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Disturbance rejection via iterative learning control with a disturbance observer for active magnetic bearing systems

Key words: Active magnetic bearings (AMBs); Iterative learning control (ILC); Disturbance observer

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Motivation

1. For most modern robust control designs, the response of the system is not fast enough, and it is not easy for them to overcome severe plant uncertainties or disturbances because they cannot directly suppress system disturbances.

2. Disturbances of the actual AMB system exist periodically or have inherent characteristics, such as harmonics, which are not easily or promptly attenuated by traditional control designs.

3. For the AMB system, the variation in the perturbation caused by shaft vibration would be hard for ILC to deal with alone.

Main idea

1. We propose an AMB system with exogenous mismatched disturbance.

2. The framework of a traditional ILC is proposed and tracking performance is analysed based on the Laplace transform of the system.

3. A universal ESO is proposed to estimate and attenuate the mismatched disturbance of the AMB system.

4. A composite ILC design is constructed combined with the proposed universal ESO design. The novel ILC algorithm has the ability to attenuate iterative variant disturbances.

Tracking performance analysis

$$
\begin{cases} s\boldsymbol{X}_k(s) = \boldsymbol{A}\boldsymbol{X}_k(s) + \boldsymbol{B}_u U_k(s) + \boldsymbol{B}_d D_k(s), \\ Y_k(s) = \boldsymbol{C}\boldsymbol{X}_k(s), \end{cases}
$$

$$
\begin{aligned}
Y_k(s) =& \boldsymbol{C}\boldsymbol{X}_k(s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B}_u U_k(s) \\
& + \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B}_d U_k(s) \\
=& \Theta_1(s)U_k(s) + \Theta_2(s)D_k(s),
\end{aligned}
$$

Tracking error represented as:

$$
\begin{aligned}
E_k(s) =& Y_k(s) - Y_{k-1}(s) + E_{k-1}(s) \\
=& E_{k-1}(s) - (\Theta_1(s)U_{k-1}(s) + \Theta_2(s)D_{k-1}(s)) \\
& + (\Theta_1(s)U_k(s) + \Theta_2(s)D_k(s)) \\
=& E_{k-1}(s) + \Theta_1(s)(U_k(s) - U_{k-1}(s)) \\
& + \Theta_2(s)(D_k(s) - D_{k-1}(s)).
\end{aligned}
$$

Applied ILC control therefore

$$
\begin{aligned}
& ||e_k(t)||_2 \\
\leq & ||(1 + \Theta_1(s)P)^{-1}(1 - \Theta_1(s)Q)||_\infty ||e_{k-1}(t)||_2 \\
& + ||(1 + \Theta_1(s)P)^{-1}\Theta_2(s)||_\infty ||d_k(t) - d_{k-1}(t)||_2.
\end{aligned}
$$

Assume

$$
||(1 + \Theta_1(s))^{-1}\Theta_2(s)||_\infty ||d_k(t) - d_{k-1}(t)||_2 \leq S_1.
$$

Therefore

$$
\begin{aligned}
||e_k(t)||_2 & \leq R||e_{k-1}(t)||_2 + S_1 \\
& \leq R^2||e_{k-2}(t)||_2 + RS_1 + S_1 \\
& \leq R^3||e_{k-3}(t)||_2 + R^2S_1 + RS_1 + S_1 \\
& \vdots \\
& \leq R^k||e_0||_2 + S_1 \sum_{n=0}^{k-1} R^n.
\end{aligned}
$$

Theorem 1 For the designed ILC system, the tracking error converges asymptotically to a specific value if the learning parameters P and Q are set properly to ensure that $||(1 + \Theta_1(s)P)^{-1}(1 - \Theta_1(s)Q)||_\infty \leq 1$.

Generalised ESO-based ILC

A similar form of the composite disturbance controller can be written as

$$\bar{u}_k(t) = u_k(t) + T\hat{d}_k(t),$$

It can be derived that

$$\begin{aligned}
&||e_k(t)||_2 \le \\
&||(1+\Theta_1(s)P)^{-1}(1-\Theta_1(s)Q)||_\infty ||e_{k-1}(t)||_2 \\
&+ ||d_{k-1}(t) - d_k(t) - \hat{d}_{k-1}(t) + \hat{d}_k(t))||_2 \\
&\cdot ||(1+\Theta_1(s))^{-1}\Theta_2(s)||_\infty \\
&+ ||d_{k-1}(t) - d_k(t)||_2 \\
&\cdot ||(1+\Theta_1(s)P)^{-1}(T\Theta_1(s)+\Theta_2(s))||_\infty .
\end{aligned}$$

Assume

$$\begin{aligned}
&||d_{k-1}(t) - d_k(t) - \hat{d}_{k-1}(t) + \hat{d}_k(t))||_2 \\
&\cdot ||(1+\Theta_1(s))^{-1}\Theta_2(s)||_\infty + ||d_{k-1}(t) - d_k(t)||_2 \\
&\cdot ||I + \Theta_1(s)P^{-1}(T\Theta_1(s)+\Theta_2(s))||_\infty \le S_2 .
\end{aligned}$$

Disturbance rejection gain satisfy

$$T = \left(\boldsymbol{C}(s\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}_u\right)^{-1}\boldsymbol{C}(s\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}_d,$$

