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# Estimation of vehicle states and tire-road friction using parallel extended Kalman filtering*

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**Abstract:** A model-based estimator design and implementation is described in this paper to undertake combined estimation of vehicle states and tire-road friction coefficients. The estimator is designed based on a vehicle model with three degrees of freedom (3-DOF) and the dual extended Kalman filter (DEKF) technique is employed. Effectiveness of the estimation is examined and validated by comparing the outputs of the estimator with the responses of the vehicle model in CarSim in three typical road adhesion conditions (high-friction, low-friction, and joint-friction roads). Simulation results demonstrate that the DEKF estimator algorithm designed is able to obtain vehicle states (e.g., yaw rate and roll angle) as well as road friction coefficients with reasonable accuracy.

**Key words:** Vehicle dynamics, State estimation and system identification, Active safety and passive safety
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## 1 Introduction

Many active safety control systems have been developed to promote vehicle dynamics. These systems are designed using the real driving environment to decide the control algorithm of vehicles properly. Obtaining road friction coefficient is of great significance for the design of control logics in vehicle chassis electronic control systems, and the vehicle states estimation precision (Yu and Gao, 2009). The states estimation procedure with constant road friction coefficient will result in large errors when the vehicle is driving on a changing road surface.

Recently, several technical approaches in the area of vehicle state estimation have been proposed and presented. Best and Gordon (2000) employed an extended Kalman filter (EKF) to realize the combined estimation of vehicle states and parameters, but the change of road adhesion conditions and its effects on the validation of estimation have not yet been taken into account. Ray (1995) proposed a method using an extended Kalman-Bucy filter coupled with a Bayesian hypothesis selection technology to estimate vehicle motions, forces, and road friction coefficients, but real-time estimation cannot be realized for the complexity of the algorithm. Wenzel *et al.* (2006) applied the dual extended Kalman filter (DEKF) technique to estimate the vehicle states and parameters, such as vehicle mass, but the effect caused by mass changing is less important than that of road friction changing for a car.

Since the road adhesion condition always changes at a relatively low frequency, the tire-road friction coefficient can be considered as a vehicle parameter which is hidden in the tire model, and therefore it can be estimated in a way similar to that of estimating the vehicle motion states. This paper intends to estimate the vehicle states and tire-road lateral friction coefficient synchronously using DEKF technique from a steering angle and several lateral

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acceleration sensor sets. This method is expected to improve the precision of vehicle state estimation on adhesion-changing roads with relatively low investment in equipment sensors.

## 2 Vehicle model

Based on the study of vehicle handling dynamics by Guo (1991), a three degrees of freedom (3-DOF) double track vehicle model considering the motions of lateral, yaw, and roll is employed (Fig. 1).

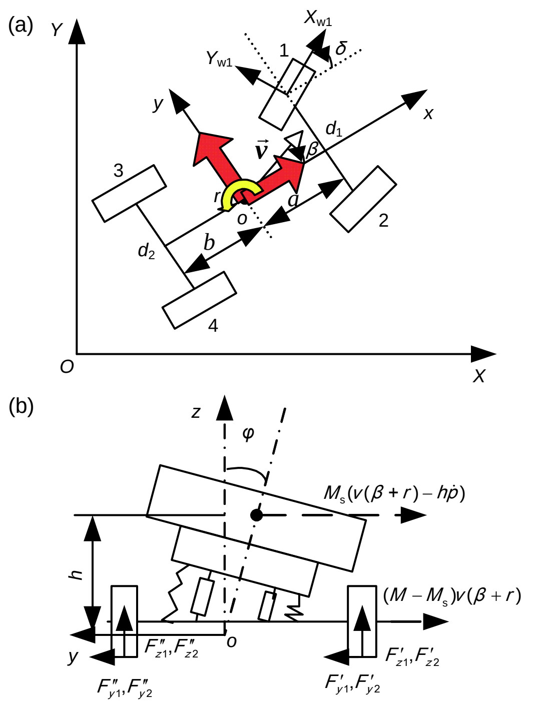


**Fig. 1 Top (a) and front (b) views of the vehicle model**
1, 2, 3, 4: left-front, right-front, left-rear, and right-rear wheels of the vehicle, respectively; $d_1$, $d_2$: wheel threads of the front and rear axles, respectively

