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An improved subspace weighting method using random matrix theory

Key words: Direction of arrival; Signal subspace; Random matrix theory

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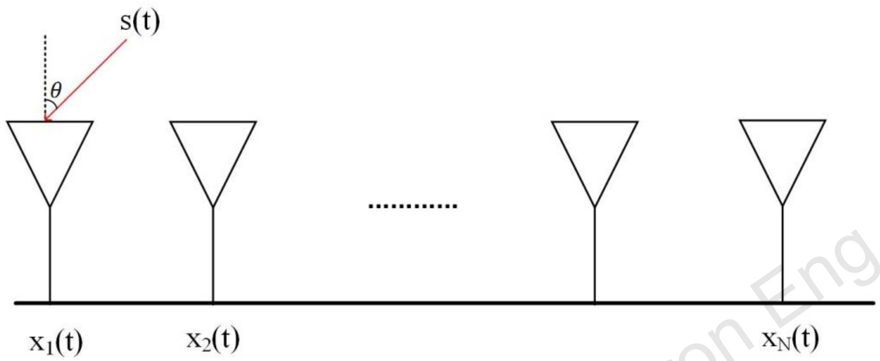
Motivation

For correlated signals and uncorrelated signals, the current algorithms have poor performance in estimating the direction of arrival, especially in the case of low signal-to-noise ratios and few snapshots.

We use the random matrix theory to improve those methods.

Method



Considering a linear array with *M* sensors receiving *N* narrow-band signals from the far field, the *M*-dimensional output ***x*** of the array is modeled as

$$\boldsymbol{x} = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s} + \boldsymbol{n}$$

The eigen structure of array covariance ***R*** can be written as

$$\boldsymbol{R} = \boldsymbol{U}_{\mathrm{s}}\boldsymbol{\Lambda}_{\mathrm{s}}\boldsymbol{U}_{\mathrm{s}}^{\mathrm{H}} + \sigma^2\boldsymbol{U}_{\mathrm{n}}\boldsymbol{U}_{\mathrm{n}}^{\mathrm{H}}$$

Normally, ***R*** is obtained through the sample covariance $\hat{\boldsymbol{R}}$, which is calculated as

$$\hat{\boldsymbol{R}} = \frac{1}{L}\sum_{t=1}^{L}\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)$$

Method (Cont'd)

As $M,\ L \to \infty$ and $M/L \to c \in (0, \infty)$, the equation below is obtained from RMT:

$$\hat{U}_{\mathrm{s}}^{\mathrm{H}} U_{\mathrm{s}} U_{\mathrm{s}}^{\mathrm{H}} \hat{U}_{\mathrm{s}} \overset{\text{a.s.}}{\to} \operatorname{diag}\left(|\langle \boldsymbol{u}_1,\ \hat{\boldsymbol{u}}_1 \rangle|^2,\right.$$
$$\left.|\langle \boldsymbol{u}_2,\ \hat{\boldsymbol{u}}_2 \rangle|^2,\ \ldots,\ |\langle \boldsymbol{u}_N,\ \hat{\boldsymbol{u}}_N \rangle|^2\right),$$

where $|\langle \boldsymbol{u}_i,\ \hat{\boldsymbol{u}}_i \rangle|^2$ can be calculated by

$$|\langle \boldsymbol{u}_i,\ \hat{\boldsymbol{u}}_i \rangle|^2$$
$$\overset{\text{a.s.}}{\to} \alpha_i^2 = \begin{cases} 1 - \dfrac{c\,(1+\lambda_i)}{\lambda_i\,(\lambda_i + c)}, & \lambda_i > c^{1/2} \\ 0, & \text{otherwise} \end{cases}$$

From

$$\hat{U}_{\mathrm{s}}^{\mathrm{H}} U_{\mathrm{n}} U_{\mathrm{n}}^{\mathrm{H}} \hat{U}_{\mathrm{s}} + \hat{U}_{\mathrm{s}}^{\mathrm{H}} U_{\mathrm{s}} U_{\mathrm{s}}^{\mathrm{H}} \hat{U}_{\mathrm{s}} = \boldsymbol{I},$$

we have

$$\hat{U}_{\mathrm{s}}^{\mathrm{H}} U_{\mathrm{n}} U_{\mathrm{n}}^{\mathrm{H}} \hat{U}_{\mathrm{s}} \overset{\text{a.s.}}{\to} \operatorname{diag}\left(1-\alpha_1^2,\ 1-\alpha_2^2,\ \ldots,\ 1-\alpha_N^2\right).$$

