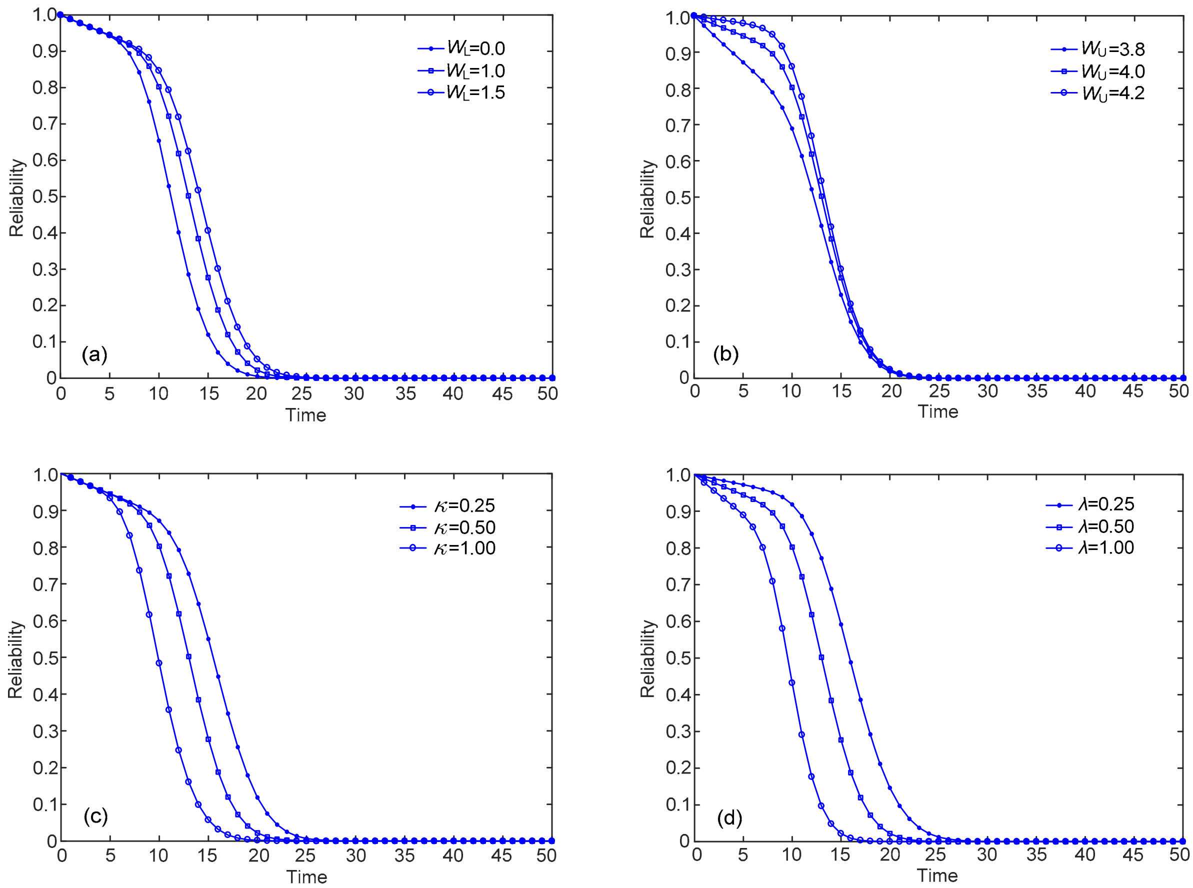

Fig. 4 Sensitivity analysis of $R(t)$ on different model parameters: (a) W_L; (b) W_U; (c) κ; (d) λ

very promising, especially in the context of CBM optimization problems.

The remaining useful lifetime of a system is defined as a residual duration for which the system fails with a given probability (Si et al., 2011).

In an RUL-based inspection, the next inspection time is chosen, such that the failure probability of a system prior to the next inspection remains lower than a limit Q, $0<Q\leq 1$, i.e. the next inspection time is determined by

$$t_i = t_{i-1} + m(X_S^+(t_{i-1}), Q), \tag{14}$$

where

$$m(X_S^+(t_{i-1}), Q) = \min\{\Delta t_i : \Pr\{X_S^-(t_{i-1}+\Delta t_i) \geq H \mid X_S^+(t_{i-1})\} = Q\}, \quad i = 0, 1, 2, \cdots. \tag{15}$$

The probability in the brace is calculated based on Eq. (13) in which H is replaced with $H - X_S^+(t_{i-1})$ and $\alpha(t)$ is replaced with $\alpha_{i-1}(t)$, where $\alpha_{i-1}(t)$ is the shape function of the gamma process in $(t_{i-1}, t_i]$, and the superscripts "−" and "+" represent the state immediately before and after a maintenance action, respectively. Note that

$$X_S^-(t_i) = X_S^+(t_{i-1}) + \Delta X(t_{i-1}, t_i) + \Delta S(t_{i-1}, t_i), \tag{16}$$

where $X_S^-(t_i)$ is the total degradation amount at t_i, $\Delta X(t_{i-1}, t_i)$ represents the continuous degradation increment over $(t_{i-1}, t_i]$, and $\Delta S(t_{i-1}, t_i)$ denotes the cumulative sudden damage increments caused by the random shocks occurring over $(t_{i-1}, t_i]$ (Eq. (9)).

3.2 Selection rules for maintenance actions

In CBM modeling, based on the magnitudes of shock loads occurring in $(t_{i-1}, t_i]$ and the total deterioration level $X_S^-(t_i)$ at inspection time t_i, the following maintenance decision rules are applied.

