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Performance of a single-stage auto-cascade refrigerator operating with a rectifying column at the temperature level of −60 °C*

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Abstract: This paper proposes a new approach to the performance optimization of an auto-cascade refrigerator (ACR) operating with a rectifying column and six types of binary refrigerants (R23/R134a, R23/R227ea, R23/R236fa, R170/R290, R170/R600a, and R170/R600) at a temperature level of −60 °C. Half of the six binary refrigerants are nonflammable, of which the 0.5 and the 0.6 mole fractions of R23 for the R23/R236fa possess the most prospective composition for the medium and low suction pressure compressors, respectively. The remaining three binary refrigerants are flammable but with low global warming potentials, of which the 0.6 mole fraction of R170 for the R170/R600 is the most prospective one. The results show that the overall matching as well as local matching of heat capacity rates of hot and cold refrigerants in the recuperators are important for the improvement of coefficient of performance of the cycle, which can be adjusted through the simultaneous optimization of the pressure level and composition. The new approach proposed also offers a wider range of applications to the optimization in performance of the cycle using multi-component refrigerants.

Key words: Auto-cascade, Binary refrigerant, Refrigerator, Optimization
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1 Introduction

The auto-cascade refrigerator (ACR) originates from the traditional cascade cycle and was first described by Podbielniak (1936). In the 1950s, the ACR was successfully applied in the natural gas liquefaction industry by employing a cycle proposed by Kleemenko (1959). The ACR can, therefore, also be referred to as a Kleemenko refrigerator sometimes.

The early ACR employed one or two phase separators to divide the components of the non-azeotropic mixed refrigerants (Podbielniak, 1936; Kleemenko, 1959). The oil contained in the circulating refrigerant cannot be effectively removed by the separators without avoiding a plug in the expansion valve when the temperature is lower than the freezing point of the oil. Missimer (1973) proposed a single-stage ACR employing more than three separators, which could remove the oil effectively, but also complicated the system.

Little (1997) studied a single-stage ACR employing one phase separator, on the top of which a fractionator was mounted to enhance the separation of the oil and contaminants. Chen (2000) proposed the use of a rectifying column to displace the phase separator in the single-stage ACR. This column possessed an oil removal effect similar to that of the multi-phase separators. Due to the simple structure of the single-stage ACR operating in tandem with the rectifying column, the reliability of the ACR is greatly improved, and a no-failure life is considerably extended.

For applications such as low temperature freezers, the reliability of the ACR is extremely important due to the considerable expense generated if the system breaks down accidentally. Compared to

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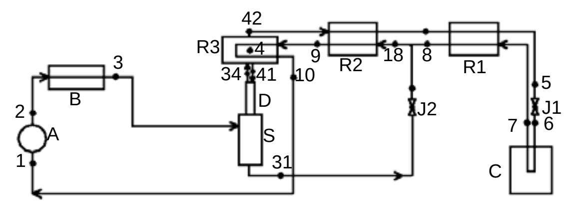
multi-component refrigerants, binary refrigerants offer more convenience in the design, production, and maintenance of the refrigerator (especially during a partial recharge of the system following a leak). The single-stage ACR operating with a rectifying column and a binary refrigerant, therefore, holds promising application prospects in those areas where a long no-failure ACR life is required to provide a −60 °C environment, such as within the food, medical, and life science industries of present day.

However, reports on the performance of the ACR at temperatures levels less than −60 °C are few (Wang and Chen, 2003; Rozhentsev and Naer, 2009; Wang *et al.*, 2010; Zhang *et al.*, 2010). This paper proposed a new empirically-based approach to optimize the performance of a single-stage ACR operating with six pairs of binary refrigerants at the temperature level of −60 °C. The results obtained will be an important guide for future research applications.

2 Analyses

2.1 Cycle description

Fig. 1 shows the flow chart of a single-stage ACR operating with a rectifying column. The non-azeotropic Refrigerant 1 discharged from the compressor partially condenses in the aftercooler, and then flows into the vapor-liquid separator at the bottom of the rectifying column, where the vapor and liquid phases of Refrigerant 1 are separated.



