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Optimized design of LED freeform lens for uniform circular illumination*

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Abstract: An optimization method is proposed for designing an LED freeform lens which produces a uniform circular pattern with high energy efficiency. This method is composed of three main aspects: design of the initial guess, parameterization of the freeform surface, and construction of the merit function. The initial guess is created by solving an ordinary differential equation numerically. An approach of selecting optimization points is introduced for parameterization of the freeform surface. The merit function is constructed by use of the irradiance uniformity and the efficiency of the lens. Design examples are given, and the results show that the irradiance distribution is well controlled with a maximum uniformity (the relative standard deviation of irradiance, RSD) of 0.0122 and a maximum efficiency of 93.88%. This optimization method can be generalized to design freeform lenses with different lighting patterns or without rotational symmetry.

Key words: Illumination design, Lenses, Light emitting diode (LED), Non-imaging optics

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1 Introduction

Compared with conventional light sources, light emitting diodes (LEDs) have many advantages for general lighting, such as long lifetime, low energy consumption, small size, and design flexibility (Chi and George, 2006; Krames *et al*., 2007). In recent years, LEDs have been used more and more widely in our daily life due to its extraordinary light efficiency (Parkyn and Pelka, 2006; Wang, 2007; Chen *et al*., 2009; Wu *et al*., 2011b). In various applications of LED lighting, LED products with uniform circular illumination, which are applied in many fields such as sensor lighting and indoor lighting, are still the first choice of designers because such an illumination pattern is easier to produce compared with other illumination patterns. Therefore, this illumination pattern is still playing a key role in promoting energy saving lighting. Usually, an LED light source can be considered as a Lambertian emitter, and its maximum emission angle is almost 90°. Undoubtedly, a uniform circular illumination cannot be ensured when the LED light source is used for lighting directly. Thus, the secondary optical design must be employed in solving this problem.

Due to its high degree of design freedom, freeform surfaces can simplify the structure of the optical system and satisfy complex illumination requirements. With the development of designing and machining of freeform surfaces, this technique has been applied in many fields, such as road or searching lighting (Feng *et al*., 2010; Luo *et al*., 2010; Zhao *et al*., 2011), lighting in projectors (Pan *et al*., 2007; Zhao *et al*., 2007; Ding *et al*., 2008; Fournier and Rolland, 2008), liquid crystal display (LCD) backlighting (Bol'shukhin *et al*., 2011), automotive head-

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lamps (Cvetkovic *et al*., 2006; Chen *et al*., 2010), and optical lithography systems (Wu *et al*., 2011c). In general, there are two main methods for designing a freeform surface for uniform circular illumination: partial differential equation (PDE) method (Ries and Muschaweck, 2001; Ries, 2002; Oliker, 2007; Rubinstein and Wolansky, 2007; Zheng *et al*., 2009; Luo *et al*., 2010) and multi-parameter optimization (MPO) method (Chen *et al*., 2009; Cao *et al*., 2011; Luo *et al*., 2011; Moiseev *et al*., 2011). A freeform lens with total internal reflection (TIR) structure, which is preferred for improving energy efficiency, can be efficiently designed by using the PDE method. However, a predetermined design result usually cannot be ensured because of the size of the LED light source. Irradiance mutation would occur at the joint of the two illumination zones produced by the reflective freeform surface and the refractive freeform surface of the lens, respectively (this will be demonstrated in Section 3). By changing the values of optimization variables, which are employed to characterize the freeform surface, the MPO method can find an optimal solution of the freeform lens with a certain optimization algorithm. Since an actual light source can be used, the MPO method is more practical compared with the PDE method. Although some optimization methods have been proposed (Luo Y *et al*., 2010; Wang *et al*., 2010; Luo XX *et al*., 2011; Situ *et al*., 2011), it is still difficult to obtain a satisfactory design by using these methods. For example, Luo *et al*. (2011) proposed a feedback modification algorithm to produce a uniform circular illumination. The uniformity of the central illumination area can reach 90%, while that of the desired illumination area is only about 44.8%. So, an efficient and robust optimization method for designing uniform circular illumination is still urgently needed.