The double track model is then simplified with a single track model while the calculate precision is ensured. The dynamic equations of the model are described as follows:

$$I_z\dot{r}+I_{xz}\dot{p}=-aF_{y1}+bF_{y2}, \tag{1}$$

$$Mv(r+\dot{\beta})-M_s h\dot{p}=-(F_{y1}+F_{y2}), \tag{2}$$

$$\begin{aligned} I_x\dot{p}-M_s hv(r+\dot{\beta})+I_{xz}\dot{r} \\ =-(D_f+D_r)p-(C_{\varphi1}+C_{\varphi2}-M_s hg)\varphi, \end{aligned} \tag{3}$$

where $I_z$ and $I_{xz}$ are the moments of inertia around the vertical axis and the longitudinal together with vertical axis, respectively; $F_{y1}$ and $F_{y2}$ are the lateral forces of the front and rear axles, respectively; $a$ and $b$ are the longitudinal distance of the gravity center to the front and rear axles, respectively; $r$ is the vehicle yaw rate, $\beta$ is the vehicle side slip angle, $\varphi$ is the vehicle roll angle, and $p$ is the vehicle roll rate, which is the derivative of $\varphi$; $v$ is the longitudinal velocity, and $h$ is the height of the gravity center of sprung mass above the roll axis; $M$ and $M_s$ are the mass and sprung mass of the vehicle, respectively; $D_f$ and $D_r$ are the front and rear dampings, respectively; $C_{\varphi1}$ and $C_{\varphi2}$ are the front and rear roll stiffnesses, respectively.

From the equations above, it can be found that all the three separate motion states, namely the yaw rate $r$, slip angle $\beta$, and roll rate $p$ depend very much on accurate knowledge of the tire forces, and conversely, these three states would also react on the calculations of tire forces.

## 3 Tire model

For the purpose of characterizing the tire-road behaviors with great accuracy as well as indicating the property of road friction, the magic formula tire model is employed to calculate the tire forces (Bakker and Pacejka, 1989):

$$y(x)=D\sin\{C\arctan[Bx-E(Bx-\arctan Bx)]\}, \tag{4}$$

$$Y(X)=y(x)+S_v, \tag{5}$$

$$x=X+S_h, \tag{6}$$

where $Y$ represents the tire forces $F_y$ and $X$ denotes tire slip angle $\alpha$. $S_v$ and $S_h$ are the shifting values of the tire model, which shift the tire performance curves vertically and horizontally, respectively. $B$, $C$, $D$, and $E$ are functions of the wheel load, slip angle, slip ratio, and camber. Parameter $D$ can be calculated using the vertical tire force $F_z$ and road friction $\mu$, which is the particular parameter to be estimated. The variables needed for calculation of the tire forces $F_y$ can be obtained using the following equations:

$$\begin{cases} F_{z1}=Mgb/l, \\ F_{z2}=Mga/l, \end{cases} \tag{7}$$

$$\begin{cases} \alpha_1 = \beta + ar/v - E_{\mathrm{f}}\varphi - \delta, \\ \alpha_2 = \beta - br/v - E_{\mathrm{r}}\varphi, \end{cases} \tag{8}$$

where the wheelbase of the vehicle $l=a+b$, $\delta$ is the front wheel steer angle, $\alpha_1$ and $\alpha_2$ are the tire side-slip angles of the front and rear axle, $E_{\mathrm{f}}$ and $E_{\mathrm{r}}$ are the front and rear roll steering stiffnesses, respectively.