A: compressor; B: aftercooler; C: evaporator; D: rectifying column; J1, J2: expansion valve; R1–R3: recuperator; S: vapor-liquid separator; all the figures are state points

Fig. 1 Flow chart of an auto-cascade refrigerator operating with a rectifying column

The gaseous phase of Refrigerant 1, which is rich in the volatile component, goes through the rectifying column, where the less volatile component in the vapor is separated and returned to the bottom of the rectifying column by a continuous return flow. After the vapor flows out from the top of the rectifying column (i.e., point 34), it passes through Recuperator 3 mounted on the top of the column. Here the vapor is cooled and enters the two-phase state (i.e., point 4). The liquid refrigerant flows back to the top of rectifying column at point 41, providing the coldest return flows, while the vapor (rich in the volatile component) flows out from the top of Recuperator 3. This vapor can be assigned the label Refrigerant 2.

Refrigerant 2 flows into Recuperators 2 and 1 in succession and partially condenses before reaching the expansion valve J1. After pressure drops in expansion valve J1, Refrigerant 2 partially evaporates in the evaporator, thereby halting evaporation in the two-phase state due to the wide temperature difference between the dew point and bubble point of the refrigerant. Refrigerant 2 flows into Recuperator 1 to cool itself a second time on the high pressure side of Recuperator 1.

The liquid phase of Refrigerant 1, rich in less volatile component, can be labeled Refrigerant 3, and flows out from the bottom of the vapor-liquid separator. After pressure drops in expansion valve J2, Refrigerant 3 mixes with Refrigerant 2 that originates from Recuperator 1, to compose Refrigerant 1 again. Refrigerant 1 flows into the low pressure side of Recuperator 2 to cool Refrigerant 2, which is located in the high pressure side, and then flows from Recuperator 2 into Recuperator 3 to provide cold capacity for the rectifying column. After leaving Recuperator 3, Refrigerant 1 flows back to the compressor.

2.2 Simulation model

The discharge and suction pressures, P_2 and P_1, of the compressor represent two independent design variables in a refrigerator using non-azeotropic refrigerant. Because the pressure ratio, $P_r=P_2/P_1$, has a strong influence on the performance of a single-stage compressor, it is proposed as one of the most important design variables in this study. Analyses will be conducted based on the following assumptions:

1. The compositions of the binary Refrigerants 1 and 2 are specified.

2. The temperature of heat sink T_H and heat source T_L are specified and isothermal.

3. P_r is specified. There is an absence of pressure loss in both the aftercooler and the high pressure sides of the recuperators. The pressure loss in the

evaporator is specified as ΔP_E. The pressure losses in the low pressure sides of Recuperators R1, R2, and R3 are specified as ΔP_{R1}, ΔP_{R2}, and ΔP_{R3}.

4. The minimum temperature differences in the aftercooler and evaporator are specified as $\Delta T_{A,min}$ and $\Delta T_{E,min}$, which occur at the cold end of the aftercooler and the hot end of the evaporator, respectively.

5. The minimum temperature differences in Recuperators R1, R2, and R3 are specified as $\Delta T_{R1,min}$, $\Delta T_{R2,min}$ and $\Delta T_{R3,min}$. These temperature differences occur at the hot or cold end of the recuperator. No heat loss occurs in the recuperator.

6. The total efficiency of the compressor ξ_C is specified. The suction temperature is equal to that of the heat sink.

7. The throttling process is isenthalpic.

8. The rectifying column is fed by the vapor-liquid separator, which is located underneath the column. The temperature of the vapor-liquid separator is equal to the heat sink.

From common thermodynamic knowledge, the coefficient of performance (COP) can be written as

$$\mathrm{COP} = q_E / w_p = f_m (h_7 - h_6) / (h_2 - h_1), \tag{1}$$

where q_E is the specific refrigerating effect, J/mol; w_p is the specific work of polytropic compression, J/mol; h is the specific enthalpy of the refrigerant, J/mol, the subscriptions 1, 2, 6, and 7 are the state points as shown in Fig. 1; and f_m is the flow rate ratio between Refrigerants 2 and 1.