Method (Cont'd)

A certain column of $\hat{U}_\mathrm{s}$ is $\hat{u}_i$, which is Gaussian independent and identically distributed (IID). Let $\hat{c}_i(\theta) = U_\mathrm{n}^\mathrm{H}\hat{u}_i$; the mean and variance are easily given by

$$E\left(\hat{c}_i\right) \approx \mathbf{0},$$
$$E\left(\hat{c}_i\hat{c}_i^\mathrm{H}\right) \approx \frac{1-\alpha_i^2}{M-N} I_{M-N}.$$

The maximum of the log likelihood function that obtains the argument $\boldsymbol{\theta}$ can be written as

$$\begin{aligned}&\max_{\boldsymbol{\theta}} l\left(\hat{c}_1,\ \hat{c}_2,\ \ldots,\ \hat{c}_N|\theta\right)\\ =&\min_{\boldsymbol{\theta}} \sum_{i=1}^{N} \hat{c}_i^\mathrm{H}\left(\frac{1-\alpha_i^2}{M-N}\right)^{-1}\hat{c}_i\\ =&\max_{\boldsymbol{\theta}} \mathrm{tr}\left\{P_A\hat{U}_\mathrm{s}W_\mathrm{RMT}\hat{U}_\mathrm{s}^\mathrm{H}\right\}\end{aligned}$$

Then the weighting matrix is

$$\begin{aligned}W_\mathrm{RMT} &= \mathrm{diag}(w_1,\ w_2,\ \ldots,\ w_N)\\ &= \mathrm{diag}\left(\frac{1}{1-\alpha_1^2},\ \frac{1}{1-\alpha_2^2},\ \ldots,\ \frac{1}{1-\alpha_N^2}\right).\end{aligned}$$

Major results

RMT achieves better performance at a low SNR and with few snapshots.

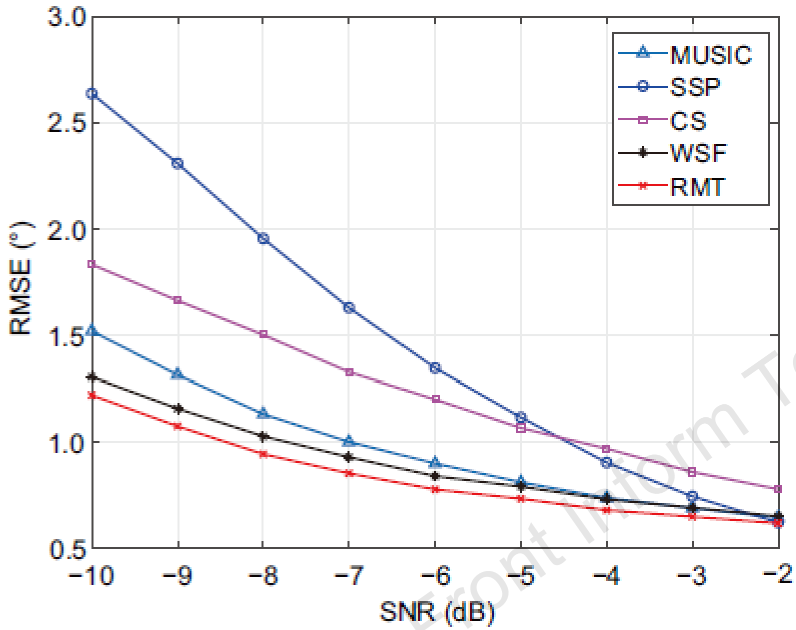


Fig. 3 RMSE versus SNR varying from −10 to −2 dB under correlated signals with coefficient 0.8 (number of snapshots: 10)