## 4 Dual extended Kalman filter

Considering the nonlinear characteristics of the 3-DOF vehicle model (mainly caused by the tire model), the EKF technique is selected to estimate vehicle states and road friction coefficients. Specifically, a DEKF is designed including two extended Kalman filters (Fig. 2). The two filters communicate with each other, correct the estimation result of each other, and take advantage of all the information available to estimate the vehicle states and road friction synchronously.

Eqs. (1)–(8) are used to define the system equations of the filters $f()$ and $h()$ as follows:

$$\dot{\boldsymbol{x}}_{\mathrm{s}}(t) = f(\boldsymbol{x}_{\mathrm{s}}(t), \boldsymbol{x}_{\mathrm{p}}(t), \boldsymbol{u}(t), \boldsymbol{w}(t)), \tag{9}$$

$$\boldsymbol{y}(t) = h(\boldsymbol{x}_{\mathrm{s}}(t), \boldsymbol{x}_{\mathrm{p}}(t), \boldsymbol{v}(t)), \tag{10}$$

where the state vector $\mathbf{x}_{\mathrm{s}}(t)=[r\ \beta\ p\ \varphi\ a_y]$, where $a_y$ is the lateral acceleration; the parameters vector $\mathbf{x}_{\mathrm{p}}(t)=[\mu_{y\_\mathrm{f}}\ \mu_{y\_\mathrm{r}}]$, where $\mu_{y\_\mathrm{f}}$ and $\mu_{y\_\mathrm{r}}$ are the lateral road friction coefficients of the front and rear axles, respectively; the input vector $\boldsymbol{u}(t)=[\delta]$; $\boldsymbol{w}(t)$ and $\boldsymbol{v}(t)$ are the process and the measurement noises, respectively. The lateral acceleration sensor equipped at the gravity centre of vehicle provides the measurement output for both the filters, so the expression $\boldsymbol{y}(t)=[a_y]$ is valid.

The Jacobian matrices $\boldsymbol{F}_{\mathrm{s}}$, $\boldsymbol{H}_{\mathrm{s}}$, and $\boldsymbol{H}_{\mathrm{p}}$ are calculated using $f()$ and $h()$:

$$\boldsymbol{F}_{\mathrm{s}} = \begin{bmatrix} \dfrac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{x}_{\mathrm{s}1}} & \cdots & \dfrac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{x}_{\mathrm{s}m}} \\ \vdots & & \vdots \\ \dfrac{\partial \boldsymbol{f}_m}{\partial \boldsymbol{x}_{\mathrm{s}1}} & \cdots & \dfrac{\partial \boldsymbol{f}_m}{\partial \boldsymbol{x}_{\mathrm{s}m}} \end{bmatrix}, \tag{11}$$

$$\boldsymbol{J}_{\mathrm{s}} = \mathrm{e}^{\boldsymbol{F}_{\mathrm{s}}\Delta T} \approx \boldsymbol{I} + \boldsymbol{F}_{\mathrm{s}}\Delta T, \tag{12}$$

$$\boldsymbol{H}_{\mathrm{s}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{13}$$

$$\boldsymbol{H}_{\mathrm{p}} = \begin{bmatrix} \dfrac{\partial a_y}{\partial \mu_{y\_\mathrm{f}}} & \dfrac{\partial a_y}{\partial \mu_{y\_\mathrm{r}}} \end{bmatrix}, \tag{14}$$

where $\boldsymbol{J}_s$ is the state transition matrix, and $\Delta T$ is the sampling time.