From assumptions 6 and 7, it is known that

$$\mathrm{COP} = f_m \xi_C (h_7 - h_5) / (h_{2s} - h_1), \tag{2}$$

where 2s denotes the final point of the isentropic compression.

From the mass balance of the rectifying column, f_m can be calculated as

$$f_m = (z_F - z_W) / (z_R - z_W), \tag{3}$$

where z_F, z_R, and z_W are the mole fractions of the volatile components of binary Refrigerants 1, 2, and 3, respectively. From the assumptions 1–3 and 8, it is observed that f_m varies with the suction pressure P_1.

From the assumptions 2–6, it can be seen that

$$T_1 = T_H, \tag{4}$$

$$P_{2s} = P_5 = P_H = P_1 P_r, \tag{5}$$

$$P_7 = P_1 + \Delta P_{R1} + \Delta P_{R2} + \Delta P_{R3}, \tag{6}$$

$$T_5 = T_7 + \Delta T_{R1,min}, \tag{7}$$

$$T_7 = T_L - \Delta T_{E,min}, \tag{8}$$

where P_H is the high pressure of the cycle, kPa.

Combining Eqs. (1) and (2), it can be found that COP varies with the suction pressure P_1 alone.

Based on many calculations for different binary refrigerants and temperature levels, it was found that the maximum COP occurs at the pressure level whereby the temperature differences at the cold end of Recuperator 1 and the hot end of Recuperator 3 are approximately equal to the minimum temperature of the two recuperators combined (i.e., $\Delta T_{R1,cold}=T_5-T_7=\Delta T_{R1,min}$, and $\Delta T_{R3,hot}=T_{34}-T_{10}=\Delta T_{R3,min}$) (Wang and Chen, 2003). This indicates that the heat capacity rates of the high and low pressure refrigerants are matched overall in Recuperators R1, R2, and R3.

This characteristic, therefore, simplifies the formula used to optimize an ACR operating with a rectifying column. For the given mixture composition and operating pressure ratio, the properties at four state points 34, 10, 5, and 7 only need to be calculated, and comparisons of $\Delta T_{R1,cold}$ with $\Delta T_{R1,min}$, and $\Delta T_{R3,hot}$ with $\Delta T_{R3,min}$ need to be drawn to obtain the optimum pressure level. Once the optimum pressure level is determined, what remains is a simple calculation of the mixture properties at all points and the cycle's performance. Computing time can, therefore, be significantly reduced.

When the suction pressure P_1 is given, properties at the state point 34 can be determined from the design parameters of the rectifying column as follows. Because the composition of Refrigerant 2, z_R, has been specified and $z_{42}=z_R$, while $P_{42}=P_H$, the temperature of state point 42, T_{42}, can be fixed by the vapor-liquid equilibrium of the saturated mixture at point 42. From the following equations, the quality of point 4 can be determined:

$$r_{min}=(z_R-z_{Fd})/(z_{Fd}-z_{Fb}), \tag{9}$$

$$r=d_r r_{min}, \tag{10}$$

$$q_4=1/(1+r), \tag{11}$$

where z_{Fd} and z_{Fb} are the dew point mole fraction and bubble point mole fraction of the volatile component of binary Refrigerant 1, respectively; r and r_{min} are the reflux ratio and minimum reflux ratio in the rectifying column, respectively; d_r is a parameter determined by experience; and q is the quality of the refrigerant. Moreover, combining $T_4=T_{42}$, and $P_4=P_H$, the composition of point 4, z_4, can be fixed by the vapor-liquid equilibrium of the mixture in Recuperator 3. Finally, for $z_{34}=z_4$, $P_{34}=P_H$, and $q_{34}=1$, the temperature of state point 34, T_{34}, can be fixed by the vapor-liquid equilibrium of the mixture at point 34.

