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Stochastic gradient algorithm for a dual-rate Box-Jenkins model based on auxiliary model and FIR model*

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Abstract: Based on the work in Ding and Ding (2008), we develop a modified stochastic gradient (SG) parameter estimation algorithm for a dual-rate Box-Jenkins model by using an auxiliary model. We simplify the complex dual-rate Box-Jenkins model to two finite impulse response (FIR) models, present an auxiliary model to estimate the missing outputs and the unknown noise variables, and compute all the unknown parameters of the system with colored noises. Simulation results indicate that the proposed method is effective.

Key words: Parameter estimation, Auxiliary model, Dual-rate system, Stochastic gradient, Box-Jenkins model, FIR model

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1 Introduction

Identification of dynamic systems plays an integral role in many applications (Wu *et al.*, 2009; Deboucha and Taha, 2010; Zong *et al.*, 2011). There exist a lot of identification methods for dynamic systems, including recursive least squares (RLS) algorithms (Cattivelli *et al.*, 2008; Ding, 2013a; 2014), stochastic gradient (SG) algorithms (Liu, 2009; 2010a; Ding, 2013b), and iterative algorithms (Vörös, 2010; Ding, 2013c; Ding *et al.*, 2013). It is well known that the SG algorithm has a smaller convergence rate than the RLS algorithm. To improve the convergence rate of the SG algorithm, Chen and Ding (2011) and Chen *et al.* (2012) proposed an SG algorithm with a forgetting factor and a modified SG algorithm by introducing a convergence index.

Box-Jenkins models are often used to model a wide class of linear dynamic systems. Identification of Box-Jenkins models has received much attention (Forssell and Ljung, 2000; BuHamra *et al.*, 2003). For example, Liu *et al.* (2010b) presented a least squares based iterative algorithm for Box-Jenkins models; Wang *et al.* (2010) proposed a gradient-based iterative parameter estimation for Box-Jenkins models. These works assumed that the input-output data of the Box-Jenkins models at every sampling instant is available. When the input and output signals of the systems have different sampling rates, such systems are called irregularly sampled data systems, e.g., dual-rate systems. Dual-rate systems can find many applications, e.g., in digital signal processing (Nakamori *et al.*, 2007), communications (Shi *et al.*, 2006), sensor networks (Kadu *et al.*, 2008), process control, and estimation (Sägfors and Toivonen, 1997). The polynomial transformation technique has become a standard tool for dual-rate system identification, and the idea is to use a polynomial to derive a dual-rate model that directly utilizes all available data: both the fast input and slow output data. However, the polynomial transformation technique

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may lead to significant unknown parameters (Chen, 2014).

Based on the work in Ding and Ding (2008), in this paper we develop a modified SG parameter estimation algorithm for a dual-rate Box-Jenkins model using an auxiliary model. The contributions of this paper are as follows:

1. Simplify the complex dual-rate Box-Jenkins model to two finite impulse response (FIR) models.

2. Present an auxiliary model to estimate the missing outputs and the unknown noise variables.

3. Compute all the unknown parameters of the system with colored noises.

2 Problem formulation and the auxiliary model based SG algorithm

Consider the following dual-rate Box-Jenkins model:

$$y(t)=\frac{B(z)}{A(z)}u(t)+\frac{D(z)}{C(z)}v(t), \tag{1}$$

where $y(t)$ is the system output, $u(t)$ is the system input, $v(t)$ is a stochastic white noise with zero mean, and $A(z)$, $B(z)$, $C(z)$, and $D(z)$ are the polynomials in the unit backward shift operator $z^{-1}y(t)=y(t-1)$,

$$\begin{aligned} A(z) &= 1+a_1z^{-1}+a_2z^{-2}+\cdots+a_nz^{-n},\\ B(z) &= b_1z^{-1}+b_2z^{-2}+\cdots+b_nz^{-n},\\ C(z) &= 1+c_1z^{-1}+c_2z^{-2}+\cdots+c_nz^{-n},\\ D(z) &= 1+d_1z^{-1}+d_2z^{-2}+\cdots+d_nz^{-n}. \end{aligned}$$

All the input data $u(t)$ $(t=0,1,\cdots)$ and only the scarce output data $y(tq)$ $(t=0,1,\cdots;\ q\geq 2)$ of the dual-rate Box-Jenkins are known. The intersample outputs or missing outputs $y(tq+j)$ $(j=1,2,\cdots,q-1)$ are unavailable.

