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Active steering control strategy for articulated vehicles*

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Abstract: To improve maneuverability and stability of articulated vehicles, we design an active steering controller, including tractor and trailer controllers, based on linear quadratic regulator (LQR) theory. First, a three-degree-of-freedom (3-DOF) model of the tractor-trailer with steered trailer axles is built. The simulated annealing particle swarm optimization (SAPSO) algorithm is applied to identify the key parameters of the model under specified vehicle speed and steering wheel angle. Thus, the key parameters of the simplified model can be obtained according to the vehicle conditions using an online look-up table and interpolation. Simulation results show that vehicle parameter outputs of the simplified model and TruckSim agree well, thus providing the ideal reference yaw rate for the controller. Then the active steering controller of the tractor and trailer based on LQR is designed to follow the desired yaw rate and minimize their side-slip angle of the center of gravity (CG) at the same time. Finally, simulation tests at both low speed and high speed are conducted based on the TruckSim-Simulink program. The results show significant effects on the active steering controller on improving maneuverability at low speed and lateral stability at high speed for the articulated vehicle. The control strategy is applicable for steering not only along gentle curves but also along sharp curves.

Key words: Articulated vehicle, Sharp curve, Lateral stability, Linear quadratic regulator (LQR)
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1 Introduction

In the past few years, with the requirements of high-efficiency goods transportation, articulated vehicles have undergone rapid development. The most popular type of such vehicles consists of a tractor unit coupled to a long, non-steering trailer via a fifth wheel (Kamnik *et al*., 2003). The conventional tractor-trailer is usually fitted with a trailer that does not steer, which has many negative consequences, such as bad maneuverability at low speed and jack-knifing, trailer swing, and rollover at high speed, which are key factors leading to fatal traffic accidents (Rangavajhula and Tsao, 2008; He *et al*., 2010). Since 1989, many passive steering systems of articulated vehicles have been developed, such as self-steering systems, command steering systems, and pivotal bogie systems (LeBlanc *et al*., 1989; Jujnovich and Cebon, 2002; Cheng, 2009). Such systems steer some trailer wheels according to a geometrical relationship. They can reduce tire wear and help improve low-speed steady-state maneuverability of the articulated vehicles. However, they cannot achieve good performance in transient steering at high speeds (Jujnovich and Cebon, 2013).

To overcome the negative consequences above, many researchers are aware that handling of articulated vehicles can benefit a lot from active steering. Their research can be categorized into two groups.

In the first group, the focus is on the active steering of a single trailer to improve the maneuverability and stability of the trailer. Hata *et al*. (1989) presented a control method to reduce tail swing on a rigid truck to make the rear of the truck body follow

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ORCID: Kyong-il KIM, http://orcid.org/0000-0001-8664-2095 the path of the front axle. Their results showed that this could effectively reduce swept path without tail swing. Their theory laid the foundation for the path-following controller. Notsuet *et al.* (1991) studied the turning behavior of the tractor-trailer with steerable wheels. The trailer followed the trajectory of the front unit by active trailer steering. The controller performed well at low speeds and along gentle curves; however, the performance along sharp curves and at high speeds was not presented. Rangavajhula and Tsao (2007) proposed an optimal linear quadratic regulator (LQR) controller for active trailer steering to minimize the rear amplification ratio, as a surrogate for minimum off-tracking. Cheng and Cebon (2008) proposed a virtual trailer steering control model (virtual driver model of the trailer) for the trailer based on the LQR theory. The idea of lateral position deviation preview LQR controller has also been researched (Islam and He, 2011; Roebuck *et al.*, 2013). The active steering controller based on path-following performs well along gentle curves; however, it cannot work along sharp curves, especially along the rectangular curves.

In the second group, since some researchers hold that the driver's steering wheel control depends mainly on his/her perception and feeling of the tractor unit's behavior, the improvement of the tractor is crucial. The second group focuses mainly on active steering control to improve the tractor handling behavior. Palkovics and El-Gindy (1996) proposed an LQR to minimize the tractor yaw rate, tractor side-slip angle, etc. by active steering. The idea of minimization using an LQR controller was followed by El-Gindy *et al.* (2001) and Hac *et al.* (2008). However, the minimization of yaw rate may not be an optimal choice, and following the desired yaw rate is better.