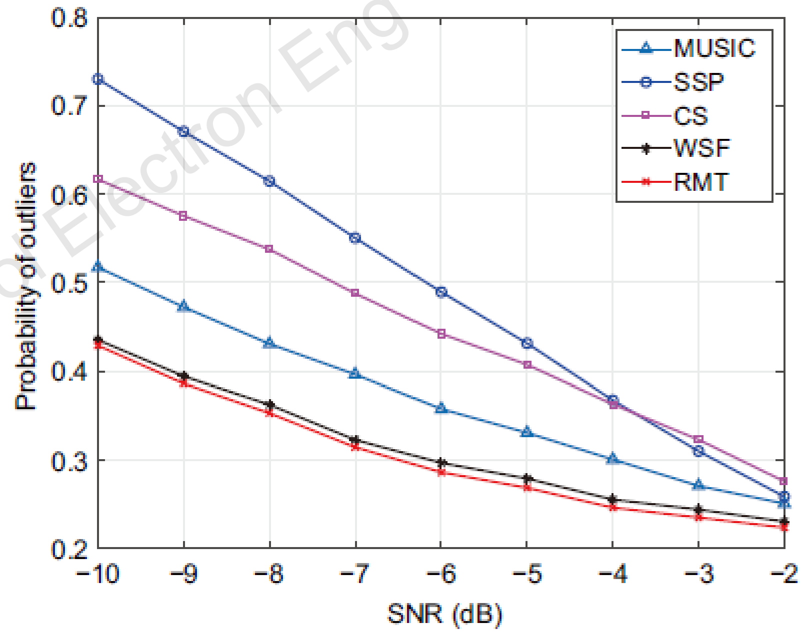


Fig. 5 Probability of outliers versus SNR varying from −10 to −2 dB under correlated signals with coefficient 0.8 (number of snapshots: 10)

Major results (Cont'd)

RMT achieves better performance at a low SNR and with few snapshots.

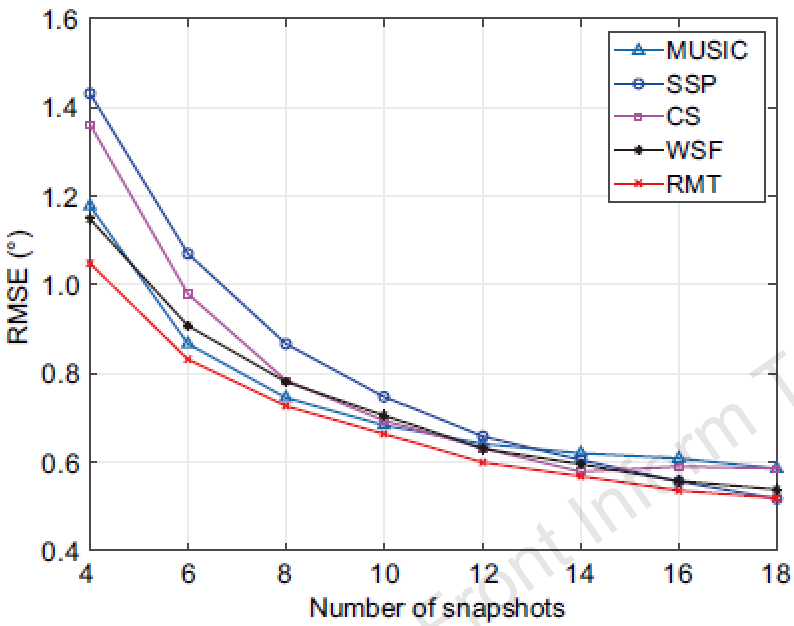


Fig. 4 RMSE versus the number of snapshots varying from 4 to 18 under correlated signals with coefficient 0.8 (SNR: −3 dB)

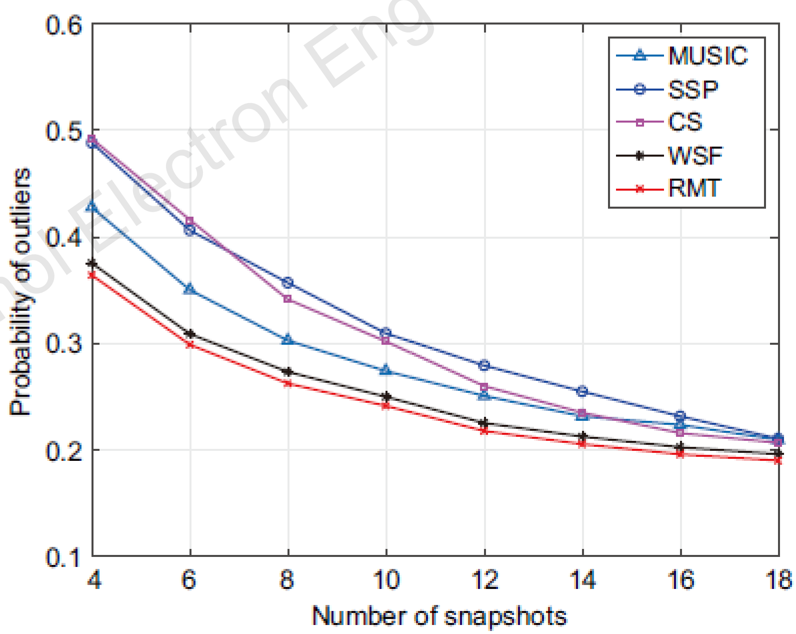


Fig. 6 Probability of outliers versus the number of snapshots varying from 4 to 18 under correlated signals with coefficient 0.8 (SNR: −3 dB)

Major results (Cont'd)

RMT achieves better performance at a low SNR and with few snapshots.