In this paper, a new optimization method is proposed for designing an LED freeform lens which produces a uniform circular pattern with high irradiance uniformity and energy efficiency. Based on the design of the initial guess, appropriate parameterization of the freeform surface, and elaborate construction of the merit function, the desirable results can be easily obtained. This paper also provides a new idea for designing other axis rotationally symmetric lighting systems with different lighting patterns.

2 Design principle

The optimized design method is composed of three main aspects: design of the initial guess, parameterization of the freeform surface, and construction of the merit function (Fig. 1). Usually, both the refractive structure and the TIR structure are used in LED secondary optics design. To explore the generality of this optimization design method, only the design of TIR freeform lens will be given in this paper.

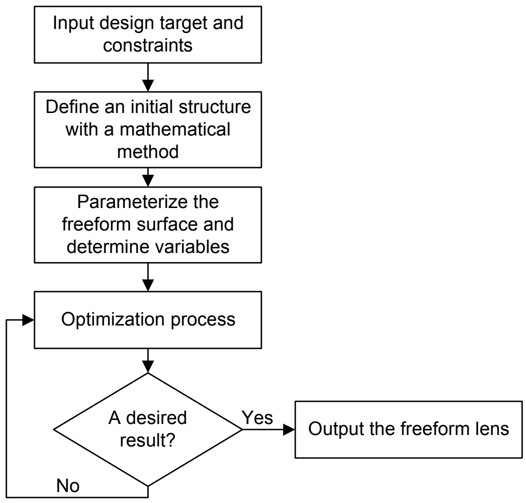

Fig. 1 Flow chart of the design method

2.1 Design of the initial guess

Whether a global or a local optimization algorithm is employed, the optimization efficiency can be improved significantly by use of an appropriate initial design. Tai and Schwarte (2000) proposed a method for designing a refractive freeform lens which produces a uniform circular illumination on a specific target plane. In this study, we generalize the Tai method to design a TIR freeform lens. The design principle of the generalized Tai method is illustrated in Fig. 2.

An arbitrary ray SB, emanating from the source S, intersects the freeform refractive surface of the lens at point B, and is refracted by this surface into the output ray BC (Fig. 2a). θ is the emission angle of source S, and ω is the angle formed between the ray BC and the optical axis SO. Assume that the distance between source S and the vertex of the refractive surface is h, the distance between this vertex and point O is d, and n is the refractive index of the lens. Based on Snell's

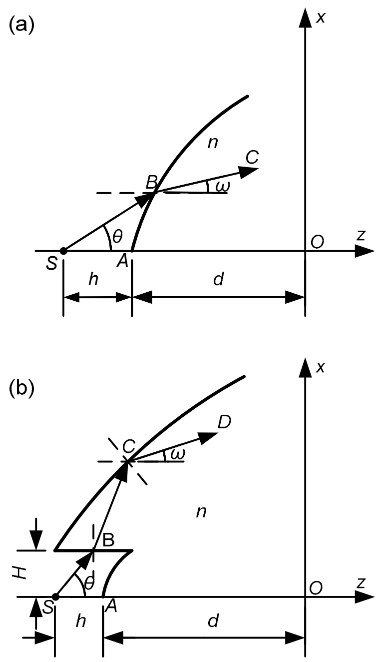

Fig. 2 Relationship of angles at the refractive surface (a) and the reflective surface (b)

law, an ordinary differential equation, by which the profile of the refractive surface is governed, is obtained by

$$\frac{\mathrm{d}z}{\mathrm{d}\theta}=(h+z+d)\cdot\frac{\dfrac{\sin\theta-n\sin\omega}{n\sin\omega-\cos\theta}}{1-\dfrac{\sin 2\theta}{2}\cdot\dfrac{\sin\theta-n\sin\omega}{n\sin\omega-\cos\theta}}. \quad (1)$$