As indicated in the studies mentioned above, much effort has been made in active steering control for tractor or trailer and great progress has been achieved. The current research on active steering of articulated vehicles focuses mainly on improving the tractor's or trailer's handling behavior. Little research has presented an active steering controller for improving the maneuverability and stability behavior of both the tractor and the trailer. It can be reasoned that if both the tractor's and the trailer's handling indexes are controlled in an acceptable range, the stability of the whole tractor-trailer combination is satisfactory. In addition, since the current path-following control strategies are applicable only along gentle curves, there is a vital need for devising a control system applicable not only along gentle curves but also along sharp curves, especially the rectangular curves.

In this study, an active steering controller based on the LQR theory is designed to improve the maneuverability at low speed and lateral stability at high speed of the whole tractor-trailer combination. A three-degree-of-freedom (3-DOF) linear model of the tractor-trailer with steered trailer axles is built, and the simulated annealing particle swarm optimization (SAPSO) algorithm is applied to identify the key parameters of the model, thus providing the accurate, desired reference yaw rate for the controller. The tractor and trailer active steering controller is designed to follow the desired yaw rate and minimize the side-slip angle of the tractor's center of gravity (CG) and trailer's CG at the same time, which is very important in the vehicle's stability evaluation. The control strategy is validated by simulation tests based on the TruckSim-Simulink program. The results show that the active steering controller has the potential to improve the maneuverability and lateral stability of the articulated vehicles.

2 Linear 3-DOF reference model and parameter identification

In this study, a tractor-trailer is selected as an example to represent the articulated vehicle for design and discussion. A linear simplified 3-DOF tractor-trailer model (Fig. 1) is derived and used to design the control strategy. To ensure that the linear model can reflect the plant model more accurately, the SAPSO

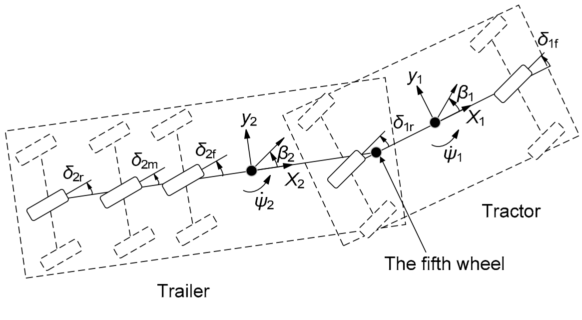

Fig. 1 The simplified 3-DOF tractor-trailer model

algorithm is used to identify the key parameters of the simplified model, including the tire cornering stiffness of the front axle and rear axle of the tractor and the trailer's axles.

The articulated vehicle model consists of two rigid bodies: the tractor and the trailer. The tractor has two degrees of freedom including lateral and yaw motions, while the trailer has the freedom to yaw relative to the tractor.

The notations are listed in Table 1 (we always use j=1 for tractor and j=2 for trailer in this study) and the assumptions are given below:

1. There are no longitudinal forces on any tire.
2. The transmission system of the steering system is ignored, and the front wheel steering angle is used directly as the system input.
3. The angular displacements and the articulation angle of the fifth wheel are small.
4. The relationship between the tire force and side-slip angle is linear.

2.1 Equations of the linear tractor-trailer vehicle model

The equations of motions of the tractor unit are

$$m_1 u_1(\dot{\beta}_1 + \dot{\psi}_1) = F_{y1}\cos\delta_{1\mathrm{f}} + F_{y2} + F_{\mathrm{C}}, \tag{1}$$

$$I_{1zz}\ddot{\psi}_1 = F_{y1} a \cos\delta_{1\mathrm{f}} - F_{y2} b - F_{\mathrm{C}} c, \tag{2}$$

where F_{y1} and F_{y2} are the lateral forces acting on the tractor's front and rear axles, respectively. The equations of motions of the trailer are as follows:

$$\begin{aligned} m_2 u_2(\dot{\beta}_2 + \dot{\psi}_2) = {} & F_{y3}\cos\delta_{2\mathrm{f}} + F_{y4}\cos\delta_{2\mathrm{m}} \\ & + F_{y5}\cos\delta_{2\mathrm{r}} - F_{\mathrm{C}}\cos\Gamma, \end{aligned} \tag{3}$$