As shown in Fig. 2, $\boldsymbol{\Phi}_{\mathrm{s}}$, $\boldsymbol{\Phi}_{\mathrm{p}}$ are the estimation error covariance matrices, $\boldsymbol{R}_{\mathrm{s}}$, $\boldsymbol{R}_{\mathrm{p}}$ are progress noise matrices, $\boldsymbol{K}_{\mathrm{s}}$, $\boldsymbol{K}_{\mathrm{p}}$ are the Kalman gain matrices of state and parameters estimators, and $\boldsymbol{P}_{\mathrm{s}}$, $\boldsymbol{P}_{\mathrm{p}}$ are measurement noise covariance matrices of state and parameters estimators, respectively. The measurement noise covariance $\boldsymbol{P}$ can be observed generally and it is the

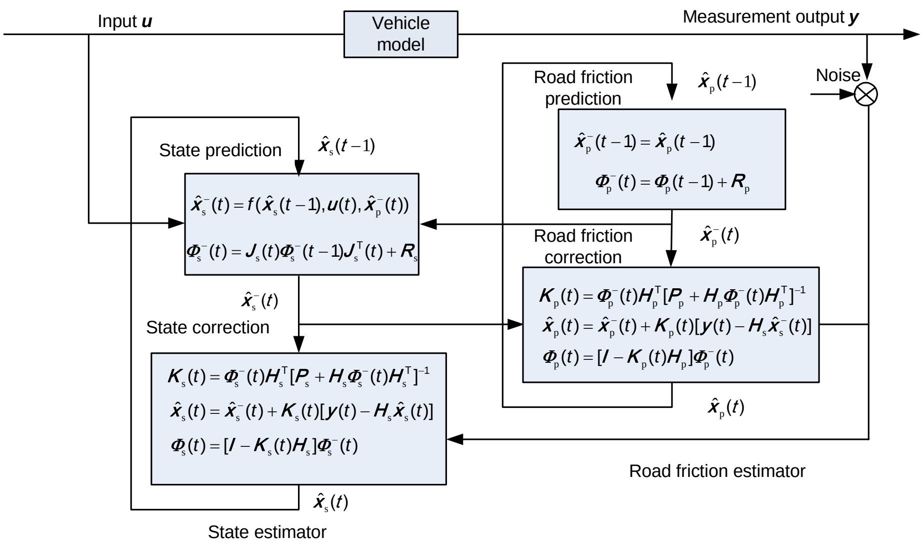


**Fig. 2 Scheme of the dual extended Kalman filter**

known conditions of the filters. However, it is difficult to determine the value of the estimation error covariance $\boldsymbol{R}$ since the process signal $\boldsymbol{x}_s$ or $\boldsymbol{x}_p$ cannot be observed directly. Regardless of whether there is a reasonable standard to select the coefficient, better performance can usually be obtained by adjusting the filters' coefficient (Welch and Bishop, 1995). The values of these matrices are shown as follows: $\boldsymbol{R}_s=10\,000\cdot\boldsymbol{I}_{5\times5}$, $\boldsymbol{R}_s=0.001\cdot\boldsymbol{I}_{2\times2}$, $\boldsymbol{P}_s=[10]$, and $\boldsymbol{P}_p=[10]$.

## 5 Simulation results

The whole set of algorithm is designed in the Matlab/Simulink environment. The CarSim software is employed here, in which the responses of the multi-body vehicle model serve as the reference in comparison, and the estimator designed in this study can be validated. Here, a B-class hatchback model is selected and some of the parameters are illustrated in Table 1. In order to render the responses of CarSim model to be similar to that of a real vehicle, white noise is added to the original model outputs.