Define

$$G(z):=\frac{B(z)}{A(z)},\qquad Q(z):=\frac{D(z)}{C(z)}.$$

Then Eq. (1) can be written as

$$y(t)=G(z)u(t)+Q(z)v(t). \tag{2}$$

Let g_i $(i=1,2,\cdots)$ be the impulse response parameters for $G(z)$, and q_i be the impulse response parameters for $Q(z)$. Using the long division to expand $G(z)$ and $Q(z)$ gives

$$\begin{aligned} G(z) &= \frac{B(z)}{A(z)}\\ &= g_1z^{-1}+g_2z^{-2}+\cdots+g_iz^{-i}+\cdots, \end{aligned} \tag{3}$$

$$\begin{aligned} Q(z) &= \frac{D(z)}{C(z)}\\ &= 1+q_1z^{-1}+q_2z^{-2}+\cdots+q_iz^{-i}+\cdots. \end{aligned} \tag{4}$$

For stable $G(z)$ and $Q(z)$, as i goes to infinity, g_i and q_i converge to zero. Thus, Eqs. (3) and (4) can be approximated by two FIR models with p parameters:

$$\begin{aligned} \frac{B(z)}{A(z)} &\approx G'(z)=g_1z^{-1}+g_2z^{-2}+\cdots+g_pz^{-p},\\ \frac{D(z)}{C(z)} &\approx Q'(z)=1+q_1z^{-1}+q_2z^{-2}+\cdots+q_pz^{-p}. \end{aligned}$$

As long as p is sufficiently large, $G'(z)$ and $Q'(z)$ are very close to $G(z)$ and $Q(z)$.

Eq. (1) can be approximately as

$$y(t)=G'(z)u(t)+Q'(z)v(t). \tag{5}$$

Define parameter vector $\boldsymbol{\theta}$ and information vector $\boldsymbol{\varphi}(t)$ as

$$\boldsymbol{\theta} := [g_1,g_2,\cdots,g_p,q_1,q_2,\cdots,q_p]^{\mathrm{T}}\in\mathbb{R}^{2p}, \tag{6}$$

$$\begin{aligned} \boldsymbol{\varphi}(t) := {}& [u(t-1),u(t-2),\cdots,u(t-p),v(t-1),\\ & v(t-2),\cdots,v(t-p)]^{\mathrm{T}}\in\mathbb{R}^{2p}. \end{aligned} \tag{7}$$

Then Eq. (5) can be rewritten as a concise form:

$$y(t)=\boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta}+v(t).$$

Replacing t with tq gives

$$y(tq)=\boldsymbol{\varphi}^{\mathrm{T}}(tq)\boldsymbol{\theta}+v(tq). \tag{8}$$

Let $\hat{\boldsymbol{\theta}}(tq)$ be the estimate of $\boldsymbol{\theta}$. Defining and minimizing the cost function

$$J(\boldsymbol{\theta}):=[y(tq)-\boldsymbol{\varphi}^{\mathrm{T}}(tq)\boldsymbol{\theta}]^2$$

give the following SG algorithm:

$$\hat{\boldsymbol{\theta}}(tq)=\hat{\boldsymbol{\theta}}(tq-q)+\frac{\boldsymbol{\varphi}(tq)}{r(tq)}e(tq), \tag{9}$$

$$\hat{\boldsymbol{\theta}}(tq-i)=\hat{\boldsymbol{\theta}}(tq-q),\ i=1,2,\cdots,q-1, \tag{10}$$

$$e(tq)=y(tq)-\boldsymbol{\varphi}^{\mathrm{T}}(tq)\hat{\boldsymbol{\theta}}(tq-q), \tag{11}$$

$$\begin{aligned} \boldsymbol{\varphi}(tq)=&[u(tq-1),u(tq-2),\cdots,u(tq-p),\\ &v(tq-1),v(tq-2),\cdots,v(tq-p)]^{\mathrm{T}}, \end{aligned} \tag{12}$$