$$I_{2zz}\ddot{\psi}_2 = -F_{\mathrm{C}} e \cos\Gamma - (F_{y3} d_1 + F_{y4} d_2 + F_{y5} d_3), \tag{4}$$

where F_{y3}, F_{y4}, and F_{y5} are the lateral forces acting on the trailer's front, middle, and rear axles, respectively. The kinematic constraint equation of the fifth wheel is

$$\dot{\beta}_1 - \dot{\beta}_2 - \frac{c}{u_1}\ddot{\psi}_1 - \frac{e}{u_2}\ddot{\psi}_2 + \dot{\psi}_1 - \dot{\psi}_2 = 0, \tag{5}$$

where

$$\begin{cases} F_{y1} = k_1(\beta_1 + a\dot{\psi}_1 / u_1 - \delta_{1\mathrm{f}}), \\ F_{y2} = k_2(\beta_1 - b\dot{\psi}_1 / u_1 - \delta_{1\mathrm{r}}), \\ F_{y3} = k_3(\beta_2 - d_1\dot{\psi}_2 / u_2 - \delta_{2\mathrm{f}}), \\ F_{y4} = k_4(\beta_2 - d_2\dot{\psi}_2 / u_2 - \delta_{2\mathrm{m}}), \\ F_{y5} = k_5(\beta_2 - d_3\dot{\psi}_2 / u_2 - \delta_{2\mathrm{r}}). \end{cases} \tag{6}$$

2.2 Identification of key parameters of the simplified model

The key parameters required for identification include tire cornering stiffness of the front axle and rear axle of the tractor and the trailer. The tire cornering stiffness of the three axles of the trailer is assumed to be the same. The key parameters of the

Table 1 Notations

Symbol	Meaning	Symbol	Meaning
m_j	Total mass	a	Distance between tractor's CG and the front axle
u_j	Longitudinal velocity	b	Distance between tractor's CG and rear axle
β_j	Side-slip angle of the vehicle body	c	Distance between tractor's CG and the fifth wheel
$\dot{\psi}_j$	Yaw rate of the vehicle body	e	Distance between trailer's CG and the fifth wheel
k_1	Tire cornering stiffness of the tractor's front axle	d_1	Distance between trailer's CG and its front axle
k_2	Tire cornering stiffness of the tractor's rear axle	d_2	Distance between trailer's CG and its middle axle
k_3	Tire cornering stiffness of the trailer's front axle	d_3	Distance between trailer's CG and its rear axle
k_4	Tire cornering stiffness of the trailer's middle axle	$\delta_{1\mathrm{f}}$	Steer angle of the tractor's front wheel
k_5	Tire cornering stiffness of the trailer's rear axle	$\delta_{1\mathrm{r}}$	Steer angle of the tractor's rear wheel
F_{C}	Lateral component of directional forces at the articulation point	$\delta_{2\mathrm{f}}$	Active steer angle of the trailer's front axle
		$\delta_{2\mathrm{m}}$	Active steer angle of the trailer's middle axle
Γ	Articulation angle	$\delta_{2\mathrm{r}}$	Active steer angle of the trailer's rear axle
I_{jzz}	Yaw moment of the inertia of sprung mass	i_{sw}	Steering ratio

simplified model are identified offline under step steering at specified vehicle speeds and lateral accelerations. The values in other conditions can be obtained by online linear interpolation according to the vehicle states.

The particle swarm optimization (PSO) algorithm is an evolutionary technology, stemming from the predatory behavior of a group of birds. Compared to the genetic algorithm, it is also an iterative optimization tool; however, it does not include the crossover and mutation processes. Due to its simplicity and rapidness, PSO has been widely applied in function optimization, neural networks, fuzzy system control, and other genetic algorithm application fields. However, the basic PSO algorithm can easily fall into a local extreme solution and has slow convergence speed and poor accuracy in the late evolution stage. Since the SAPSO algorithm combines the global search ability of the PSO algorithm and the advantage of jumping out of the local optimal solution of the simulated annealing algorithm (Gao and Xie, 2004), here we use it to identify the key parameters of the simplified tractor-trailer model. The fitness function is