If any $W_j>W_U$, $j=1, 2, \ldots, N_i$, where N_i is the occurrence number of shocks occurring in the ith interval, i.e. $t\in(t_{i-1}, t_i]$, a hard failure occurs, and a corrective replacement action is performed at time t_i. After the replacement, the deterioration level is reset to 0, and the mean deterioration speed is reset to the initial value $v_0=\alpha(t)\cdot\beta/t$.

If $X_S^-(t_i) < M$, the system is in a working state, and no maintenance action is performed. M is called the preventive maintenance threshold, and it is a decision variable to be optimized.

If $M \leq X_S^-(t_i) < H$, although the system is still functioning, its deterioration level is considered as a "warning". Hence, a preventive maintenance action is performed. Without loss of generality, it is assumed that this preventive maintenance action is the kth preventive maintenance action since the last perfect maintenance or replacement. If $k=K$ (K is called the perfect preventive threshold, which is also a decision variable to be optimized), the kth preventive maintenance action takes a perfect one. The deterioration level of the system $X_S^+(t_i)$ after maintenance is reset to 0, and the mean deterioration speed is restored to v_0. In contrast, if $k<K$, the kth preventive maintenance action is an imperfect one. This imperfect preventive maintenance action can restore the system to a state between "as bad as old" and "as good as new" (for more details, please refer to Section 3.3).

If $X_S^-(t_i) \geq H$, a soft failure occurs, and then a corrective replacement action is performed at time t_i. After the replacement, the system is "as good as new".

In addition, it is assumed that all of the maintenance durations are neglected.

3.3 Effects of imperfect maintenance

Suppose that imperfect maintenance actions on a system produce impacts on both the deterioration level and the deterioration rate.

It is assumed that the kth intervention gain Z_k is a continuous random variable, $0 \leq Z_k \leq X_S^-(t_i)$, and follows a truncated normal distribution with density (Do Van and Bérenguer, 2012; Do et al., 2015):

$$g_{\mu,\sigma,a,b}^{TN}(x) = \frac{\frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)} I_{[a,b]}(x), \tag{17}$$

where $I_{[a, b]}(x)=1$ if $a\leq x\leq b$ and $I_{[a, b]}(x)=0$ otherwise.

$\phi(\xi)=\frac{1}{\sqrt{2\pi}}\cdot\exp\left(-\frac{1}{2}\xi^2\right)$ is the probability density function of a standard normal distribution, and $\Phi(\cdot)$ is its cumulative distribution function.

$$\mu=X_S^-(t_i)/2,\quad \sigma=X_S^-(t_i)/6,$$
$$a=\mu-3\sigma=0,\quad b=\mu+3\sigma=X_S^-(t_i).$$

Due to the imperfect maintenance model, the deterioration level of the system immediately after the kth maintenance becomes:

$$X_S^+(t_i)=X_S^-(t_i)-Z_k. \tag{18}$$

The imperfect maintenance action may accelerate the speed of a deterioration process, for example, welding can reduce crack length but it may damage some physical properties of materials (El-Ferik and Ben-Daya, 2006; Coria et al., 2015; Do et al., 2015). A non-negative continuous random variable ε_k is used to describe this effect, which follows an exponential distribution:

$$h(x)=\gamma e^{-\gamma x}I_{\{x\geq 0\}}. \tag{19}$$

The mean value of ε_k is $E[\varepsilon_k]=1/\gamma$ ($\gamma>0$).

The kth maintenance action will result in a change in the mean deterioration rate:

$$v_k=v_{k-1}+\varepsilon_k. \tag{20}$$

It is also assumed that ε_k is measurable. For example, the mean deterioration rate after the first IPM becomes $v_1=v_0+\varepsilon_1=\frac{E[X(t_1)]}{t_1}+\varepsilon_1=\frac{ct_1^b\cdot\beta}{t_1}+\varepsilon_1$. In particular, for the stationary gamma process, i.e. b=1, then $v_1=c\beta+\varepsilon_1$.

4 Optimization of maintenance strategies

4.1 Cost modeling

Each maintenance action will incur a cost. For an IPM, this cost is usually dependent on the intervention gain Z_k. It is assumed that the kth imperfect maintenance action at inspection time t_i requires a cost expressed below:

$$C_{\mathrm{IP}_k}=C_{\mathrm{P}_0}\cdot u(t_i)^\eta, \tag{21}$$

where $u(t_i)\leq 1$ is the degradation improvement factor defined as $Z_k/X_S^-(t_i)$, C_{P_0} is the imperfect maintenance cost incurred when the deterioration level is reduced to zero, and η is a non-negative real constant number representing the imperfect preventive maintenance characteristic of the system. Different kinds of maintenance cost functions can be found depending on the value of η (Do Van and Bérenguer, 2012; Do et al., 2015).

A cost of C_P is incurred when a perfect preventive maintenance action is performed on the system. For a corrective replacement action, it is necessary to pay a cost, C_C. Moreover, for each inspection, it costs C_I. Since failure is not repaired immediately, an additional downtime cost C_d is incurred. The total maintenance cost can be expressed as (Do et al., 2015)

$$\begin{aligned}C^t(M,K)=C_\mathrm{I}\cdot N_\mathrm{I}(t)+\sum_{i=1}^{N_\mathrm{IP}(t)}C_{\mathrm{IP}_i}+C_\mathrm{P}\cdot N_\mathrm{P}(t)\\+C_\mathrm{C}\cdot N_\mathrm{C}(t)+C_\mathrm{d}\cdot D(t),\end{aligned} \tag{22}$$

where M and K are the decision parameters mentioned in Section 3.2; $N_\mathrm{I}(t)$, $N_\mathrm{IP}(t)$, $N_\mathrm{P}(t)$, and $N_\mathrm{C}(t)$ are the numbers of inspections, of imperfect and perfect maintenance, and of replacement in [0, t], respectively; $D(t)$ denotes the total downtime in [0, t].