**Table 1 Parameters of the vehicle model**

| Parameter | Value |
|---|---|
| Mass of vehicle, $M$ (kg) | 1231 |
| Sprung mass, $M_s$ (kg) | 1111 |
| Moment of inertia around vertical axis, $I_z$ (kg·m$^2$) | 1353 |
| Moment of inertia around longitudinal and vertical axis, $I_{xz}$ (kg·m$^2$) | 288 |
| Height of the gravity center of sprung mass above roll axis, $h$ (m) | 0.54 |
| Longitudinal distance of the gravity center to the front axle, $a$ (m) | 1.04 |
| Longitudinal distance of the gravity center to the rear axle, $b$ (m) | 1.56 |
| Front roll stiffness, $C_{\varphi1}$ (N·m/rad) | 55 000 |
| Rear roll stiffness, $C_{\varphi2}$ (N·m/rad) | 18 000 |
| Front roll damping, $D_f$ (N·m·s/rad) | 3000 |
| Rear roll damping, $D_r$ (N·m·s/rad) | 3000 |
| Front roll steering stiffness, $E_f$ (N·m·s/rad) | −0.114 |
| Rear roll steering stiffness, $E_r$ (N·m·s/rad) | 0 |

### 5.1 High friction road

A steering step input test with the constant speed of 60 km/h and the steady steering wheel input of 80 ° was implemented. The road friction coefficient was set 0.85 in CarSim. The initial values of the DEKF algorithm are set as follows: $\hat{\boldsymbol{x}}_{s0}^{-}=[0\ 0\ 0\ 0\ 0]$, $\hat{\boldsymbol{P}}_{s0}^{-}=\boldsymbol{I}_{5\times5}$, $\hat{\boldsymbol{x}}_{p0}^{-}=[1\ 1]$, $\hat{\boldsymbol{P}}_{p0}^{-}=0.0225\boldsymbol{I}_{2\times2}$.

The estimations of the vehicle states are shown in Fig. 3, and the road friction coefficient estimation result is presented in Fig. 4.

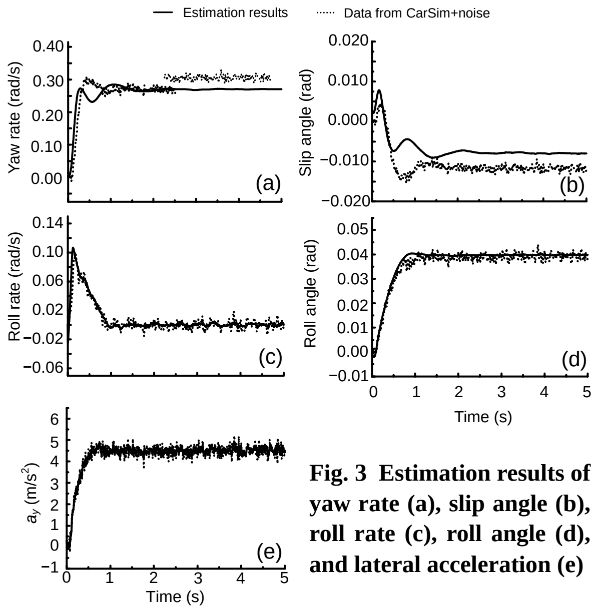


**Fig. 3 Estimation results of yaw rate (a), slip angle (b), roll rate (c), roll angle (d), and lateral acceleration (e)**

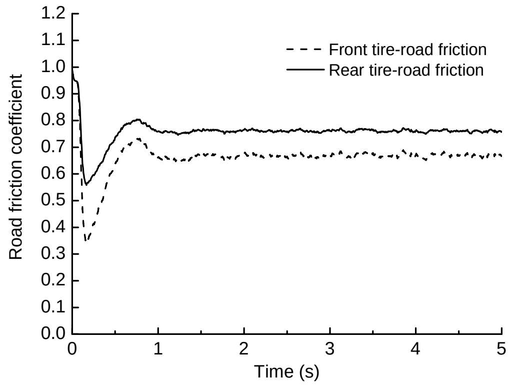


**Fig. 4 High tire-road friction coefficient**

### 5.2 Low friction road

The same maneuver condition as on the high-$\mu$ road is then implemented on a low-adhesion road with the friction coefficient set 0.2 in CarSim. The estimations of the vehicle states are shown in Fig. 5, and the road friction coefficients estimation result is presented in Fig. 6.