With reference to the geometry of Fig. 2b, similarly an ordinary differential equation, by which the profile of the total internal reflective surface is governed, is given by

$$\frac{\mathrm{d}z}{\mathrm{d}\theta}=\frac{\sqrt{n^2-\cos^2\theta}-n\sin\omega}{n\cos\omega-\cos\theta}\left(\frac{\sqrt{n^2-\cos^2\theta}}{\cos\theta}\frac{H}{\sin\theta}+\frac{h+d+z-H\cot\theta}{\sqrt{n^2-\cos^2\theta}}\frac{\sin\theta}{\cos^2\theta}n^2\right)\cdot\frac{1}{1-\dfrac{\sqrt{n^2-\cos^2\theta}-n\sin\omega}{n\cos\omega-\cos\theta}\dfrac{\sqrt{n^2-\cos^2\theta}}{\cos\theta}}, \quad (2)$$

where $\frac{\mathrm{d}z}{\mathrm{d}\theta}$ is the first-order derivative of the coordinate z with respect to θ.

To solve these two differential equations, a mapping relationship between the emission angles of source S and the coordinates of target points should be established. Considering the rotational symmetry of the freeform lens, a mapping shown in Fig. 3 is used here (Fournier *et al.*, 2010). Then, these two ordinary differential equations are solved numerically by the fourth-order Runge-Kutta formulas, and the initial guess of the freeform lens is constructed by use of these discrete data points. Due to the size of the extended LED source and the TIR structure of the freeform lens, usually target illumination cannot be ensured by the initial guess. The optical performance of the freeform lens should be further improved by use of the optimization process.

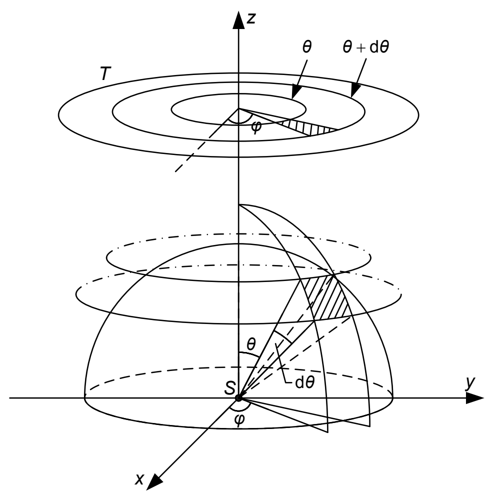

Fig. 3 Mapping relationship

The incident flux of a solid angle defined by φ (angle between incident ray and positive direction of the x axis) and θ (angle between incident ray and positive direction of the z axis) is refracted or reflected into a ring area confined by two circles with the radii of x_n and x_{n+1}, respectively

2.2 Parameterization of the freeform surface and merit function

Due to the rotational symmetry of the TIR lens, we can just focus on optimizing the profile of the freeform lens. Model parameterization and construction of the merit function are two important aspects of setting up this optimization. Model parameterization, actually, is a process of choosing an appropriate representation form for the profile of the freeform lens. There are many ways to represent a curve, such as multinomial and B-spline curve, and a different representation form usually results in a different variable and a different optimization result. In this study, the B-spline curve is employed because of its stability and flexibility. Meanwhile, some discrete data points on the profile are chosen as the optimization points. The downhill simplex method, which is a local optimization algorithm, is used in this method (Koshel, 2005; Zhang *et al*., 2010). Considering the search efficiency of the optimization design method, the number of optimization points is usually small. Intuitively, we can determine these optimization points in the way that the angles formed between each two neighboring optimization points are equal (Zhang *et al*., 2010), as shown in Fig. 4a. Due to the small number of the optimization points, the distribution of the points could be very nonuniform. For example, $\overset{\frown}{A_4A_5} > \overset{\frown}{A_1A_2}$ (Fig. 4a). In this case, the target profile cannot be ensured by using this set of points (Piegl and Tiller, 1997). Hence, we introduce a new approach for determining the optimization points with equal arc-length between each two neighboring points (Fig. 4b). When the lengths between optimization points with the equal angle method vary greatly, the equal length method can reconstruct the model more precisely and shows more power in line controlling. This is validated by a simulation (Fig. 5). To better illustrate this issue, an initial lens model constructed directly from a lot of discrete data points generated from the numerical solution of Eqs. (1) and (2) is chosen and the ray-tracing result is shown in Fig. 5a. Because of the local optimization algorithm employed in this design method, the initial guess of freeform lens is vital to the optimization result. Thus, a model reconstruction method can be evaluated by the similarity of the reproduced model to the initial one. Compared with the ray-tracing result of the equal angle method (Fig. 5b), it is clear that the irradiance map of the equal arc-length method (Fig. 5c) is closer to Fig. 5a. However, sometimes there is no distinct difference between the equal angle method and the equal arc-length method when the number of optimization points increases and the lengths between the optimization points decided by both methods are almost the same (Fig. 6). In this way, no matter how the design parameters change, the equal length method will not show worse performance compared with the equal angle method. Next, we will use the approach of equal arc-length to determine the optimization points for the TIR lens shown in Fig. 7.