$$v(tq-i)=y(tq-i)-\boldsymbol{\varphi}^{\mathrm{T}}(tq-i)\hat{\boldsymbol{\theta}}(tq-i), \tag{13}$$

$$r(tq)=r(tq-q)+\|\boldsymbol{\varphi}(tq)\|^2,\ r(0)=1, \tag{14}$$

where $\|\boldsymbol{X}\|^2 := \mathrm{tr}[\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X}]$. Since the outputs $y(tq-i)$ $(i=1,2,\cdots,q-1)$ and the noise terms $v(tq-1)$, $v(tq-2)$, $\cdots$, $v(tq-p)$ are unknown, the SG algorithm is impossible to implement. The solution is using an auxiliary model to estimate these unknown variables.

The unknown missing outputs $y(tq-i)$ $(i=1,2,\cdots,q-1)$ and the unknown noise variables $v(tq-i)$ $(i=1,2,\cdots,p)$ are replaced with the outputs $\hat{y}(tq-i)$ and $\hat{v}(tq-i)$ of the following auxiliary model:

$$\begin{aligned}\hat{y}(tq-i) =& \hat{\boldsymbol{\varphi}}^{\mathrm{T}}(tq-i)\hat{\boldsymbol{\theta}}(tq-i)+\hat{v}(tq-i-1),\\ \hat{v}(tq-i) =& \hat{y}(tq-i)-\hat{\boldsymbol{\varphi}}^{\mathrm{T}}(tq-i)\hat{\boldsymbol{\theta}}(tq-i),\\ \hat{\boldsymbol{\varphi}}(tq-i) =& [u(tq-i-1),u(tq-i-2),\cdots,\\ & u(tq-i-p),\hat{v}(tq-i-1),\\ & \hat{v}(tq-i-2),\cdots,\hat{v}(tq-i-p)]^{\mathrm{T}},\end{aligned}$$

where $\hat{y}(tq-i)$ represents the estimate of $y(tq-i)$, $\hat{v}(tq-i)$ represents the estimate of $v(tq-i)$, and $\hat{\boldsymbol{\theta}}(tq-i)$ represents the estimate of $\boldsymbol{\theta}$ at time $tq-i$. Then it is easy to obtain the following auxiliary model based modified SG algorithm (the AM-M-SG algorithm for short):

$$\hat{\boldsymbol{\theta}}(tq) = \hat{\boldsymbol{\theta}}(tq-q)+\frac{\hat{\boldsymbol{\varphi}}(tq)}{r^{\epsilon}(tq)}e(tq), \frac{1}{2}<\epsilon\leq 1, \tag{15}$$

$$\hat{\boldsymbol{\theta}}(tq-i) = \hat{\boldsymbol{\theta}}(tq-q),\ i=1,2,\cdots,q-1, \tag{16}$$

$$e(tq) = y(tq)-\hat{\boldsymbol{\varphi}}^{\mathrm{T}}(tq)\hat{\boldsymbol{\theta}}(tq-q), \tag{17}$$

$$\begin{aligned}\hat{\boldsymbol{\varphi}}(tq-i+1) =& [u(tq-i),u(tq-i-1),\cdots,\\ & u(tq-i-p+1),\hat{v}(tq-i),\\ & \hat{v}(tq-i-1),\cdots,\\ & \hat{v}(tq-i-p+1)]^{\mathrm{T}},\end{aligned} \tag{18}$$

$$\hat{y}(tq-i) = \hat{\boldsymbol{\varphi}}^{\mathrm{T}}(tq-i)\hat{\boldsymbol{\theta}}(tq-i)+\hat{v}(tq-i-1), \tag{19}$$

$$\hat{v}(tq-i) = \hat{y}(tq-i)-\hat{\boldsymbol{\varphi}}^{\mathrm{T}}(tq-i)\hat{\boldsymbol{\theta}}(tq-i), \tag{20}$$

$$r(tq) = r(tq-q)+\|\hat{\boldsymbol{\varphi}}(tq)\|^2, r(0)=1. \tag{21}$$

The steps of computing the parameter estimate $\hat{\boldsymbol{\theta}}(tq)$ by the AM-M-SG algorithm are listed in the following:

1. Let $y(-j)=0, v(-j)=0, u(-j)=0$, $j=0,1,\cdots,p-1$ and give a small positive number ε and a positive number ϵ, $1/2<\epsilon\leq 1$.