Eq. (22) cannot be evaluated for an infinite horizon, where the operating time of the system is undefined. In such a case, the average long-run total maintenance cost per unit time is alternatively computed for determining the optimal maintenance policy. From the basic renewal theory (Ross, 1996; Li and Pham, 2005), the average long-run total maintenance cost per unit of time is (Do et al., 2015)

$$\mathrm{MC}(M,K)=\lim_{t\to\infty}\frac{C^t(M,K)}{t}=\frac{E[C^L(M,K)]}{E[L]}, \tag{23}$$

where L is the length of a cycle, and $E[\cdot]$ represents the mathematical expectation. A cycle is defined as either a time interval since the installation of a system to the first replacement or a time interval between two consecutive replacements (Peng et al., 2010). The successive cycles together with the costs incurred in each cycle constitute a renewal process.

4.2 Formulation and solution algorithm for minimizing the cost rate

A maintenance optimization model is proposed as

$$\begin{aligned} &\text{Find: } \mathrm{MC}^*=\min_{M,K}\mathrm{MC}(M,K),\\ &\text{s.t. } M\in(0,H];\\ &\quad K\in[1,K_{\max}],\ \text{and } K \text{ is integer};\\ &\quad R(t)\geq 1-Q,\quad t\in[0,L];\\ &\quad \Delta t_{i+1}:\Pr\{X_S^-(t_i+\Delta t_{i+1})\geq H\mid X_S^+(t_i)\}=Q,\\ &\qquad\qquad i=0,1,2,\cdots,\ t_0=0. \end{aligned} \tag{24}$$

In Eq. (24), K_{max} is the maximum value of K. The model features the constraint that an RUL-based inspection policy is applied according to the reliability requirement, taking into consideration both soft failure and hard failure, as mentioned in Section 3.1. The third constraint means that the system reliability should not be lower than the predefined threshold at any time t during its operation process.

In determining the decision parameters (M, K) that minimize the cost rate, particle swarm optimization (PSO) (Ge et al., 2008; Nickabadi et al., 2011; de Fátima Araújo and Uturbey, 2013) is used in this study. For a given set of (M, K), N Monte-Carlo simulations (MCSs) are executed to evaluate the fitness function MC(M, K).

The optimization procedure is stated as follows.

1. Initialize n particles (M_i, K_i), i=1, 2, …, n.

2. Evaluate the fitness function MC(M_i, K_i) for each particle (M_i, K_i) via MCS:

(2.1) For j=1 to N;

(2.2) Determine the next scheduled inspection time t_i;

(2.3) Simulate the continuous degradation increment and the shocks occurring over (t_{i-1}, t_i];

(2.4) Compare the shocks with the failure threshold W_U; if failure occurs, go to (2.6);

(2.5) Compare the total deterioration level with these two thresholds (i.e., M and H) and determine its maintenance cost;

(2.6) Evaluate the mean cost rate, i.e. MC$_j$(M_i, K_i);

(2.7) End;

(2.8) $\mathrm{MC}(M_i,K_i)=\dfrac{1}{N}\sum_{j=1}^{N}\mathrm{MC}_j(M_i,K_i).$

3. Determine the best solution, P_{best}, of each particle and the best solution, G_{best}, of the population.

4. If a stopping criterion is satisfied, then stop and output the optimal solution (M^*, K^*).

5. Update each particle in the population based on P_{best} and G_{best}, and then go to Step 2.

The detailed descriptions for Steps (2.2) to (2.6), i.e. the single MCS process, will be presented in Sections 4.3 and 4.4.

4.3 Single MCS process to evaluate the cost rate over an infinite time span

Three methods are available for simulating a gamma process: (1) gamma-increment sampling, (2) gamma-bridge sampling, and (3) approximating a gamma process as a limit of a compound Poisson process (van Noortwijk, 2009). The gamma-increment sampling approach is used here to simulate the continuous degradation process (i.e. gamma process), since it is straightforward that independent increments with respect to very small units of time are directly simulated.

Let CM$_i$ be the total cost caused by the maintenance actions during the time interval between the 1st inspection and the ith inspection, CP$_i$ be the ith maintenance cost and CF be the downtime cost from the 1st inspection to the ith inspection. These definitions are also applied in the following section (Section 4.4). The procedure to evaluate MC$_j$(M_i, K_i) over an infinite time span for a given set of (M_i, K_i) is described as follows.

1. Initialize the decision variables:

Given the values of H, Q, W_L, W_U, M, K, and cost parameters.

2. Initialize the process parameters:

i=0, t_i=0, k=0, CM$_0$=0, CF=0, $T_{f1}=+\infty$, $T_{f2}=+\infty$, $X_S^-(t_i)=X_S^+(t_i)=0$, $v_0=c\beta$, and $i=i+1$.

3. Compute the next scheduled inspection time $t_i=t_{i-1}+\Delta t_i$, where $\Delta t_i=m(X_S^+(t_{i-1}),Q)$, see Eq. (15).

4. Simulate the random shocks occurred in (t_{i-1}, t_i]:

Generate N_i random shocks W_j (j=1, 2, …, N_i) occurring in (t_{i-1}, t_i] as mentioned in Section 2.4, and the corresponding time intervals $\Delta\tau_j$, following an exponential distribution as mentioned in Section 2.3.

5. Discretize the time intervals $\Delta\tau_j$ (j=1, 2, …, N_i) into smaller intervals and evaluate the total deterioration level at each time instant including the time at which shock loads occur.