According to the new selection method mentioned above, the discrete optimization points on contour line *AB* are obtained by

$$\overset{\frown}{AQ_1} = \overset{\frown}{Q_iQ_{i+1}} = \overset{\frown}{Q_{i+1}Q_{i+2}} = \overset{\frown}{Q_{N_1-2}B}, \tag{3}$$

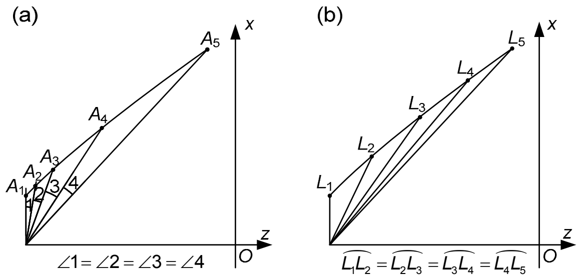

Fig. 4 Methods of optimization points selection: (a) equal angle method; (b) equal arc-length method

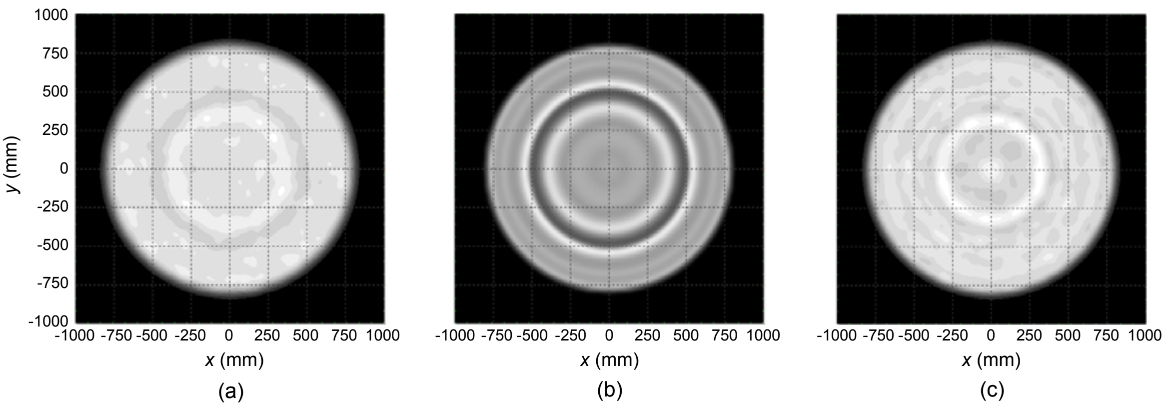

Fig. 5 A comparison between the equal angle method and the equal arc-length method

(a) The irradiance map generated by the lens model constructed directly from profile data points in software Rhinoceros; (b) The irradiance map generated by the equal angle method; (c) The irradiance map generated by the equal arc-length method