2. Let $t=1$, $r(0)=1$ and $\hat{\boldsymbol{\theta}}(0)=\mathbf{1}/p_0$ with $\mathbf{1}$ being a column vector whose entries are all unity and $p_0=10^6$.

3. Collect the input data $u(tq)$, $u(tq-1)$, $\cdots$, $u(tq-p)$, and collect the output data $y(tq)$.

4. Let $i=q-1$ and compute $\hat{y}(tq-i)$ by Eq. (19).

5. Compute $\hat{v}(tq-i)$ by Eq. (20).

6. Form $\hat{\boldsymbol{\varphi}}(tq-i+1)$ by Eq. (18).

7. Decrease i by 1. If $i\geq 1$, go to step 4; otherwise, go to the next step.

8. Compute $e(tq)$ and $r(tq)$ by Eqs. (17) and (21), respectively.

9. Update the parameter estimation vector $\hat{\boldsymbol{\theta}}(tq)$ by Eq. (15).

10. Compare $\hat{\boldsymbol{\theta}}(tq)$ and $\hat{\boldsymbol{\theta}}(tq-q)$: if $\|\hat{\boldsymbol{\theta}}(tq)-\hat{\boldsymbol{\theta}}(tq-q)\|\leq\varepsilon$, then terminate the procedure and obtain the $\hat{\boldsymbol{\theta}}(tq)$; otherwise, increase t by 1 and go to step 3.

3 Determining the unknown parameters of the dual-rate Box-Jenkins model

In this section, we use the estimate $\hat{\boldsymbol{\theta}}(t)$ to determine the original parameters a_i, b_i, c_i, and d_i. Apply the estimate $\hat{\boldsymbol{\theta}}(t)$ to construct

$$\hat{G}'(z) := \hat{g}_1 z^{-1}+\hat{g}_2 z^{-2}+\cdots+\hat{g}_p z^{-p}, \tag{22}$$

$$\hat{Q}'(z) := 1+\hat{q}_1 z^{-1}+\hat{q}_2 z^{-2}+\cdots+\hat{q}_p z^{-p}. \tag{23}$$

Define the estimated polynomials

$$\begin{aligned}\hat{A}(z) :=& 1+\hat{a}_1 z^{-1}+\hat{a}_2 z^{-2}+\cdots+\hat{a}_n z^{-n},\\ \hat{B}(z) :=& \hat{b}_1 z^{-1}+\hat{b}_2 z^{-2}+\cdots+\hat{b}_n z^{-n},\\ \hat{C}(z) :=& 1+\hat{c}_1 z^{-1}+\hat{c}_2 z^{-2}+\cdots+\hat{c}_n z^{-n},\\ \hat{D}(z) :=& 1+\hat{d}_1 z^{-1}+\hat{d}_2 z^{-2}+\cdots+\hat{d}_n z^{-n}.\end{aligned}$$

Letting $\hat{B}(z)=\hat{A}(z)\hat{G}(z)$ and $\hat{D}(z)=\hat{C}(z)\hat{Q}(z)$, we have

$$\begin{aligned}\hat{b}_1 z^{-1}+&\hat{b}_2 z^{-2}+\cdots+\hat{b}_n z^{-n}\\ =&(1+\hat{a}_1 z^{-1}+\hat{a}_2 z^{-2}+\cdots+\hat{a}_n z^{-n})\\ &\cdot(\hat{g}_1 z^{-1}+\hat{g}_2 z^{-2}+\cdots+\hat{g}_p z^{-p}),\end{aligned} \tag{24}$$

$$\begin{aligned}1+\hat{d}_1 z^{-1}+&\hat{d}_2 z^{-2}+\cdots+\hat{d}_n z^{-n}\\ =&(1+\hat{c}_1 z^{-1}+\hat{c}_2 z^{-2}+\cdots+\hat{c}_n z^{-n})\\ &\cdot(1+\hat{q}_1 z^{-1}+\hat{q}_2 z^{-2}+\cdots+\hat{q}_p z^{-p}).\end{aligned} \tag{25}$$