6. Check whether a soft failure occurred in (t_{i-1}, t_i] or not:

If $X_S^-(t_i) \geq H$, soft failure occurs before t_i and the failure time $T_{f1} \in (t_{i-1}, t_i]$ is determined.

7. Check whether a hard failure occurred in (t_{i-1}, t_i] or not:

Suppose the kth ($1 \leq k \leq N_i$) shock is the first one larger than W_U. Then, the failure time is T_{f2}= $t_{i-1} + \sum_{j=1}^{k} \Delta\tau_j$.

8. If a failure occurred, then compute the failure time T_f=min{T_{f1}, T_{f2}} and CF=C_d·(t_i−T_f). Compute the cost rate $\mathrm{MC} = (i \cdot C_I + \mathrm{CM}_{i-1} + \mathrm{CP}_i + \mathrm{CF})/t_i$ with CP_i=C_C. Stop and output MC.

9. Compare the deterioration level $X_S^-(t_i)$ with M to determine the corresponding maintenance action:

(9.1) If $X_S^-(t_i) \geq M$, then k=k+1. For k<K, an imperfect PM is applied, $\mathrm{CP}_i = C_{P_0} \cdot u(X_S^-(t_i), Z(t_i))^\eta$, CM_i=CM_{i-1}+CP_i, $X_S^+(t_i) = X_S^-(t_i) - Z(t_i)$ and i=i+1, v_i=v_{i-1}+ε_i (Eqs. (18) and (20)), and then go to Step 3; otherwise, a perfect PM is applied, CP_i=C_P, CM_i=CM_{i-1}+CP_i, $X_S^+(t_i) = 0$, and i=i+1, v_i=v_0, k=0, and then go to Step 3.

(9.2) If $X_S^-(t_i) < M$, then CP_i=0, CM_i=CM_{i-1}+CP_i, $X_S^+(t_i) = X_S^-(t_i)$, and i=i+1, v_i=v_{i-1}, and then go to Step 3.

Fig. 5 presents the corresponding flowchart.

4.4 Single MCS process to evaluate the cost rate over a finite time span

For a finite horizon, the average maintenance cost per unit of time becomes MC(M, K)=C^T(M, K)/T, where C^T denotes the total maintenance cost, and T is the time span of the system, specified by the user.

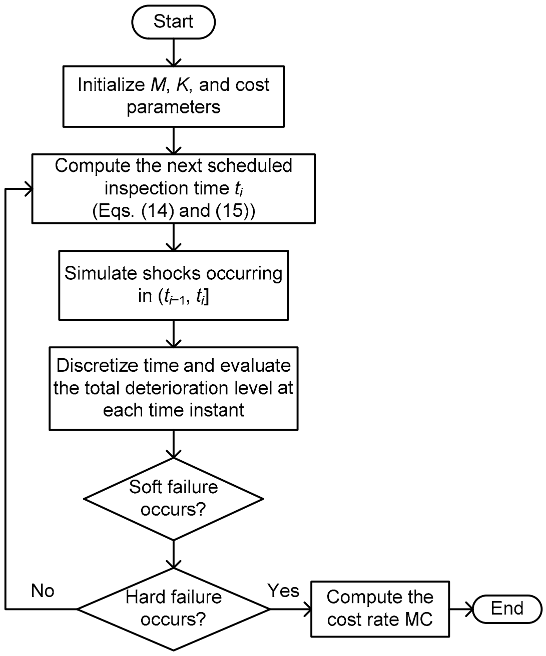

Fig. 5 Flowchart of evaluating MC_j(M_i, K_i) over an infinite horizon

The procedure to evaluate MC_j(M_i, K_i) over a finite time span T for a given set of (M_i, K_i) is described as follows.

1–3. Steps 1–3 are the same as the Steps 1–3 in Section 4.3, respectively.

4. Check whether the stop condition t_i>T is satisfied. If it is, set t_i=T and evaluate the cost of the last interval (t_{i-1}, T] as with other intervals. Then, stop and output MC=(i·C_I+CM_{i-1}+CP_i+CF)/T; otherwise, go to Step 5.

5–8. Steps 5–8 are the same as the Steps 4–7 in Section 4.3, respectively.

9. If a failure occurred, then compute the failure time T_f=min{T_{f1}, T_{f2}} and CF_i=C_d·(t_i−T_f). CP_i=C_C, CM_i=CM_{i-1}+CP_i, CF=CF+CF_i, $X_S^+(t_i) = 0$, and i=i+1, v_i=v_0, k=0. Then, go to Step 3.

10. Compare the deterioration level $X_S^-(t_i)$ with M to determine the corresponding maintenance action:

(10.1) If $X_S^-(t_i) \geq M$, then k=k+1. If k<K, then imperfect PM is applied $\mathrm{CP}_i = C_{P_0} \cdot u(X_S^-(t_i), Z(t_i))^\eta$, CF_i=0, CM_i=CM_{i-1}+CP_i, CF=CF+CF_i, $X_S^+(t_i) = X_S^-(t_i) - Z(t_i)$, and i=i+1, v_i=v_{i-1}+ε_i (Eqs. (18) and

(20)). Then, go to Step 3; otherwise, perfect PM is applied. $CP_i=C_P$, $CF_i=0$, $CM_i=CM_{i-1}+CP_i$, $CF=CF+CF_i$, $X_S^+(t_i)=0$, and $i=i+1$, $v_i=v_0$, $k=0$. Then, go to Step 3.

(10.2) If $X_S^-(t_i)<M$, then $CP_i=0$, $CF_i=0$, $CM_i=CM_{i-1}+CP_i$, $CF=CF+CF_i$, $X_S^+(t_i)=X_S^-(t_i)$, and $i=i+1$, $v_i=v_{i-1}$. Then, go to Step 3.