Based on the simulation model mentioned above, the compositions of the binary Refrigerant 1 and the pressure ratio can be varied to locate the optimum pressure ratio and composition. For the binary refrigerant, only the mole fraction of one component is independent. The optimization steps for each binary refrigerant are as follows:

1. The pressure ratio is specified as 4, 7, and 10, respectively.

2. The mole fraction of one component is specified as falling within 0.2–0.6 at an interval of 0.2, respectively.

3. The optimum pressure level is obtained through the use of the simulation model described above.

In this way, the optimum composition and optimum pressure ratio can be obtained based on the corresponding optimum pressure level for each binary refrigerant. Also obtained is a regular pattern of COP.

3 Simulations

The signing of the 1987 Montreal Protocol made it important to replace chlorofluorocarbons (CFCs) with alternatives that possess an ozone depletion potential (ODP) of zero (Missimer, 1997) in the old systems and use mixed refrigerants with an ODP of zero in the new systems. Nonflammable mixed refrigerants are also of significant concern in many applications (Khatri and Boiarski, 2008). To date, it is difficult to find, however, these 'ideal' nonflammable mixed refrigerants bearing an ODP of zero and a low global warming potential (GWP) with high efficiency in the ACR.

Two groups of binary refrigerants were selected from eight pure refrigerants: (a) R23/R134a, R23/R227ea, R23/R236fa; (b) R170/R290, R170/R600a, R170/R600. The physical and environmental data of their components is listed in Table 1 (Desmarteau and Beyerlein, 1996; Calm and Hourahan, 2007).

Table 1 Physical and environmental data of eight pure refrigerants (Desmarteau and Beyerlein, 1996; Calm and Hourahan, 2007)

Pure refrigerant	NBP (°C)	LFL (%, v/v)	ODP	GWP
R23	−82.1	None	0	12000
R134a	−26.1	None	0	1300
R227ea	−15.6	None	0	3500
R236fa	−1.4	None	0	9800
R170	−88.9	2.9	0	~20
R290	−42.2	2.1	0	~20
R600a	−11.7	1.7	0	~20
R600	−0.5	1.5	0	~20

NBP: normal boiling point; LFL: lower flammability limit; ODP: ozone depletion potential; GWP: global warming potential

The above environmental data shows that the ODP of each selected binary refrigerant is zero. The binary refrigerants in group (a) are nonflammable fluorinated hydrocarbons with a high GWP; whereas the binary refrigerants in group (b) all consist of natural refrigerants with a low GWP, and yet are flammable.

Based on the simulation models presented in Section 2, performances of a single-stage ACR operating with a rectifying column were simulated at two pressure ratios of 7 and 10, which represent the medium and low suction pressure compressors, respectively, as well as the following operation conditions: T_H=26.85 °C; T_L=−60 °C; $\Delta T_{R1,min}=\Delta T_{R2,min}=\Delta T_{R3,min}=\Delta T_{A,min}=\Delta T_{E,min}$=2 °C; $\Delta P_{R1}=\Delta P_{R2}=\Delta P_{R3}$=2.5 kPa; ΔP_E=5 kPa; ξ_C=0.4; z_R=0.8; d_r=1.2. REFPROP (NIST, 2007) is used to calculate the thermodynamic properties of binary refrigerants in these simulations.

3.1 Optimization of pressure levels

Fig. 2 illustrates six typical variations of COP, $\Delta T_{R3,hot}$ and $\Delta T_{R1,cold}$ with the suction pressures at a P_r of 7. An optimum COP exists for each pair of the binary refrigerants, and that the COP may undergo a dramatic change as it approaches the optimum COP. An optimum COP occurs when $\Delta T_{R1,cold}$ and $\Delta T_{R3,hot}$ are minimized, approaching $\Delta T_{R1,min}$ and $\Delta T_{R3,min}$, respectively. These numerical calculation results confirm the results in Section 2, namely, the overall matching of the heat capacity of the hot and cold fluids with a specified composition and a pressure ratio leads to an optimum COP. This suggests that an optimization of the pressure level should be conducted prior to an optimization of the composition; the result will, otherwise, lack significance if the composition is optimized at the same fixed pressure level for different binary refrigerants.

