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A survey on complex dynamical networks with impulsive effects

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Desynchronizing impulses; Pinning control; Time delay

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Motivation

- 1. In the past decades, there has been much research on complex networks such as applications of complex networks, modeling and statistical analysis, dynamics and structures of complex dynamical networks (CDNs), and synchronization of CDNs.
- 2. As a kind of discontinuous control, impulsive control is often seen as an efficient and economic method for networks. Then the studies of CDNs with impulsive effects are very important for understanding the dynamics of most complex real-world networks.
- 3. Due to the fact that impulsive perturbation may affect the dynamical behaviors of CDNs, it is necessary to study the impulse effects on CDNs.

Introduction

- 1. The stability and ability to synchronize CDNs can be guaranteed by designing small impulses at some impulsive instants, which can increase the robustness and reduce information redundancy in the processing of transmitted signals.
- 2. If a given system is stable without impulsive effects, then it can still be stable even with impulsive interference, which is viewed as an impulsive perturbation issue.
- 3. Time delay should be considered and it is significant to investigate complex networks with time delay. Synchronization of neural networks with mixed delay was investigated, including time-varying delays and distributed delays.
- 4. Impulsive effects and stochastic effects exist in CDNs during signal transmission between dynamical nodes. Some results have been derived for synchronization of CDNs with stochastic and impulsive effects.

Method

- 1. Impulsive control methods are supplied to achieve synchronous, globally asymptotically synchronous, and even exponentially synchronous CDNs.
- 2. An adaptive-impulsive synchronization method for uncertain CDNs was proposed, which has the advantages of the adaptive method and impulsive method.
- 3. Because of the effectiveness of impulsive control, pinning synchronization for CDNs with impulsive control was considered. The pinning impulsive control method was developed on CDNs.

Major results

1. An example to illustrate the fact that impulsive perturbation may affect the dynamical behaviors of CDNs.

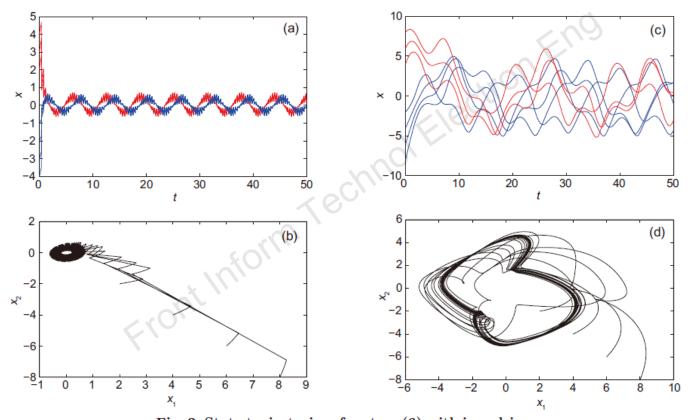
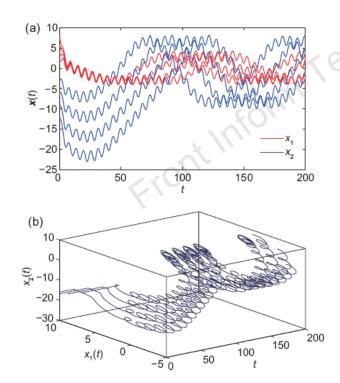


Fig. 2 State trajectories of system (6) with impulsive effect (a), phase portraits of 2π -periodic solutions of system (6) (b), state trajectories of system (6) without impulses (c), and phase portraits of system (6) without impulses (d). Reprinted from Li XD et al. (2015b), Copyright 2014, with permission from Elsevier Ltd.

2. A given CDN is synchronous or asynchronous when there are no impulsive effects, but it can become synchronous, asymptotically synchronous, and even exponentially synchronous by designing a proper impulsive controller.



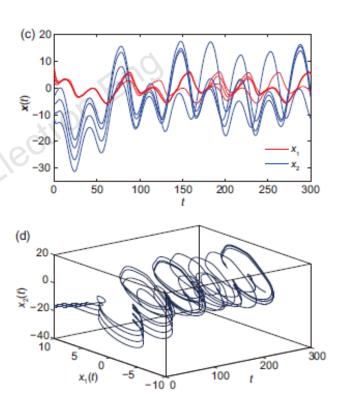


Fig. 4 State trajectories of system (16) with $\omega=6$ and without impulsive control (a), phase portraits of system (16) with $\omega=6$ and without impulsive control (b), state trajectories of system (16) with $\omega=35$ and without impulsive control (c), and phase portraits of system (16) with $\omega=35$ and without impulsive control (d). Reprinted from Li XD and Song (2013), Copyright 2013, with permission from IEEE

3. Sometimes, the impulsive perturbation may affect the dynamics of CDNs. For example, a stable equilibrium may become a periodic attractor under impulsive perturbation. In the following, an example is given to show this observation.

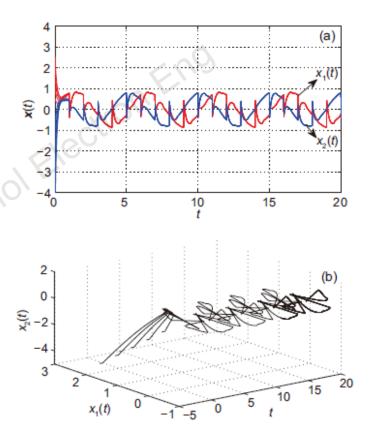


Fig. 7 State trajectories (a) and phase plots (b) of system (19) with $\omega = 5$. Reprinted from Li XD and Shen (2010), Copyright 2010, with permission from IEEE

4. In general, most of the early results on synchronization of CDNs consider a kind of network, in which the dynamics of every node and couplings are assumed to be known in advance. Some works have been derived on general uncertain CDNs, where the coupling strength of the network is unknown, and the topology structure of the network is partially known or completely unknown. Therefore, an adaptive-impulsive synchronization method for uncertain CDNs was proposed, which has the advantages of the adaptive method and impulsive method.

5. A strategy of pinning control was introduced, where only a few network nodes were directly controlled. Because of the effectiveness of impulsive control, pinning synchronization for CDNs with impulsive control was considered in many works. Pinning impulsive synchronization of CDNs with various delays was considered. Some results have been derived for synchronization of CDNs with stochastic and impulsive effects. Robust exponential stability of uncertain impulsive stochastic neural networks with delayed impulses was discussed.

Conclusions

- 1. Several definitions of synchronization of CDNs have been introduced. Moreover, we have described the general input-to-state stability, in which impulse effects can be classified as impulsive synchronization and impulsive desynchronization.
- 2. Some sufficient conditions have been established for CDNs based on the impulsive control theory. An example has been given to illustrate the existence and attractiveness of the periodic solution for neural networks with impulsive control.
- 3. The impulsive desynchronization problem for CDNs has been discussed. Some sufficient conditions are presented. From the example, it can be seen that a stable equilibrium becomes a periodic attractor under impulsive perturbation.
- 4. In view of the results obtained on impulsive effects in the last decades, the adaptive-impulsive synchronization of CDNs, pinning control of CDNs with impulsive effects, and CDNs with stochastic and impulsive effects are discussed.