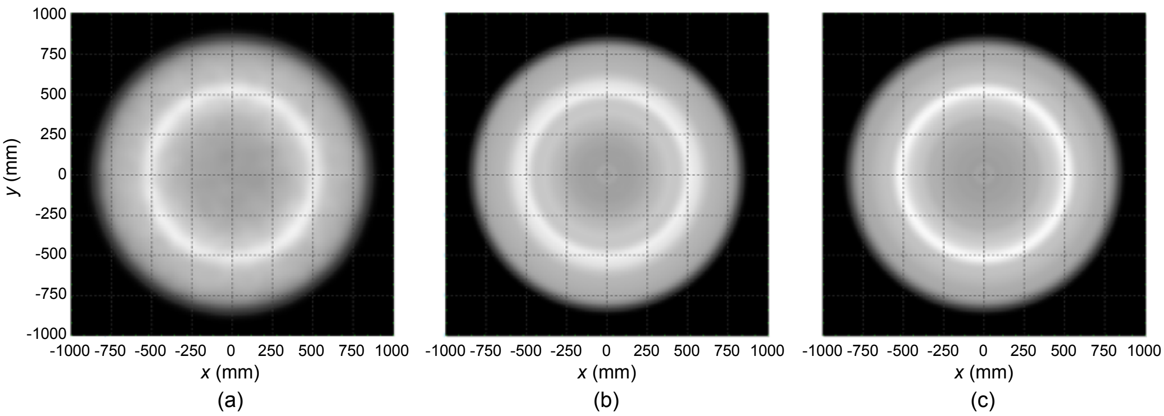

Fig. 6 Another comparison between the equal angle method and the equal arc-length method (the number of discrete optimization points on the lens' profile is 11, and other parameters are the same as in Fig. 5)

(a) The irradiance map generated by the lens model constructed directly from profile data points in software Rhinoceros; (b) The irradiance map generated by the equal angle method; (c) The irradiance map generated by the equal arc-length method

where i=1, 2, ⋯, N_1−2 and N_1 is the number of discrete optimization points on AB. Also, the optimization points on CD are generated by

$$\overset{\frown}{CP_1} = \overset{\frown}{P_jP_{j+1}} = \overset{\frown}{P_{j+1}P_{j+2}} = \overset{\frown}{P_{N_2-2}D}, \tag{4}$$

where j=1, 2, ⋯, N_2−2 and N_2 is the number of discrete optimization points on CD. The emission angles θ_{ri} (i=1, 2, ⋯, N_1) and θ_{lj} (j=1, 2, ⋯, N_2) corresponding to the optimization points can be calculated by interpolation operation. ρ_{ri} (i=1, 2, ⋯, N_1) and ρ_{lj} (j=1, 2, ⋯, N_2) shown in Fig. 7 will be chosen as the variables to be optimized. After each iteration, the coordinates of these optimization points are defined by

$$\begin{cases} x_{ri} = \rho_{ri}\sin\theta_{ri}, \\ z_{ri} = -(h+d) + \rho_{ri}\cos\theta_{ri}, \end{cases} \quad i = 1, 2, \cdots, N_1, \tag{5}$$

$$\begin{cases} x_{lj} = H + \dfrac{\rho_{lj}}{n}\sqrt{n^2 - \cos^2\theta_{lj}}, \\ z_{lj} = -(h+d) + H\cot\theta_{lj} + \dfrac{\rho_{lj}\cos\theta_{lj}}{n}, \end{cases} \quad j = 1, 2, \cdots, N_2, \tag{6}$$

where (x_{ri}, 0, z_{ri}) and (x_{lj}, 0, z_{lj}) are the coordinates of Q_i and P_j, respectively. When the optimization points are determined, the control points and the knot vectors are obtained by use of interpolation theory. Then, the profile is constructed using both the control points and the knot vectors (Wu *et al.*, 2011a), as shown in Fig. 8.

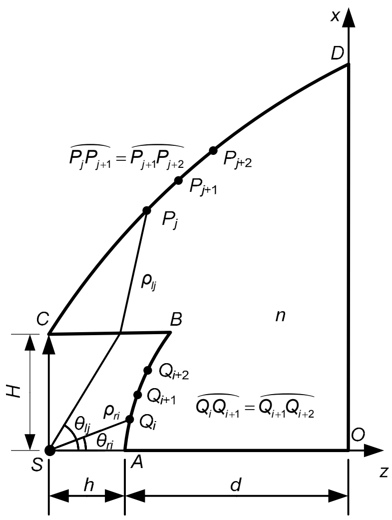

Fig. 7 Optimized variables selection of freeform lens

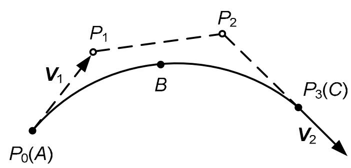

Fig. 8 Principle of curve reconstruction

Due to the axis rotational symmetry of the illumination pattern, the irradiance uniformity of the illumination area can be represented by that of an irradiance curve running through the center of this area. Based on this idea, some sample data points on the irradiance curve are chosen and the relative standard deviation (RSD) of irradiance is used to evaluate the irradiance uniformity, given by