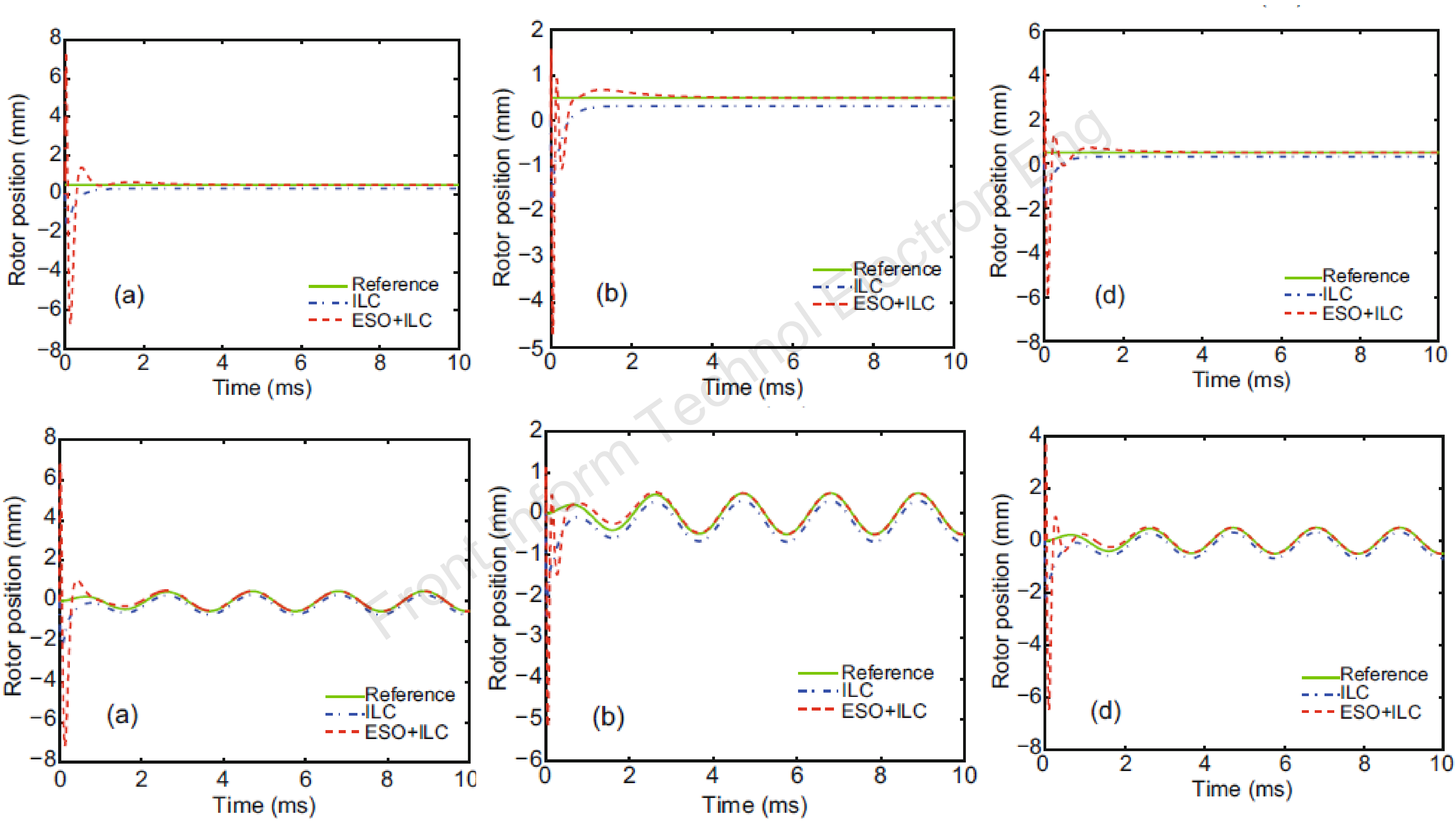
It can be derived that

$$||e_k(t)||_2 \le R^k ||e_0(t)||_2 + S_2 \sum_{n=0}^{k-1} R^n \le \frac{S_2 - S_2 R^{k-1}}{1-R}.$$

Remark 1 Although disturbances here can still be estimated, direct rejection is not applicable. Therefore, a generalized composite controller can be designed in the form $u_k^*(t) = u_k(t) + K_d\hat{\omega}$, and a new controller gain K_d is calculated to attenuate the unmatched disturbances.

Theorem 1 For the designed ILC system, the tracking error converges asymptotically to a specific value if the learning parameters P and Q are set properly to ensure that $||(1+\Theta_1(s)P)^{-1}(1-\Theta_1(s)Q)||_\infty \le 1$.

Simulation results



Tracking performance under step and harmonics reference

Conclusions

1. A linearized model of voltage controlled AMBs has been analysed under unmatched external disturbances.

2. An integrated control scheme was proposed that combines a universal extended state observer and Arimoto-ILC and solves the unmatched problem improving the iteration-variant disturbance rejection ability, compared with the standard ILC system.

3. Simulation results demonstrated that the proposed methods can achieve reference tracking while significantly attenuating the influence of unmatched-iteration-variant disturbances.