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Stochastic gradient algorithm for a dual-rate Box-Jenkins model based on auxiliary model and FIR model

基于辅助模型和有限脉冲响应模型的双率Box-Jenkins系统随机梯度辨识算法

Key words: Parameter estimation, Auxiliary model, Dual-rate system, Stochastic gradient, Box-Jenkins model, FIR model

关键词：参数估计，辅助模型，双率系统，随机梯度，Box-Jenkins模型

The following auxiliary model based modified SG algorithm is used to estimate the unknown parameters:

$$\hat{\theta}(tq)=\hat{\theta}(tq-q)+\frac{\hat{\varphi}(tq)}{r^{\varepsilon}(tq)}e(tq),\ \frac{1}{2}<\varepsilon\le 1,$$

$$\hat{\theta}(tq-i)=\hat{\theta}(tq-q),\ i=1,2,\cdots,q-1,$$

$$e(tq)=y(tq)-\hat{\varphi}^{T}(tq)\hat{\theta}(tq-q),$$

$$\hat{\varphi}(tq-i+1)=[u(tq-i),u(tq-i-1),\cdots,u(tq-i-p+1),\hat{v}(tq-i),$$
$$\hat{v}(tq-i-1),\cdots,\hat{v}(tq-i-p+1)]^{T},$$

$$\hat{y}(tq-i)=\hat{\varphi}^{T}(tq-i)\hat{\theta}(tq-i)+\hat{v}(tq-i-1),$$

$$\hat{v}(tq-i)=\hat{y}(tq-i)-\hat{\varphi}^{T}(tq-i)\hat{\theta}(tq-i),$$

$$r(tq)=r(tq-q)+\|\hat{\varphi}(tq)\|^{2},\ r(0)=1$$

The contributions of this paper are as follows:

1. Simplify the complex dual-rate Box-Jenkins model to two finite impulse response (FIR) models.

2. Present an auxiliary model to estimate the missing outputs and the unknown noise variables.

3. Compute all the unknown parameters of the system with colored noises.

4. In this method, the number of the parameters to be estimated is less than the number of the parameters in the polynomial transformation technique.

Example:

Consider the Box-Jenkins model with an updating period q = 2. The unknown parameters are as follows:

$$\theta = [g_{1,} g_2, g_3, g_4, q_1, q_2, q_3, q_4]^T$$
$$= [0.2, 0.44, 0.078, -0.0304, 0.02, 0.098, -0.0058, 0.0202]^T.$$

Table 1 The AM-M-SG algorithm estimates and errors

Parameter	Estimate						True value
	t =1000	t =2000	t =3000	t =4000	t =5000	t =6000	
g_1	0.23036	0.21579	0.19026	0.20522	0.20101	0.19457	0.20000
g_2	0.44822	0.46967	0.44050	0.46150	0.46998	0.44271	0.44000
g_3	0.09757	0.06896	0.08727	0.07937	0.08830	0.08065	0.07800
g_4	−0.00835	−0.02477	−0.02783	−0.03478	−0.04088	−0.03308	−0.03040
q_1	0.01732	0.01989	0.02088	0.02107	0.02116	0.02109	0.02000
q_2	0.03910	0.05941	0.07918	0.08697	0.08715	0.09795	0.09800
q_3	−0.01018	−0.00544	−0.00444	−0.00417	−0.00265	−0.00312	−0.00580
q_4	−0.00129	0.01043	0.02711	0.02622	0.01922	0.02572	0.02020
δ (%)	15.22116	10.61235	4.85585	5.17451	7.04411	1.89306	

When t = 6000, by the estimated parameters, we can compute the original parameters.