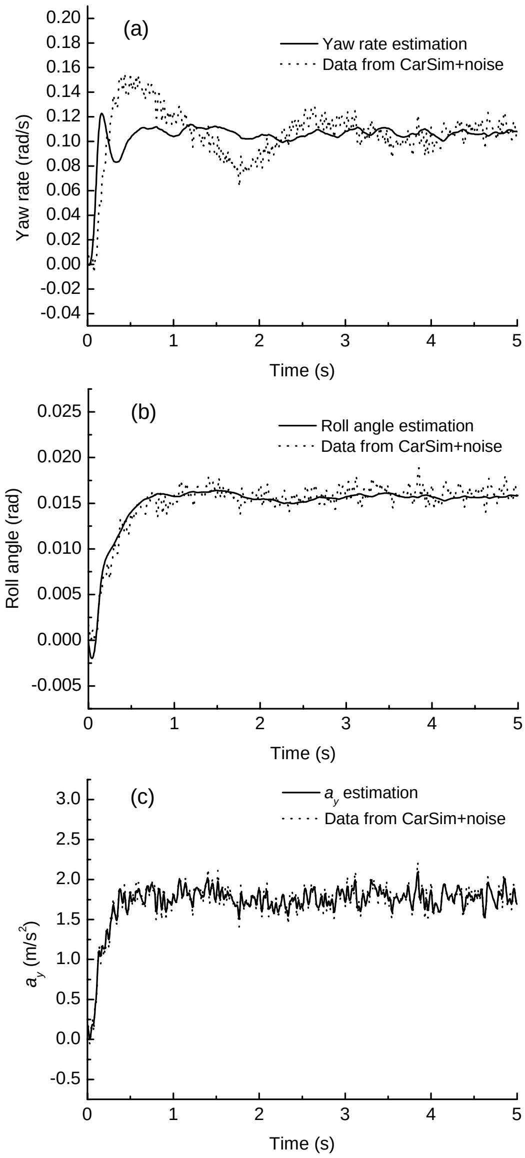


**Fig. 5 Estimation results of yaw rate (a), roll angle (b), and lateral acceleration (c)**

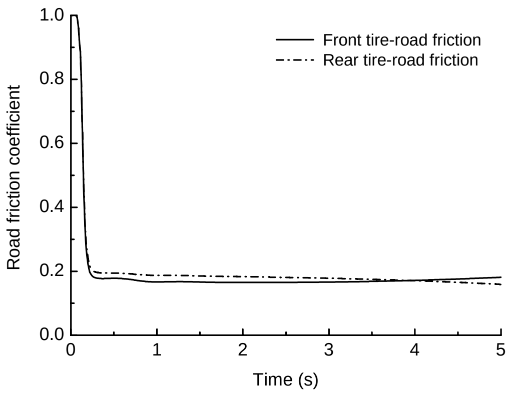


**Fig. 6 Low tire-road friction coefficient**

### 5.3 Joint friction road

The same maneuver condition as on the high-$\mu$ and low-$\mu$ roads is now implemented on a joint friction road. The feature of the road is detailed as follows: a road of a total length of 200 m with the first 40 m a high-$\mu$ section (a friction coefficient of 0.8) and the rest a low-$\mu$ section (a friction coefficient of 0.2). Fig. 7 shows the vehicle trajectory when driving on the joint friction road. The estimations of the vehicle states are shown in Fig. 8, and the road friction coefficients estimation results are presented in Fig. 9.

From the both vehicle states and road friction estimation results of high, low, and join friction roads respectively, the DEKF algorithm has shown reasonable estimates.