Assuming $p \geq 2n$ and comparing the coefficients of z^{-i} of both sides of Eqs. (24) and (25) give

$$\begin{cases} z^{-1}: \hat{b}_1 = \hat{g}_1, \\ z^{-2}: \hat{b}_2 = \hat{g}_2 + \hat{a}_1\hat{g}_1, \\ z^{-3}: \hat{b}_3 = \hat{g}_3 + \hat{a}_1\hat{g}_2 + \hat{a}_2\hat{g}_1, \\ \quad\vdots \\ z^{-n}: \hat{b}_n = \hat{g}_n + \hat{a}_1\hat{g}_{n-1} + \cdots + \hat{a}_{n-1}\hat{g}_1, \\ z^{-n-j}: 0 = \hat{g}_j\hat{a}_n + \hat{g}_{j+1}\hat{a}_{n-1} + \cdots \\ \qquad + \hat{g}_{j+n-1}\hat{a}_1 + \hat{g}_{j+n}, \\ \qquad j = 1, 2, \cdots, n, \end{cases} \tag{26}$$

and

$$\begin{cases} z^{-1}: \hat{d}_1 = \hat{c}_1 + \hat{q}_1, \\ z^{-2}: \hat{d}_2 = \hat{c}_2 + \hat{q}_2 + \hat{c}_1\hat{q}_1, \\ z^{-3}: \hat{d}_3 = \hat{c}_3 + \hat{q}_3 + \hat{c}_1\hat{q}_2 + \hat{c}_2\hat{q}_1, \\ \quad\vdots \\ z^{-n}: \hat{d}_n = \hat{c}_n + \hat{q}_n + \hat{c}_1\hat{q}_{n-1} + \cdots \\ \qquad + \hat{c}_{n-2}\hat{q}_2 + \hat{c}_{n-1}\hat{q}_1, \\ z^{-n-j}: 0 = \hat{q}_j\hat{c}_n + \hat{q}_{j+1}\hat{c}_{n-1} + \cdots \\ \qquad + \hat{q}_{j+n-1}\hat{c}_1 + \hat{q}_{n+j}, \\ \qquad j = 1, 2, \cdots, n. \end{cases} \tag{27}$$

Let

$$\begin{aligned} \hat{\boldsymbol{a}} &:= [\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_n]^{\mathrm{T}} \in \mathbb{R}^n, \\ \hat{\boldsymbol{b}} &:= [\hat{b}_1, \hat{b}_2, \cdots, \hat{b}_n]^{\mathrm{T}} \in \mathbb{R}^n, \\ \hat{\boldsymbol{\gamma}}_a &:= [\hat{g}_1, \hat{g}_2, \cdots, \hat{g}_n]^{\mathrm{T}} \in \mathbb{R}^n, \\ \hat{\boldsymbol{\gamma}}_b &:= [\hat{g}_{n+1}, \hat{g}_{n+2}, \cdots, \hat{g}_{2n}]^{\mathrm{T}} \in \mathbb{R}^n, \\ \hat{\boldsymbol{M}}_1 &:= \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \hat{g}_1 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ \hat{g}_{n-1} & \hat{g}_{n-2} & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{n\times n}, \\ \hat{\boldsymbol{M}}_2 &:= \begin{bmatrix} \hat{g}_n & \hat{g}_{n-1} & \cdots & \hat{g}_1 \\ \hat{g}_{n+1} & \hat{g}_n & \cdots & \hat{g}_2 \\ \vdots & \vdots & & \vdots \\ \hat{g}_{2n-1} & \hat{g}_{2n-2} & \cdots & \hat{g}_n \end{bmatrix} \in \mathbb{R}^{n\times n}, \end{aligned}$$