$$F=\sum_{i=1}^{N}\frac{[\beta_{1\mathrm{s}}(i)-\beta_{1\mathrm{t}}(i)]^2}{|\beta_{1\mathrm{t}}(i)|_{\max}}+\sum_{i=1}^{N}\frac{[\dot{\psi}_{1\mathrm{s}}(i)-\dot{\psi}_{1\mathrm{t}}(i)]^2}{|\dot{\psi}_{1\mathrm{t}}(i)|_{\max}}+\sum_{i=1}^{N}\frac{[\beta_{2\mathrm{s}}(i)-\beta_{2\mathrm{t}}(i)]^2}{|\beta_{2\mathrm{t}}(i)|_{\max}}+\sum_{i=1}^{N}\frac{[\dot{\psi}_{2\mathrm{s}}(i)-\dot{\psi}_{2\mathrm{t}}(i)]^2}{|\dot{\psi}_{2\mathrm{t}}(i)|_{\max}},\tag{7}$$

where N is the amount of the simulation data, $\beta_{j\mathrm{s}}(i)$ and $\dot{\psi}_{j\mathrm{s}}(i)$ are the side-slip angle and yaw rate of the linear model, respectively, and $\beta_{j\mathrm{t}}(i)$ and $\dot{\psi}_{j\mathrm{t}}(i)$ are the side-slip angle and yaw rate of TruckSim, respectively. For the detailed program of SAPSO, readers can refer to Gong and Wang (2014).

Tire cornering stiffness is the key parameter in the linear vehicle model that influences the model accuracy greatly. The map of the identified tire cornering stiffness of the front axle and rear axle of the tractor and the axle of trailer are shown in Figs. 2–4. The tire cornering stiffness of the tractor (or trailer) under the current condition can be easily obtained by interpolation according to the current lateral acceleration and velocity of the tractor (or trailer).

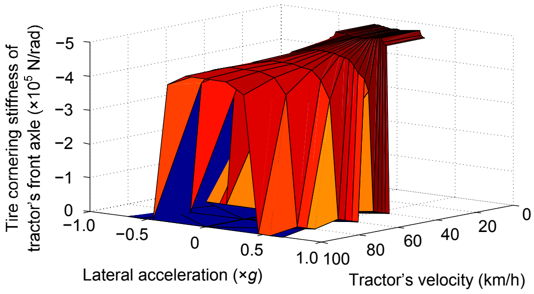

Fig. 2 Tire cornering stiffness of the tractor's front axle

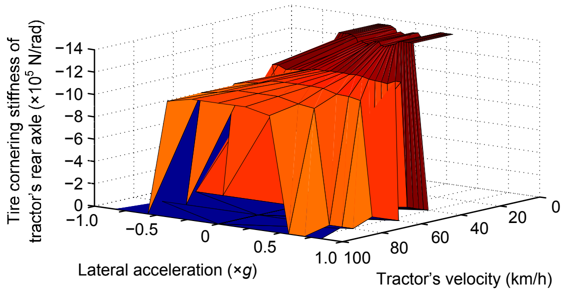

Fig. 3 Tire cornering stiffness of the tractor's rear axle

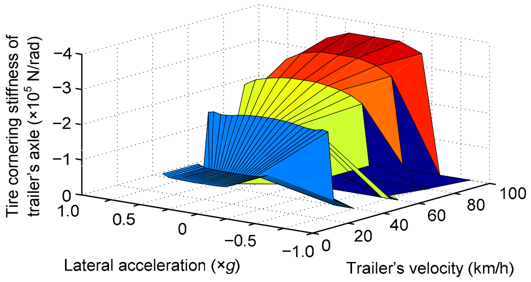

Fig. 4 Tire cornering stiffness of the trailer's axle

To validate the accuracy of the simplified model, a varying vehicle condition (including the velocity and the steering wheel angle) is simulated (Figs. 5 and 6). Figs. 7–10 show the outputs of several state variables of the simplified vehicle model and TruckSim. It can be seen that the outputs are in good agreement with the online interpolation of the key parameters according to real-time vehicle states (u_1, u_2, a_{y1}, a_{y2}). Here, a_{y1} and a_{y2} represent the lateral accelerations of the tractor and trailer, respectively. The simplified model provides a reliable basis for the control with high precision.