$$\mathrm{RSD}=\sqrt{\frac{1}{M}\sum_{i=1}^{M}\left(\frac{E_i}{\overline{E}}-1\right)^2}, \tag{7}$$

where E_i is the irradiance of each sample data point, $\overline{E}$ is the average irradiance of all the points, and M is the number of sample points. A smaller value represents a higher uniformity. Assume that transmission efficiency is the percentage of light from the source transmitted by the freeform surface within the desired illumination area. To ensure both irradiance uniformity and efficiency of the freeform lens, the merit function of the optimization process can be defined by

$$\mathrm{MF}=w_1\cdot\mathrm{RSD}+w_2\cdot\mathrm{Efficiency}, \tag{8}$$

where w_1 and w_2 are the weights, and Efficiency represents the transmission efficiency. In the following section, many design examples based on the proposed principle will be presented. All the following designs adopt the same w_1 and w_2 with $w_1>w_2$ so that the irradiance uniformity can be ensured preferentially.

3 Examples and analysis

In this section, several design examples are given to verify this optimization design method, and a Monte-Carlo ray tracing simulation approach is used to analyze the optical performance of the freeform lenses. A freeform TIR lens is first designed to explore the feasibility of this optimization design, and the design parameters are listed in Table 1, where $R_{\max}$ is the radius of target circular illumination and L is the lighting distance between LED and the target plane. Four optimization points on the refractive profile and five optimization points on the TIR profile are determined using the new selection method introduced above. A 1 mm×1 mm LED Lambertian emitter is used, and 200 000 rays are traced during each iteration. When the MF becomes saturated, two million rays are traced to reduce the statistical noise in the final simulation. The model of the freeform lens and the irradiance distribution are shown in Figs. 9 and 10, respectively.

Fig. 10a shows the irradiance distribution produced by the initial guess. Due to the size of the extended LED source and the TIR structure of the freeform lens, irradiance mutation occurs at the joint of the two illumination zones produced by the TIR surface and the refractive surface of the lens, respectively. The irradiance uniformity (RSD) is only 0.2753. Obviously, the actual irradiance distribution cannot meet the design requirements. Fig. 10b shows the optimized irradiance distribution. The irradiance uniformity (RSD) and the efficiency are 0.0122 and 93.88%, respectively. Undoubtedly, the optical performance of the freeform lens is improved significantly by use of this optimization method.

Table 1 Design parameters of TIR lens

Parameter	Value	Parameter	Value
n	1.4935	N_1	4
d	8 mm	N_2	5
h	3 mm	$R_{\max}$	800 mm
θ_r	40°	L	2000 mm

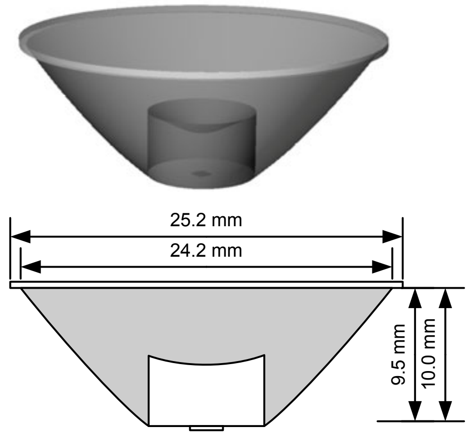

Fig. 9 Model of the freeform lens

To explore the universality of this design method, we try to design several other lenses with different $R_{\max}$ of 1000 mm, 900 mm, 700 mm, and 600 mm, and with the lighting distance h unchanged. The results are listed in Table 2.

It is clearly shown that the light beams of the extended LED source are controlled well, and that the optimized optical performance meets the design requirement quite well. Based on these detailed analyses, we can conclude that this optimization design method is feasible and practical, and is superior to the existing methods that can be used to produce the uniform circular illumination, such as the Luo method (Luo *et al.*, 2011).