Fig. 6 presents the corresponding flowchart. One of the differences between Figs. 5 and 6 lies in the stopping condition. For the infinite horizon case with application of the renewal theory, once the first failure occurs, the procedure is terminated. On the other hand, for the finite horizon, the stopping condition is based on whether or not the inspection time reaches the predefined operating time *T*. Moreover, the length of the cycle *L* in Eq. (23) is a random variable for the infinite time span, and it is replaced by the predefined lifecycle *T* for the finite horizon.

5 A numerical example

This section demonstrates how the proposed reliability model and the preventive maintenance model can be used in maintenance optimization of a deteriorating system experiencing MDCFP.

Suppose that the continuous degradation process follows a stationary gamma process (i.e. $b=1$ in Eq. (5)) with shape parameter $c=1$ and scale parameter $\beta=1$, respectively, and suppose too that the scale parameter is constant and the shape parameter is modified after each repair. That is to say, after the *i*th inspection, the shape parameter in Eqs. (1) and (5) is changed following the three criteria: (1) $\alpha_i=\alpha_{i-1}$ ($c_i=c_{i-1}$) for doing nothing; (2) $\alpha_i=\alpha_{i-1}+\varepsilon_i/\beta=\alpha_{i-1}+\varepsilon_i$ ($c_i=c_{i-1}+\varepsilon_i$) for imperfect preventive maintenance; (3) $\alpha_i=\alpha_0$ ($c_i=c_0=c$) for perfect preventive maintenance or replacement. Random shock loads can be characterized by a homogeneous Poisson process with intensity $\lambda=0.5$. Failure occurs when the total deterioration level due to the continuous degradation process and the random shocks exceeds a prescribed failure threshold *H*, or a severe shock ($W_i\geq W_U$) takes place, whichever occurs first. Parameters used in the time-dependent reliability analysis, as well as the maintenance optimization, are summarized in Table 1 of Section 2.4.

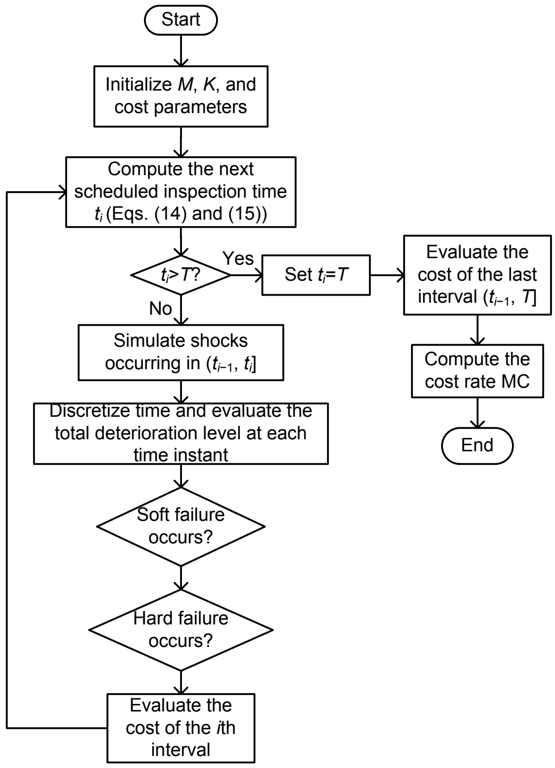

Fig. 6 Flowchart for evaluating $MC_j(M_i, K_i)$ over a finite horizon

It is noted that the deterioration level and the degradation rate are altered by imperfect preventive maintenance actions, as mentioned in Section 4. Both the perfect preventive maintenance and the corrective replacement can restore the deterioration level to an "as good as new" state and reset the deterioration rate to the nominal speed $v_0=c\beta$. In applying the RUL-based inspection policy, the failure probability threshold *Q* is set to be 0.1 in Eq. (24). The maximum number of IPM actions either between the system installation and the first replacement or between any two consecutive replacements is set to $K_{max}=10$ in Eq. (24). Table 2 shows the parameters of maintenance costs (all costs are given in arbitrary units for illustrative purpose) and IPM (Eqs. (19) and (21)). Besides, the sample size of the Monte-Carlo simulation is 10^4, and the population size and the maximum iterations of the PSO are 20 and 50, respectively, in this study.

The performance of imperfect PM actions is characterized by η in the cost function (Eq. (21)). Optimal maintenance policies of the deteriorating system with an infinite horizon for different values of η are reported in Table 3.

The following conclusions can be drawn from the results in Table 3 and Fig. 7.

1. When $\eta \leq 1$, for case 2 in which the deterioration process is induced by the continuous degradation process only, the optimal preventive maintenance corresponds to the perfect one, i.e. $K=1$. For case 1, in which the system experiences a shock process and a degradation process simultaneously, the optimal preventive maintenance is also the perfect one, except for $\eta=1$. For $\eta>1$, however, the imperfect preventive maintenance is preferred over the perfect one.

Table 2 Data of costs and impact of IPM (Do et al., 2015)

C_I	C_C	C_P	C_{P_0}	C_d	η	γ
10	100	90	70	20	3.0	0.2

Table 3 Optimal maintenance planning for different values of η with an infinite time span

η	Case	M	K	MC
0.3	Case 1	17.1962	1	12.1665
	Case 2	16.4163	1	6.7527
0.5	Case 1	17.1224	1	12.1629
	Case 2	16.3407	1	6.7474
1.0	Case 1	16.7549	4	11.9307
	Case 2	16.2092	1	6.7327
2.0	Case 1	13.8248	10	10.5591
	Case 2	14.2260	5	6.3936
3.0	Case 1	11.5145	10	9.8365
	Case 2	13.3556	8	5.9669

Case 1: gamma process and shocks; Case 2: gamma process only

2. In both case 1 and case 2, the optimal preventive maintenance threshold M and cost rate MC are insensitive to η when $\eta<1$. In the region where $\eta>1$, the optimal maintenance threshold M decreases greatly with the increase of η for both cases. In addition, the cost rate MC decreases with η, since the single IPM cost decreases with η for the same intervention gain.