Using the method described in Section 2, the COP at the optimum pressure level, COP_{opt}, was calculated in a mole fraction range of z_F=0.2–0.6 and P_r of 7 and 10 for six pairs of binary components. The results are shown in Fig. 3. ΔT_R is the temperature difference between the refrigerants at the hot or cold end of the recuperator.

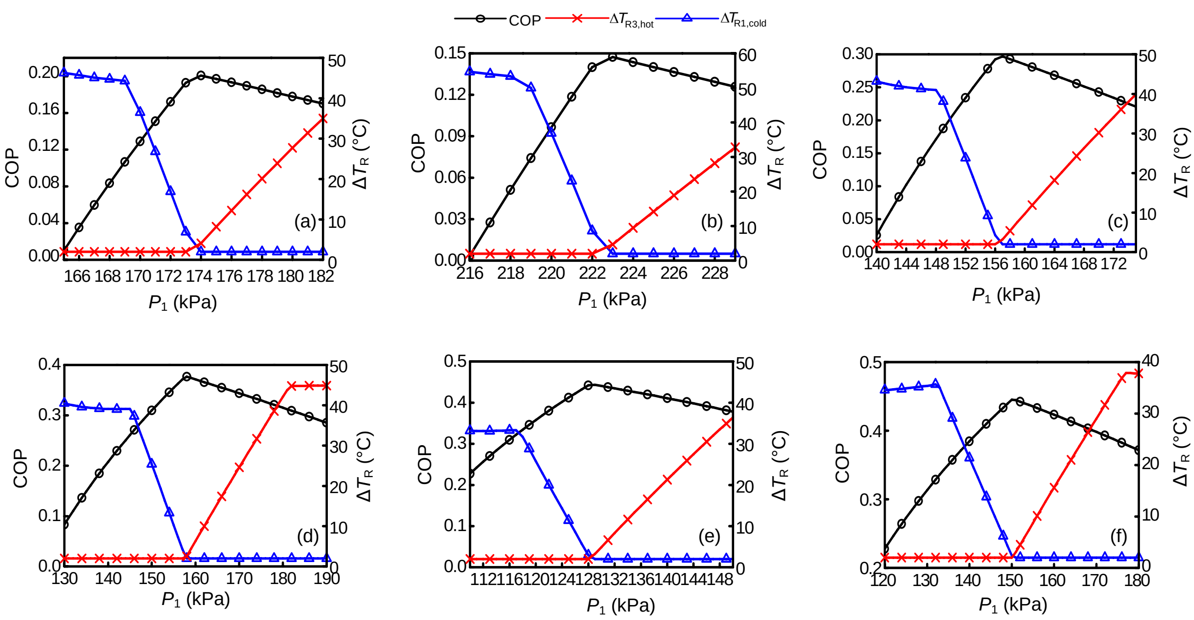


Fig. 2 Variations of COP, $\Delta T_{R3,hot}$, and $\Delta T_{R1,cold}$ with the suction pressure

(a) R23/R134a (z_F=0.35, P_r=7); (b) R170/R290 (z_F=0.35, P_r=7); (c) R23/R227ea (z_F=0.45, P_r=7); (d) R170/R600a (z_F=0.55, P_r=7); (e) R23/R236fa (z_F=0.5, P_r=7); (f) R170/R600 (z_F=0.6, P_r=7)