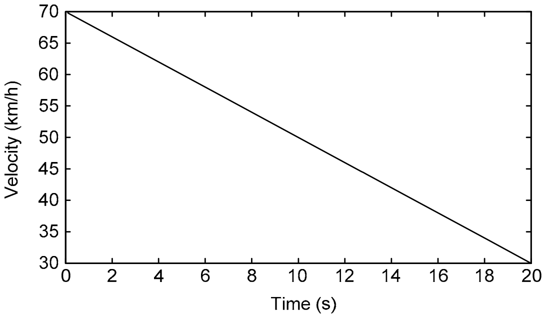

Fig. 5 Velocity of the vehicle

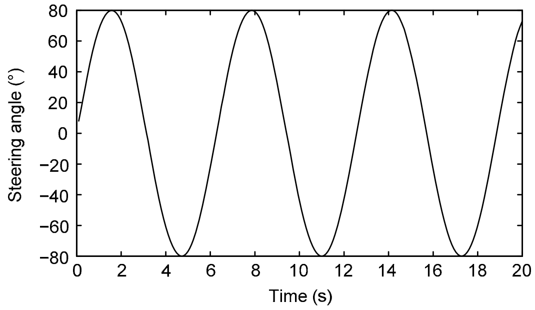

Fig. 6 Steering wheel angle of the vehicle

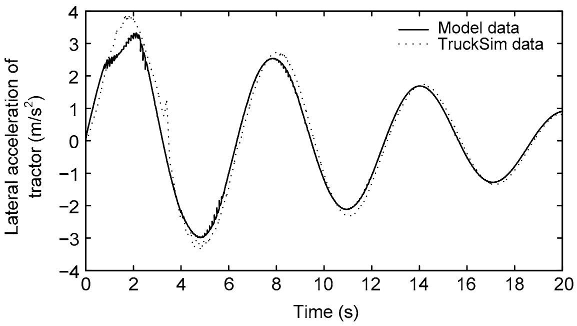

Fig. 7 Lateral acceleration of the tractor

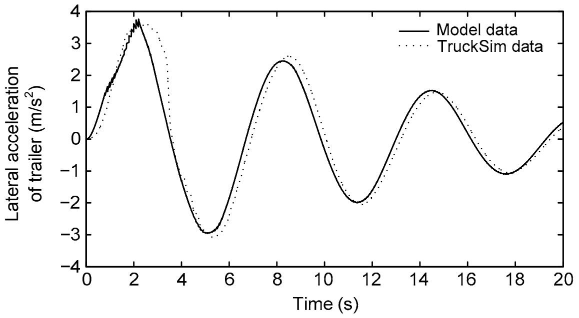

Fig. 8 Lateral acceleration of the trailer

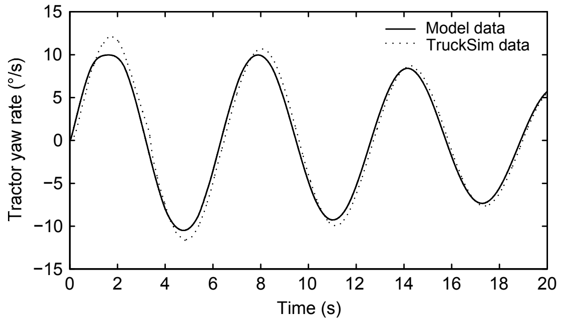

Fig. 9 Yaw rate of the tractor

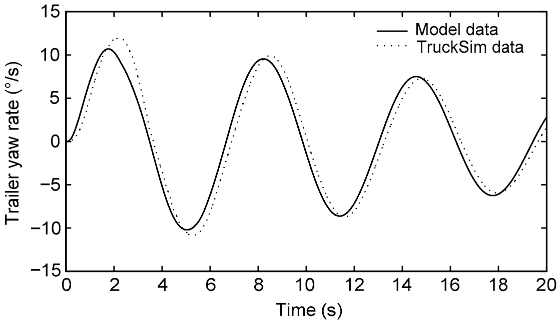

Fig. 10 Yaw rate of the trailer

3 Active steering controller for tractor-trailer

In the stability control of passenger cars, the yaw rate and the side-slip angle are universally chosen as the control variables. The control strategies focus mainly on making the actual yaw rate follow the ideal yaw rate and ensuring the side-slip angle as much as possible. Usually, the reference yaw rate is obtained from the bicycle model according to the steering wheel angle.