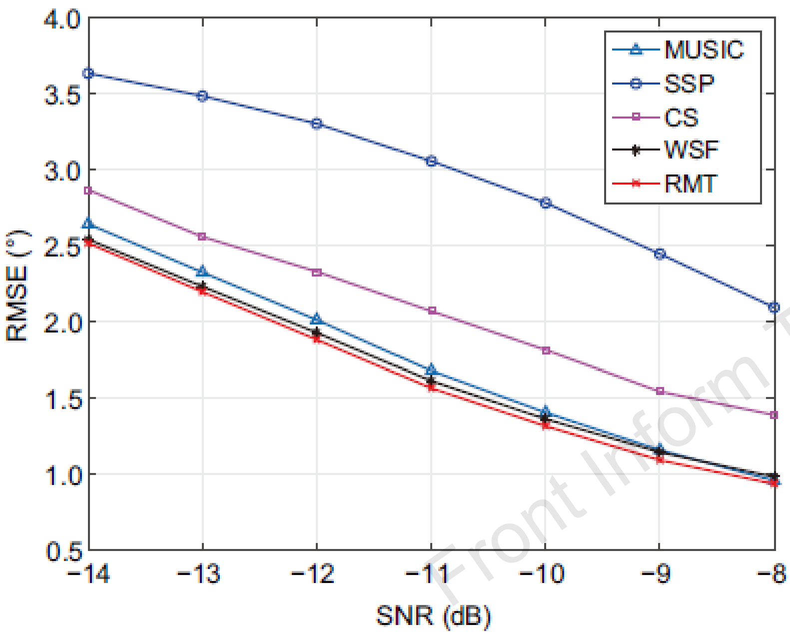


Fig. 7 RMSE versus SNR varying from −14 to −8 dB with uncorrelated signals (number of snapshots: 10)

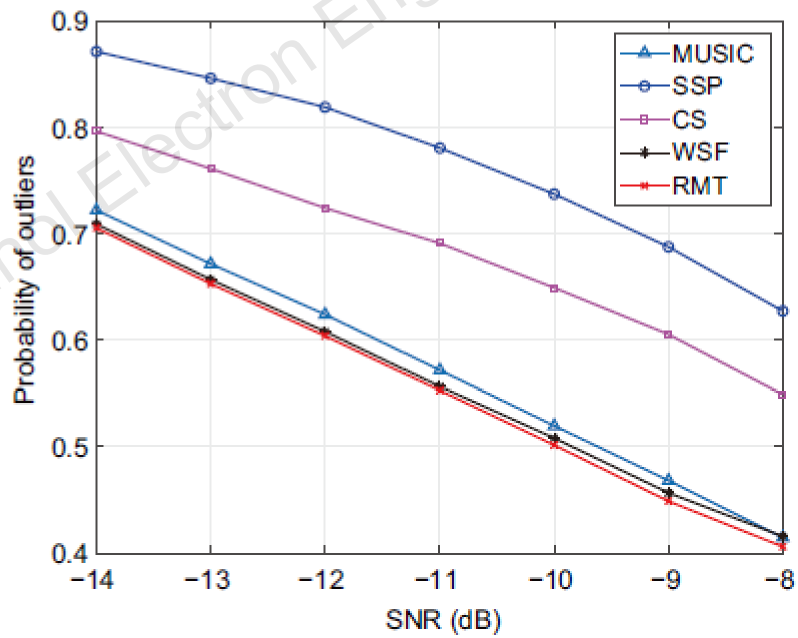


Fig. 8 Probability of outliers versus SNR varying from −14 to −8 dB with uncorrelated signals (number of snapshots: 10)

Conclusions

- We have presented a new method of DOA estimation for narrow-band signals.
- The weighting matrix is calculated to achieve a lower RMSE.
- The performance of RMT is better than those of MUSIC, SSP, CS, and WSF under the same scenario with few snapshots and at a low SNR, which was verified through numerical simulations.

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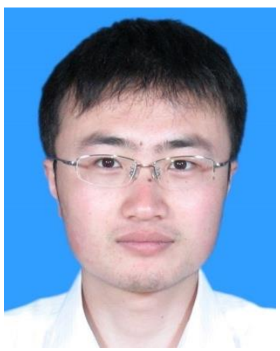

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