and

$$\begin{aligned} \hat{\boldsymbol{c}} &:= [\hat{c}_1, \hat{c}_2, \cdots, \hat{c}_n]^{\mathrm{T}} \in \mathbb{R}^n, \\ \hat{\boldsymbol{d}} &:= [\hat{d}_1, \hat{d}_2, \cdots, \hat{d}_n]^{\mathrm{T}} \in \mathbb{R}^n, \\ \hat{\boldsymbol{\beta}}_a &:= [\hat{q}_1, \hat{q}_2, \cdots, \hat{q}_n]^{\mathrm{T}} \in \mathbb{R}^n, \\ \hat{\boldsymbol{\beta}}_b &:= [\hat{q}_{n+1}, \hat{q}_{n+2}, \cdots, \hat{q}_{2n}]^{\mathrm{T}} \in \mathbb{R}^n, \\ \hat{\boldsymbol{N}}_1 &:= \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \hat{q}_1 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ \hat{q}_{n-1} & \hat{q}_{n-2} & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{n\times n}, \\ \hat{\boldsymbol{N}}_2 &:= \begin{bmatrix} \hat{q}_n & \hat{q}_{n-1} & \cdots & \hat{q}_1 \\ \hat{q}_{n+1} & \hat{q}_n & \cdots & \hat{q}_2 \\ \vdots & \vdots & & \vdots \\ \hat{q}_{2n-1} & \hat{q}_{2n-2} & \cdots & \hat{q}_n \end{bmatrix} \in \mathbb{R}^{n\times n}. \end{aligned}$$

Eqs. (26) and (27) can be equivalently expressed as

$$\begin{cases} \hat{\boldsymbol{b}} = \hat{\boldsymbol{\gamma}}_a + \hat{\boldsymbol{M}}_1\hat{\boldsymbol{a}}, \\ \hat{\boldsymbol{\gamma}}_b = -\hat{\boldsymbol{M}}_2\hat{\boldsymbol{a}}, \end{cases} \tag{28}$$

and

$$\begin{cases} \hat{\boldsymbol{d}} = \hat{\boldsymbol{\beta}}_a + \hat{\boldsymbol{N}}_1\hat{\boldsymbol{c}}, \\ \hat{\boldsymbol{\beta}}_b = -\hat{\boldsymbol{N}}_2\hat{\boldsymbol{c}}. \end{cases} \tag{29}$$

Solving Eq. (28) gives the estimates of $\boldsymbol{a}$ and $\boldsymbol{b}$:

$$\begin{bmatrix} \hat{\boldsymbol{a}} \\ \hat{\boldsymbol{b}} \end{bmatrix} = \begin{bmatrix} -\hat{\boldsymbol{M}}_1 & \boldsymbol{I}_n \\ -\hat{\boldsymbol{M}}_2 & \boldsymbol{0} \end{bmatrix}^{-1} \begin{bmatrix} \hat{\boldsymbol{\gamma}}_a \\ \hat{\boldsymbol{\gamma}}_b \end{bmatrix}; \tag{30}$$

and solving Eq. (29) gives the estimates of $\boldsymbol{c}$ and $\boldsymbol{d}$:

$$\begin{bmatrix} \hat{\boldsymbol{c}} \\ \hat{\boldsymbol{d}} \end{bmatrix} = \begin{bmatrix} -\hat{\boldsymbol{N}}_1 & \boldsymbol{I}_n \\ -\hat{\boldsymbol{N}}_2 & \boldsymbol{0} \end{bmatrix}^{-1} \begin{bmatrix} \hat{\boldsymbol{\beta}}_a \\ \hat{\boldsymbol{\beta}}_b \end{bmatrix}. \tag{31}$$

4 Example

Consider the following Box-Jenkins model with an updating period $q = 2$:

$$\begin{aligned} y(t) =& \frac{B(z)}{A(z)}u(t) + \frac{D(z)}{C(z)}v(t), \\ A(z) =& 1 + a_1z^{-1} + a_2z^{-2} = 1 - 0.2z^{-1} + 0.1z^{-2}, \\ B(z) =& b_1z^{-1} + b_2z^{-2} = 0.2z^{-1} + 0.4z^{-2}, \\ C(z) =& 1 + c_1z^{-1} + c_2z^{-2} = 1 + 0.1z^{-1} - 0.2z^{-2}, \\ D(z) =& 1 + d_1z^{-1} + d_2z^{-2} = 1 + 0.12z^{-1} - 0.1z^{-2}, \end{aligned}$$