3. For a given value of η, case 1 costs more than case 2 (the two dashed lines in Fig. 7). The presence of shocks makes the system become more vulnerable, and consequently it must be maintained more frequently, resulting in the higher cost. In addition, the first failure of case 1 will occur earlier than case 2, leading to a smaller mean cycle length L and greater cost rate.

Optimal maintenance strategies over a finite time span (T=50) for different values of η are investigated and presented in Table 4.

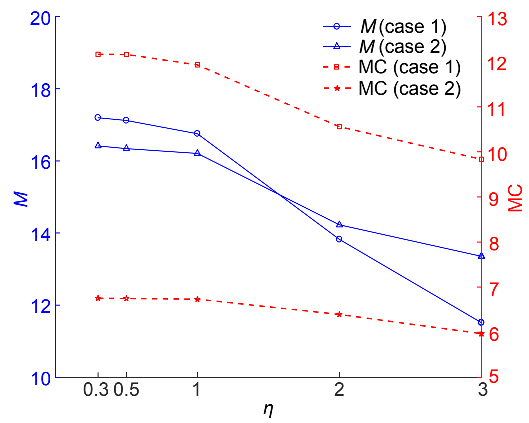

Fig. 7 Optimal maintenance threshold M and cost rate MC with respect to η (infinite time span)

Table 4 Optimal maintenance planning for different values of η with T=50

η	Case	M	K	N_F	N_{PM}	N_{PPM}	N_{IPM}	MC
0.3	Case 1	17.0817	1	1.0350	2.7052	2.7052	0	9.8905
	Case 2	17.0134	1	0.4883	1.7069	1.7069	0	5.4079
0.5	Case 1	17.0097	1	1.0244	2.7248	2.7248	0	9.8884
	Case 2	17.0127	1	0.4841	1.7106	1.7106	0	5.4034
1.0	Case 1	16.4994	2	1.2136	3.6874	1.4764	2.2110	9.7048
	Case 2	17.0034	1	0.4792	1.7135	1.7135	0	5.4020
2.0	Case 1	14.8950	10	1.6546	4.9903	0.0028	4.9876	8.3035
	Case 2	15.2259	10	1.0284	2.7710	0	2.7710	4.9037
3.0	Case 1	12.7674	10	1.6282	5.6742	0.0060	5.6682	7.4857
	Case 2	14.3712	10	1.0264	2.9120	0	2.9120	4.4758

Case 1: gamma process and shocks; Case 2: gamma process only. N_F, N_{PM}, N_{PPM}, and N_{IPM} are the expected numbers of failures, PM, PPM, and IPM, respectively

From the results reported in Table 4 and Fig. 8, the following can be determined:

1. When $\eta \leq 1$, the perfect preventive maintenances (K=1, N_{IPM}=0) should be chosen for case 1. When taking the effects of shocks into consideration, the optimal preventive maintenance is still the perfect one, except for η=1. For η>1, imperfect maintenances are preferable (K=10) for both case 1 and case 2. The reason for this is that for η>1, the imperfect preventive maintenance cost function is convex, i.e. for the same intervention gain, the imperfect one corresponds to a lower cost as compared to the perfect one.

2. For $\eta \leq 1$, the optimal preventive maintenance threshold and the cost rate in case 1 are insensitive to η. On the other hand, when η>1, M and MC are highly dependent on η for both cases, exhibiting the same tendency as in the infinite time span (Table 3 and Fig. 7). The threshold M decreases as η increases for η>1, indicating that the earlier PM should be executed. Although it will lead to more maintenance actions (e.g., when η changes from 0.3 to 3.0, N_{PM} increases from 2.7052 to 5.6742 for case 1, and N_{PM} increases from 1.7069 to 2.9120 for case 2), a smaller cost rate can be achieved for larger η due to the low cost of a single IPM action.

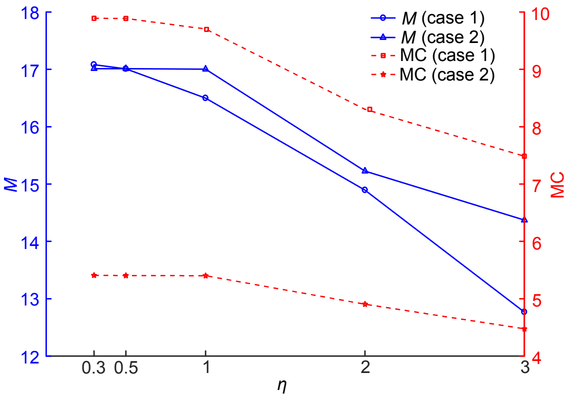

Fig. 8 Optimal maintenance threshold M and cost rate MC with respect to η (T=50)

For a given η, the cost rate of case 1 is larger than that of case 2 (the two dashed lines in Fig. 8). The reasons for this can be explained as follows. On one hand, shocks may accelerate the deterioration process, and more preventive maintenance actions have to be implemented to ensure safety throughout the time span T. Taking η=3.0 as an example, N_{PM}=5.6742 for case 1 and N_{PM}=2.9120 for case 2. On the other hand, the random shocks may result in a higher occurrence probability of hard failures that have catastrophic consequences on the structure, hence leading to a higher cost. For example, when η=3.0, N_F=1.6282 for case 1 and N_F=1.0264 for case 2.