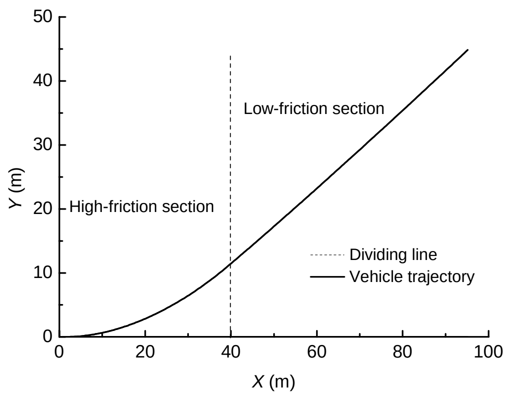


**Fig. 7 Vehicle trajectory when driving on the joint friction road**

## 6 Error caused by constant friction

Suppose the vehicle is running on the joint friction road while the estimation process is conducted with the road friction coefficient being set as a constant of 0.8. That is to say, the change of vehicle parameter, namely the tire-road friction coefficient, is not taken into consideration, and only single EKF is employed to estimate the vehicle states. The results of the estimation of vehicle yaw rate and roll angle are shown in Fig. 10.

It can be found that when the vehicle entering the low-friction section, severe estimation errors are shown. It is of great importance to estimate the road friction coefficient in the research on vehicle dynamics when the vehicle is running on the road with different adhesion conditions.

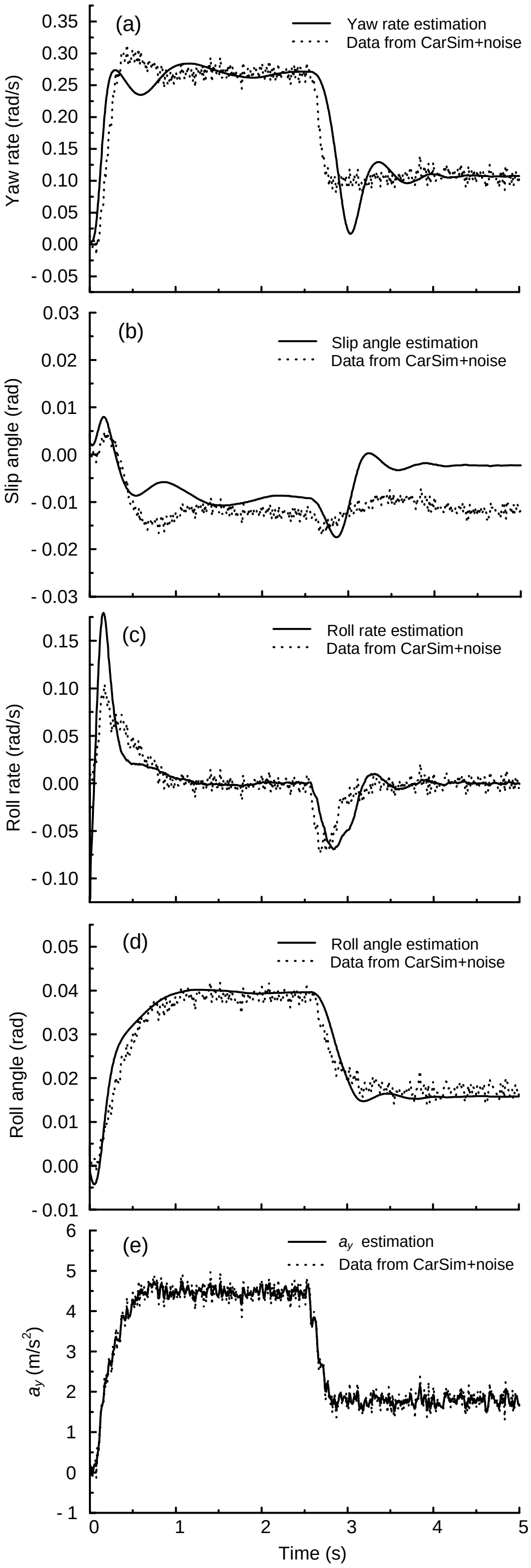


**Fig. 8 Estimation results of yaw rate (a), slip angle (b), roll rate (c), roll angle (d), and lateral acceleration (e)**