In this study, we introduce the stability control strategy into the articulated vehicles. As shown in Fig. 11, the LQR controller aims to make both the tractor and trailer follow the desired yaw angle and side-slip angle by actively steering the tractor's rear axle and the trailer's axles. The 3-DOF reference model is used to provide the desired states according to the tractor's front axle steering angle input, which is governed by the driver.

The vehicle dynamics (Eqs. (1)–(6)) can be written in the following state vector form:

$$\boldsymbol{M}\dot{\boldsymbol{X}} = \boldsymbol{N}\boldsymbol{X} + \boldsymbol{D}_0\boldsymbol{u} + \boldsymbol{D}_1\delta_{1\mathrm{f}}, \tag{8}$$

where

$$\boldsymbol{N}=\begin{bmatrix} k_1(a+c)+k_2(c-b) & -m_1u_1c+\dfrac{k_1a(a+c)-k_2b(c-b)}{u_1} & 0 & 0 \\ k_1+k_2 & -m_1u_1+\dfrac{k_1a-k_2b}{u_1} & k_3+k_4+k_5 & -m_2u_2-\dfrac{k_3d_1+k_4d_2+k_5d_3}{u_2} \\ 0 & 0 & -(k_3+k_4+k_5)e-(k_3d_1+k_4d_2+k_5d_3) & m_2u_2e+\dfrac{e(k_3d_1+k_4d_2+k_5d_3)+(k_3d_1^2+k_4d_2^2+k_5d_3^2)}{u_2} \\ 0 & 1 & 0 & -1 \end{bmatrix},$$

$$\boldsymbol{X}=\begin{bmatrix}\beta_1\\ \dot{\psi}_1\\ \beta_2\\ \dot{\psi}_2\end{bmatrix},\ \boldsymbol{u}=\begin{bmatrix}\delta_{1\mathrm{r}}\\ \delta_{2\mathrm{f}}\\ \delta_{2\mathrm{m}}\\ \delta_{2\mathrm{r}}\end{bmatrix},\ \boldsymbol{D}_1=\begin{bmatrix}-k_1(a+c)\\ -k_1\\ 0\\ 0\end{bmatrix},$$

$$\boldsymbol{M}=\begin{bmatrix} m_1u_1c & I_{1zz} & 0 & 0 \\ m_1u_1 & 0 & m_2u_2 & 0 \\ 0 & 0 & -m_2u_2e & I_{2zz} \\ -1 & c/u_1 & 1 & e/u_2 \end{bmatrix},$$

$$\boldsymbol{D}_0=\begin{bmatrix} k_2(b-c) & 0 & 0 & 0 \\ -k_2 & 0 & 0 & 0 \\ 0 & -k_3(e+d_1) & -k_4(e+d_2) & -k_5(e+d_2) \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

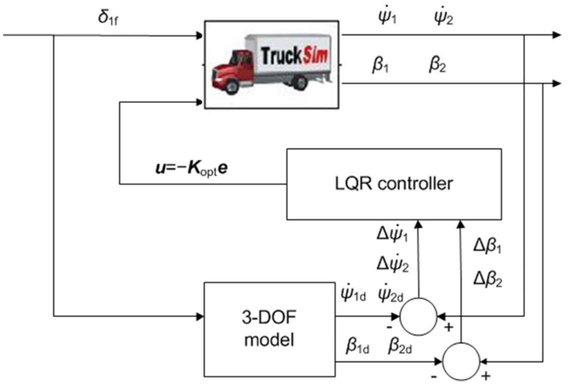

Fig. 11 Diagram of the control strategy

To design the LQR controller, Eq. (8) is rewritten as follows:

$$\dot{\boldsymbol{X}}=\boldsymbol{A}\boldsymbol{X}+\boldsymbol{B}_0\boldsymbol{u}+\boldsymbol{B}_1\delta_{1\mathrm{f}}, \tag{9}$$

where $\boldsymbol{A}=\boldsymbol{M}^{-1}\boldsymbol{N}$, $\boldsymbol{B}_0=\boldsymbol{M}^{-1}\boldsymbol{D}_0$, and $\boldsymbol{B}_1=\boldsymbol{M}^{-1}\boldsymbol{D}_1$. Then the desired state vector can be expressed as

$$\dot{\boldsymbol{X}}_\mathrm{d}=\boldsymbol{A}\boldsymbol{X}_\mathrm{d}+\boldsymbol{B}_1\delta_{1\mathrm{f}}. \tag{10}$$