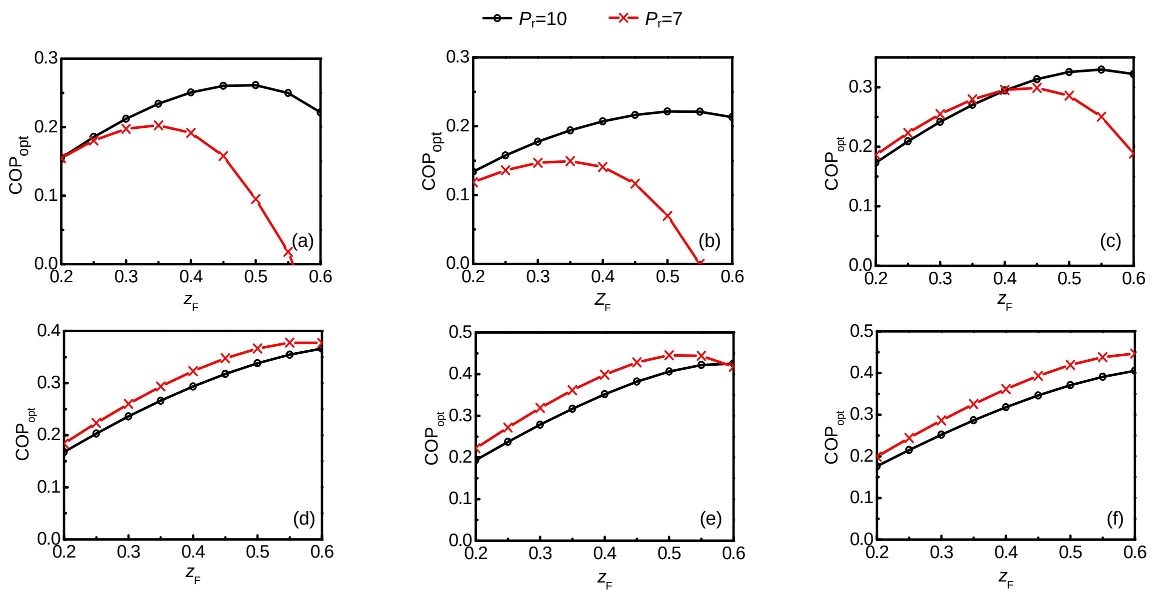


Fig. 3 Variations of COP_{opt} with the mole fraction and pressure ratio

(a) R23/R134a; (b) R170/R290; (c) R23/R227ea; (d) R170/R600a; (e) R23/R236fa; (f) R170/R600

3.2 Optimization of compositions

3.2.1 Optimization of mole fractions

Fig. 3 shows that variations of COP_{opt} with the mole fraction all bear peak displays, as well as different trends for different P_r. This suggests that optimization of the mole fractions should be performed prior to comparing the COP of different binary component refrigerants; the result will, otherwise, lack significant if the comparison is conducted at the same fixed mole fractions.

The maximum COP for each pair of component refrigerants at each specified P_r, COP_{max}, can be selected from Fig. 3. The results of this selection are listed in Table 2.

Table 2 COP_{max} for six pairs of binary component refrigerants

Binary component refrigerant	z_F	P_r	P_1 (kPa)	COP_{max}
R23/R134a	0.35	7	173.50	0.2027
R23/R227ea	0.45	7	156.50	0.2989
R23/R236fa	0.50	7	128.38	0.4452
R170/R290	0.35	7	222.45	0.1494
R170/R600a	0.55	7	157.51	0.3777
R170/R600	0.60	7	150.16	0.4466
Binary component refrigerant	z_F	P_r	P_1 (kPa)	COP_{max}
R23/R134a	0.50	10	162.18	0.2612
R23/R227ea	0.55	10	141.70	0.3295
R23/R236fa	0.60	10	129.98	0.4255
R170/R290	0.50	10	194.78	0.2214
R170/R600a	0.60	10	132.53	0.3663
R170/R600	0.60	10	110.66	0.4055

3.2.2 Optimization of components

Table 2 shows that amongst all pressure ratios, COP_{max} is the largest for the two pairs of binary refrigerants identified as R23/R236fa and R170/R600, and is the smallest for the pairs R23/R134a and R170/R290. The table, therefore, reveals that the difference between the normal boiling points (NBPs) of the two components is a useful reference for the optimization of the components.

By comparing the temperature distributions of hot and cold refrigerants in the recuperators, it can be concluded that the improvement of COP_{max} fundamentally results from a reduction in the average temperature difference between hot and cold refrigerants in the recuperators at a specified P_r. For example, Fig. 4 illustrates the temperature profiles of the three flammable binary refrigerants listed in Table 2 at a P_r of 7, whereby the pinch point in the middle of the temperature profile represents the joining of Recuperator 2 with Recuperator 3.