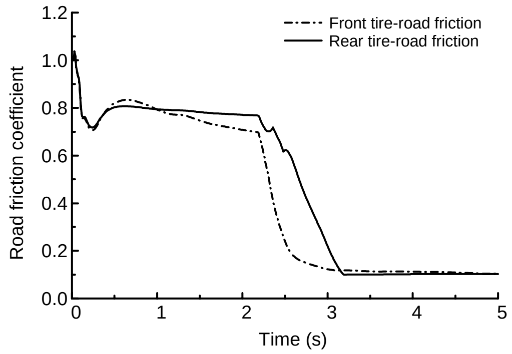


**Fig. 9 Joint friction coefficient**

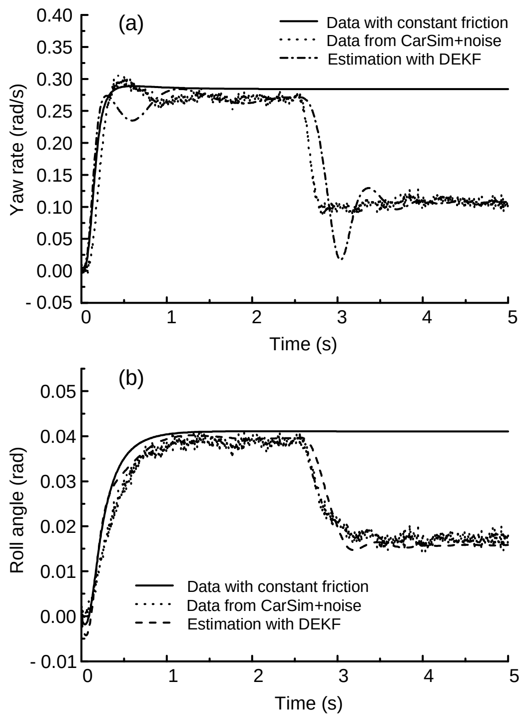


**Fig. 10 Estimation comparisons of yaw rate (a) and roll angle (b)**

## 7 Conclusions

This paper demonstrates the application of the DEKF technique to realize the estimation of the vehicle states and road friction coefficients. The estimator is designed based on a 3-DOF vehicle model coupled with the magic formula tire model. Effectiveness of the estimation is examined and validated by comparing the outputs of the estimators with the responses of the vehicle model in CarSim in three typical road adhesion conditions. Simulation results indicate that the estimation algorithm designed in this paper is able to obtain vehicle states as well as road friction coefficients, which are very difficult to obtain by common sensors with reasonable accuracy.

## References


Bakker, E., Pacejka, H., 1989. A New Tire Model with an Application in Vehicle Dynamics Studies. Autotechnologies Conference and Exposition, Monte-Carlo, Monaco.

Best, M.C., Gordon, T.J., 2000. Combined State and Parameter Estimation of Vehicle Handling Dynamics. Proceedings of 5th International Symposium on Advanced Vehicle Control, Ann Arbor, MI, USA.

Guo, K., 1991. Vehicle Handling Dynamics. Jilin Science & Technology Press, Changchun, China (in Chinese).

Ray, L.R., 1995. Nonlinear state and tire force estimation for advanced vehicle control. *IEEE Transaction on Control Systems Technology*, **3**(1):117-124. [doi:10.1109/87.370717]

Welch, G., Bishop, G., 1995. An Introduction to the Kalman Filter. Technical Report TR 95-041, Chapel Hill, NC, USA.

Wenzel, T.A., Burnham, K.J., Blundell, M.V., Williams, R.A., 2006. Dual extended Kalman filter for vehicle state and parameter estimation. *Vehicle System Dynamics*, **44**(2): 153-171. [doi:10.1080/00423110500385949]

Yu, Z.P., Gao, X.J., 2009. Review of vehicle state estimation problem under driving situation. *Chinese Journal of Mechanical Engineering*, **45**(5):20-33 (in Chinese).