# Synchronization stability between initial-dependent oscillators with periodical and chaotic oscillation

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Initial-dependent dynamical system

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = -y - z, \\ \frac{\mathrm{d}y}{\mathrm{d}t} = x + ay - kz^2 y, \quad (1) \\ \frac{\mathrm{d}z}{\mathrm{d}t} = b + z(x - c), \end{cases}$$

**Mechanism and characteristics**: Slight difference in setting initial value for variable z can induce distinct difference in dynamics, oscillation transition between periodic and chaotic states. Any changes in initial value for variable z can generate distinct parameter modulation in the nonlinear term  $z^2y$  and the nonlinear modulation is enhanced.



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#### Synchronization stability and approach

$$\begin{cases} \frac{\mathrm{d}x_{i}}{\mathrm{d}t} = -y_{i} - z_{i}, \\ \frac{\mathrm{d}y_{i}}{\mathrm{d}t} = x_{i} + ay_{i} - k_{i}z_{i}^{2}y_{i} + g_{y}(y_{i+1} + y_{i-1} - 2y_{i}), \\ \frac{\mathrm{d}z_{i}}{\mathrm{d}t} = b + z_{i}(x_{i} - c), \end{cases}$$

$$(2)$$

#### Synchronization factor R

$$R = \frac{\langle F^2 \rangle - \langle F \rangle^2}{\frac{1}{N} \sum_{i=1}^{N} \langle x_i^2 \rangle - \langle x_i \rangle^2}, \quad F = \frac{1}{N} \sum_{i=1}^{N} x_i, \quad (3)$$

Where *i* is the subscript for oscillator in the network,  $g_y$  is the coupling intensity, R is the factor of synchronization. Perfect synchronization is stabilized at  $R \sim 1$ , while non-perfect synchronization is reached at lower factor  $R \sim 0$ .



### Conclusions

- The synchronization approach is dependent on the initial setting for variable *z*.
- Two different oscillators are activated when different initials are applied for the variable z.
- Two periodical oscillators can be coupled to reach chaotic synchronization.
- Chaotic oscillator can be suppressed when it is coupled with another periodical oscillator.
- Diversity and difference in the initial setting for variable z of oscillators can generate heterogeneity in the network, and the synchronization approach becomes difficult.
- This network model can be further used for pattern selection and image encryption.