Fig. 4 also shows that, for the three flammable binary refrigerants, the pinch points are located at the ends of the recuperators, which indicates an overall matching of the heat capacity rates for the hot and cold refrigerants in the recuperators at an optimal pressure level. But the average temperature difference between hot and cold refrigerants in the recuperator for R170/R600 is significantly smaller than those for R170/R600a and R170/R290. The average temperature difference can be represented by the area between the horizontal ordinate and the temperature difference profile of the hot and cold refrigerants in the recuperator. A similar relationship between the average temperature difference and the NBP

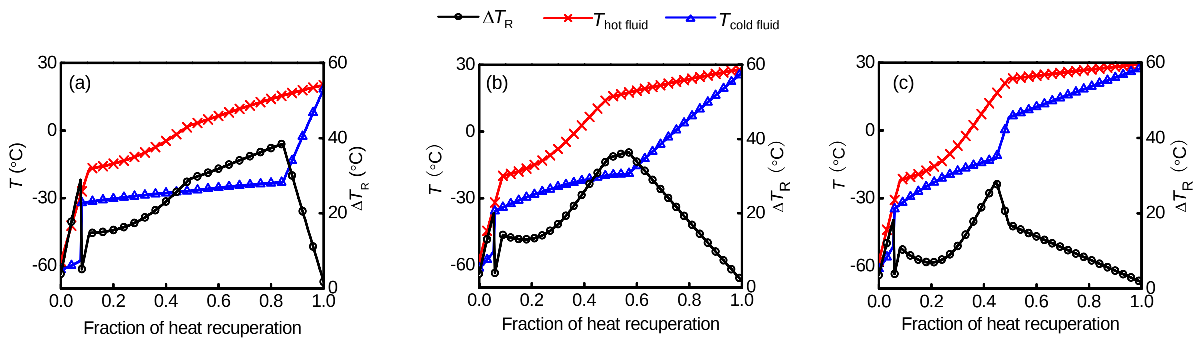


Fig. 4 Recuperator temperature profiles for the three flammable binary refrigerants
(a) R170/R290 (z_F=0.35, P_r=7); (b) R170/R600a (z_F=0.55, P_r=7); (c) R170/R600 (z_F=0.6, P_r=7)

difference is also seen for R23/R236fa, R23/R227ea, and R23/R134a. This clearly indicates that both the overall matching and local matching of heat capacity rates of the hot and cold refrigerants in the recuperators are equally necessary for the improvement of COP_{max}. In reference to Table 2, the best component refrigerants for groups (a) and (b) at the two specified pressure ratios are R23/R236fa and R170/R600, respectively.

4 Conclusions

The following conclusions can be drawn from this study:

1. The performance of an ACR operating with a rectifying column is primarily determined by the compositions of the binary Refrigerants 1 and 2, as well as by the pressure level and ratio. It is, therefore, necessary to consider all these design parameters inclusively when goaling for the optimization of the refrigerator.

2. At specific composition and pressure ratios, the optimum suction pressure for COP occurs when $\Delta T_{R1,cold}$ and $\Delta T_{R3,hot}$ are approximately equal to $\Delta T_{R1,min}$ and $\Delta T_{R3,min}$, respectively when in Recuperator 1 and 3. Within these conditions, the results indicate that the heat capacity rates of the hot and cold refrigerants in the recuperators matched overall.

3. The mole fraction and binary components of Refrigerant 1 have a great influence on the COP_{opt} at specific pressure ratios, which is fundamentally determined by the loss of available energy in the recuperator.

4. Since the pressure ratio of a single-stage compressor is designed to function optimally within a narrow range, the 0.5 and 0.6 mole fractions of R23 for R23/R236fa are the most prospective compositions in group (a) of the nonflammable refrigerants for medium and low suction pressure compressors (P_r=7 and 10), respectively. Similarly, of the flammable natural refrigerants with low GWP within group (b), the 0.6 mole fraction of R170 for R170/R600 is the most prospective for both kinds of compressors.

Finally, the new approach proposed in this study is of applicable value for the performance optimization of the ACR operating with a rectifying column and multi-component refrigerants.

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