Impacts of time span T on the optimal maintenance policy, as well as the cost rate, are investigated using the parameters given in Table 2. The results are shown in Table 5 and depicted in Fig. 9.

The results presented in Table 5 and Fig. 9 indicate that the optimal maintenance policy is largely dependent on the design life T. When T changes from 30 to 60, the optimal PM threshold M decreases as T

Table 5 Optimal maintenance planning for different values of T with η=3.0

T	Case	M	K	N_F	N_{PM}	N_{PPM}	N_{IPM}	MC
30	Case 1	13.1545	8	0.8266	3.2586	0	3.2586	6.8717
	Case 2	15.0120	7	0.4377	1.6340	0	1.6340	3.6348
40	Case 1	12.8720	9	1.2385	4.4737	0.0035	4.4702	7.2570
	Case 2	14.3154	7	0.7388	2.3344	0	2.3344	4.1821
50	Case 1	12.7674	10	1.6282	5.6742	0.0060	5.6682	7.4857
	Case 2	14.3712	10	1.0264	2.9120	0	2.9120	4.4758
60	Case 1	12.2990	10	2.0037	7.0445	0.0119	7.0326	7.5995
	Case 2	13.7962	10	1.3040	3.6293	0	3.6293	4.6248
70	Case 1	12.5702	10	2.3958	8.1360	0.0154	8.1206	7.7457
	Case 2	13.8406	10	1.5657	4.3096	0	4.3096	4.7746
90	Case 1	12.6832	9	3.1577	10.4835	0.0513	10.4322	7.8395
	Case 2	13.9882	10	2.1376	5.5490	0.0004	5.5486	4.9251
100	Case 1	13.0806	9	3.5376	11.5066	0.0600	11.4466	7.8857
	Case 2	14.0781	8	2.4141	6.1506	0.0089	6.1417	4.9812

Case 1: gamma process and shocks; Case 2: gamma process only

increases, meaning that PM should be executed more frequently within the time interval [0, T] to maintain system safety. For T≥60, opposite trends are observed, which are attributed to the balance among the cost terms of Eq. (22) in an optimal maintenance strategy.

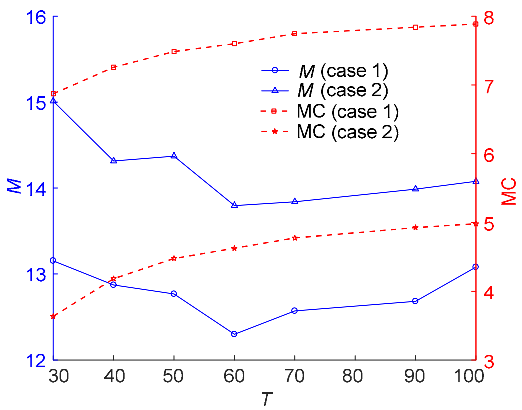

Fig. 9 Optimal maintenance threshold M and cost rate MC with respect to T

For small value of M, more frequent interventions are involved, leading to a large maintenance cost. On the other hand, if M is large, failure occurrence is more frequent, which increases the corresponding cost. Similar variation trends are noted for K. To determine the sensitivity of MC to M, the case where T=100 is taken as an example, where M^*=13.0806, and K^*=9. Cost rates and expected numbers of various actions for different values of M are presented in Table 6 and depicted in Fig. 10.

The optimal maintenance thresholds of case 1 are smaller than those of case 2 (Fig. 9), which means that PM should be applied earlier when random shocks are involved in the deterioration process, leading to more PM actions and a higher cost rate. Taking T=100 as an illustration, M=13.0806, N_{PM}=11.5066, and MC=7.8857 for case 1, while for case 2, M=14.0781, N_{PM}=6.1506, and MC=4.9812. It is noted that the possible hard failures in case 1 are responsible for a part of the total costs.

Table 6 Cost rates and expected numbers of various actions for different values of M (K=9)

M	N_F	N_{PM}	N_{PPM}	N_{IPM}	MC
8	3.5710	13.4548	0.0778	13.3770	8.0781
13	3.5376	11.5066	0.0600	11.4466	7.8857
18	4.0872	7.6537	0.0111	7.6427	8.9016

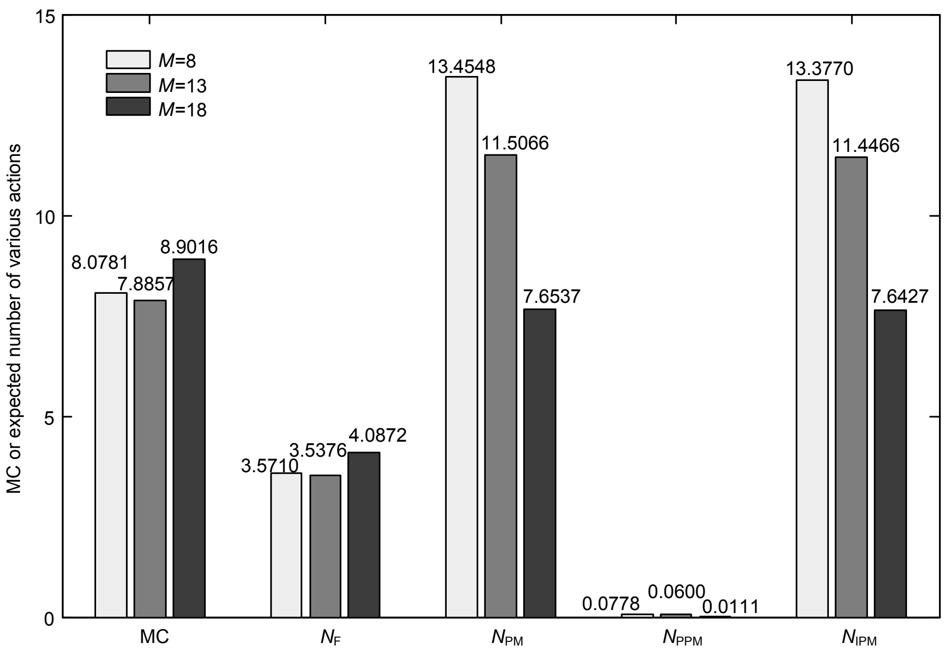

Fig. 10 Cost rates and expected numbers of various actions for different values of M (K=9)

Overall, as anticipated, the mean cost rate increases with the increase of T for both case 1 and case 2, as shown in Fig. 9, since the expected numbers of inspections, preventive maintenances, and failures increase as T increases.