the input $\{u(t)\}$ is taken as a persistent excitation signal sequence with zero mean and unit variance, $\{v(t)\}$ is a white noise sequence with zero mean and variance $\sigma^2 = 0.20^2$, and

$$\begin{aligned} \frac{B(z)}{A(z)} \approx& G'(z) = 0.2z^{-1} + 0.44z^{-2} \\ & + 0.078z^{-3} - 0.0304z^{-4}, \\ \frac{D(z)}{C(z)} \approx& Q'(z) = 1 + 0.02z^{-1} + 0.098z^{-2} \\ & - 0.0058z^{-3} + 0.0202z^{-4}. \end{aligned}$$

The unknown parameters are as follows:

$$\begin{aligned} \boldsymbol{\theta} &= [g_1, g_2, g_3, g_4, q_1, q_2, q_3, q_4]^{\mathrm{T}} \\ &= [0.2, 0.44, 0.078, -0.0304, 0.02, \\ &\quad 0.098, -0.0058, 0.0202]^{\mathrm{T}}. \end{aligned}$$

Applying the AM-M-SG algorithm to estimate the parameters, the parameter estimates and their errors are as shown in Table 1, and the parameter estimation errors $\delta := \|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\|/\|\boldsymbol{\theta}\|$ versus t are as shown in Fig. 1.

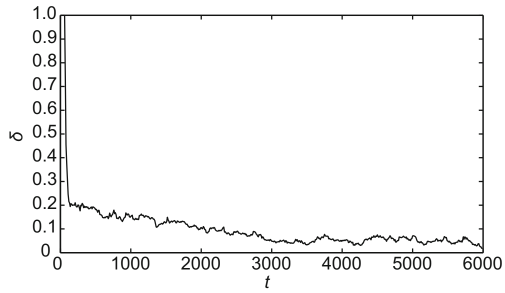

Fig. 1 The parameter estimation errors δ versus t

From Table 1 and Fig. 1, we can conclude that:

1. The parameter estimation errors become smaller and smaller and go to zero with the increase of t.

2. When $t = 6000$, by the estimated parameters g_1, g_2, g_3, g_4, q_1, q_2, q_3, q_4, the original parameters can be computed as: $a_1 = -0.2237$, $a_2 = 0.1174$, $b_1 = 0.1950$, $b_2 = 0.3973$, $c_1 = 0.0874$, $c_2 = -0.2116$, $d_1 = 0.1092$, $d_2 = -0.1360$.

5 Conclusions

In this paper we propose an auxiliary model based modified stochastic gradient algorithm for a dual-rate Box-Jenkins model. The auxiliary model can be used to estimate the missing outputs, and the modified SG algorithm can be used to improve the convergence rate; thus, we can use all the inputs and outputs to estimate the unknown parameters quickly using the proposed algorithm. Furthermore, this method can be extended to other dual-rate linear or nonlinear systems.

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Table 1 The AM-M-SG algorithm estimates and errors

Parameter	Estimate						True value
	t =1000	t =2000	t =3000	t =4000	t =5000	t =6000	
g_1	0.23036	0.21579	0.19026	0.20522	0.20101	0.19457	0.20000
g_2	0.44822	0.46967	0.44050	0.46150	0.46998	0.44271	0.44000
g_3	0.09757	0.06896	0.08727	0.07937	0.08830	0.08065	0.07800
g_4	−0.00835	−0.02477	−0.02783	−0.03478	−0.04088	−0.03308	−0.03040
q_1	0.01732	0.01989	0.02088	0.02107	0.02116	0.02109	0.02000
q_2	0.03910	0.05941	0.07918	0.08697	0.08715	0.09795	0.09800
q_3	−0.01018	−0.00544	−0.00444	−0.00417	−0.00265	−0.00312	−0.00580
q_4	−0.00129	0.01043	0.02711	0.02622	0.01922	0.02572	0.02020
δ (%)	15.22116	10.61235	4.85585	5.17451	7.04411	1.89306	

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