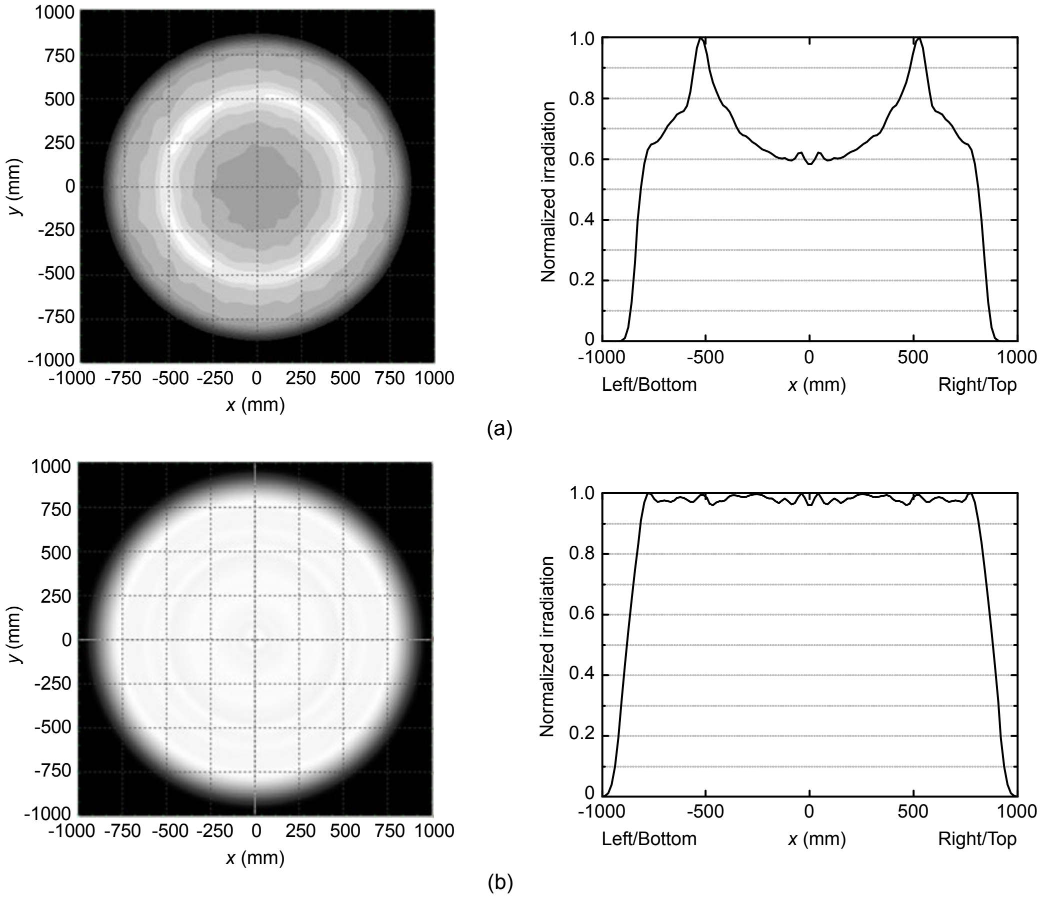

Fig. 10 Illuminance distribution of the target plane: (a) before optimization; (b) after optimization

Table 2 RSD and Efficiency of lenses corresponding to different spot radii

R_{max} (mm)	RSD	Efficiency (%)
1000	0.0130	92.74
900	0.0149	95.28
800	0.0122	93.88
700	0.0184	96.49
600	0.0131	94.22

4 Conclusions

Design of a uniform circular illumination is still popular in non-imaging optics. In this paper, we present an optimization design method for producing such an illumination mode. This optimization design method is composed of three main aspects. All aspects are detailed, and an approach for selecting optimization points is introduced. An extended LED source is employed, and a desirable result with high irradiance uniformity can be easily obtained by use of this optimization method. Other optical properties of the freeform lens, such as light efficiency, can also be obtained by employing a specific merit function. This design method is feasible and practical, and is superior to the existing methods that can be used to produce uniform circular illumination. Moreover, this method provides an idea for designing a freeform surface without rotational symmetry.

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