Considering (W_L=1.0) or ignoring (W_L=0) the structural resistance against small shock loads, the immunity of the system to small shocks produces differences in MC and maintenance actions, as shown in Fig. 11. The first reason for this is that when W_L=0, every shock induces sudden damage to the system, making the system more vulnerable and therefore increasing the expected number of failures N_F. Second, to ensure that the system is able to fulfill the predefined function, more PMs have to be executed when W_L=0, resulting in an increased cost.

6 Conclusions

In this paper, a reliability model for a single component system experiencing a continuous degradation process and a random shock process is developed. The cumulative and extreme shock models, corresponding to soft failures and hard/catastrophic failures, respectively, are considered in the reliability model. In addition, the degradation process is characterized by a stationary gamma process and the random shock process is assumed to be a homogeneous Poisson process. Based on the reliability analysis, a CBM model with aperiodic inspections, in which the inspection time is determined based on the RUL-based inspection policy, is developed. The optimal maintenance policy aims to minimize the average maintenance cost rate for a system over an infinite and/or a finite time span. In the proposed CBM model, the random shocks, causing hard failures, or sudden damage increments to structures, are also taken into account, except for the impact of gradual degradation.

Numerical results demonstrate that the shock loads exert notable impacts on the optimal maintenance strategy. In addition, optimal solutions of a finite time span differ greatly from those of an infinite time span. Thus, it is essential to investigate the optimal CBM policy of a system within a finite horizon, since engineering structures usually have finite service lives and optimal maintenance policies are highly dependent on their lives.

Analyses of time-dependent reliability and of maintenance planning optimization for multi-components systems, for example, long-life infrastructures, subject to MDCFP will constitute a topic of our future work.

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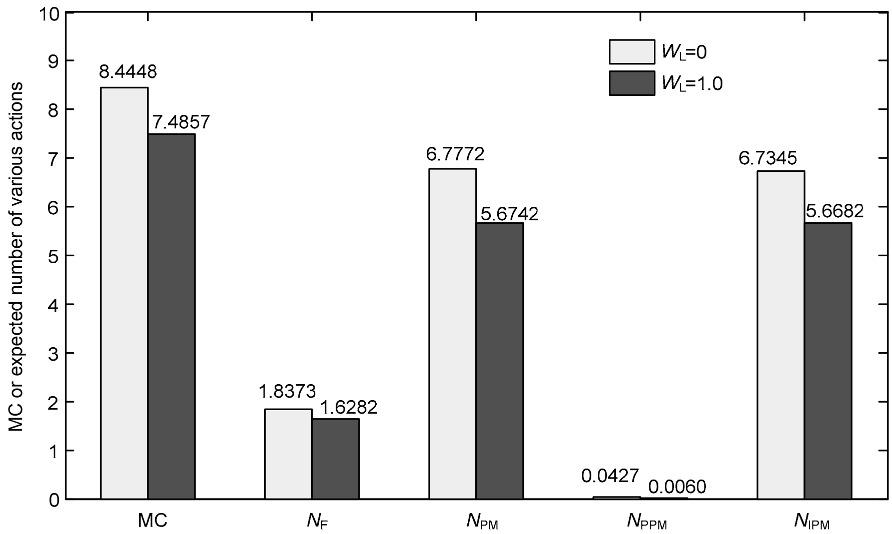

Fig. 11 Cost rates and expected numbers of various actions (T=50, η=3.0)

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中文概要

题　目： 连续退化和随机冲击下基于状态的结构维修策略优化

目　的： 结构在使用过程中，其性能往往发生劣化而导致其安全性能（可靠度）不断降低。本文旨在探讨结构在连续退化和冲击荷载共同作用下，其可靠度随时间的变化情况。此外，研究结构的最佳维修策略，使其在满足可靠度约束条件的同时，将平均费用降到最低。

创新点： 1. 在连续退化和冲击荷载的共同影响下，建立结构时变可靠度计算模型。2. 在前述可靠度分析的基础上，建立基于状态维修的非周期检测模型；基于剩余使用寿命检测策略，确定检测时间，并确定系统的最优维护策略，旨在将平均维护成本率降至最低。3. 针对无限时间域和有限时间域，分别确定对应的最佳维修策略。

方　法： 1. 通过理论推导，构建结构时变可靠度计算公式（公式（13）），分析各参数与可靠度之间的变化关系（图 4）。2. 通过仿真模拟，运用蒙特卡洛法确定结构在使用过程中的最佳维修策略（图 5 和 6）。

结　论： 1. 与仅考虑连续退化的情况相比，随机冲击荷载的存在，使得系统的可靠度降低，更容易发生失效。2. 冲击载荷的存在，对最佳维修策略具有显著影响。3. 有限时间域的最优解与无限时间域的最优解之间存在很大的不同，因此，有必要对这两种情况分别进行研究。

关键词： 软失效；硬失效；剩余寿命；可靠度；维修；成本率；有限时域