Since the desired states are steady responses of the state vector (Hac *et al.*, 2008), in this situation, $\dot{\boldsymbol{X}}_\mathrm{d}=\boldsymbol{0}$. Therefore, the desired states can be obtained as follows:

$$\boldsymbol{X}_\mathrm{d}=(\beta_{1\mathrm{d}}\ \ \dot{\psi}_{1\mathrm{d}}\ \ \beta_{2\mathrm{d}}\ \ \dot{\psi}_{2\mathrm{d}})^\mathrm{T}=-\boldsymbol{A}^{-1}\boldsymbol{B}_1\delta_{1\mathrm{f}}. \tag{11}$$

To design the LQR controller, combined with Eqs. (9) and (10), the error vector is written as follows:

$$\dot{\boldsymbol{e}}=\boldsymbol{A}\boldsymbol{e}+\boldsymbol{B}_0\boldsymbol{u}, \tag{12}$$

where

$$\boldsymbol{e}=\boldsymbol{X}-\boldsymbol{X}_\mathrm{d}. \tag{13}$$

The LQR controller aims to make both the tractor and the trailer follow the desired yaw angle and minimize the side-slip angle at the same time. Thus, the cost function is

$$\begin{aligned} J=\sum\big(&q_1\beta_1^2+q_2(\dot{\psi}_1-\dot{\psi}_{1\mathrm{d}})^2\\ &+q_3\beta_2^2+q_4(\dot{\psi}_2-\dot{\psi}_{2\mathrm{d}})^2\\ &+r_1\delta_{1\mathrm{r}}^2+r_2\delta_{2\mathrm{f}}^2+r_3\delta_{2\mathrm{m}}^2+r_4\delta_{2\mathrm{r}}^2\big), \end{aligned} \tag{14}$$

where q_1–q_4 and r_1–r_4 are the weighting factors. The optimal steering angle can be calculated by

$$\boldsymbol{u}=-\boldsymbol{K}_\mathrm{opt}\boldsymbol{e}, \tag{15}$$

where $\boldsymbol{K}_\mathrm{opt}$ is the feedback matrix, which is calculated by the Riccati equation. Eq. (14) can also be written as the following matrix form:

$$J=\int_0^\infty(\boldsymbol{e}^\mathrm{T}\boldsymbol{Q}\boldsymbol{e}+\boldsymbol{u}^\mathrm{T}\boldsymbol{R}\boldsymbol{u})\mathrm{d}t, \tag{16}$$

where $\boldsymbol{Q}=\mathrm{diag}(q_1, q_2, q_3, q_4)$ and $\boldsymbol{R}=\mathrm{diag}(r_1, r_2, r_3, r_4)$.

4 Simulation and discussion

TruckSim is a commercial software with high precision and has been widely used in vehicle virtual simulations. The vehicle model is provided by TruckSim, and the LQR controller is achieved in MATLAB/Simulink. Both of them consist of the complete simulation model. With an appropriate setting of weighting factors, the performance of articulated vehicles can be improved. In our study, the weighting factors are chosen as q_1=0.1, q_2=0.08, q_3=1, q_4=0.1, and r_1=r_2=r_3=r_4=1. These values are set through repeated experimental investigations.

The virtual vehicle for the experiment in TruckSim is a demo tractor-trailer with five axles to evaluate the steering control strategies. The essential vehicle system parameters are given in Table 2.

Table 2 Values for the essential vehicle system parameters

Parameter	Value	Parameter	Value
m_1 (kg)	5762	c (m)	1.435
m_2 (kg)	25 910	e (m)	4.346
I_{1zz} (kg·m^2)	7700	d_1 (m)	2.354
I_{2zz} (kg·m^2)	82 182	d_2 (m)	3.554
a (m)	1.565	d_3 (m)	4.754
b (m)	2.335	i_{sw}	25

4.1 Simulation test along a sharp curve and at a low speed

At a low speed, the stability indexes of the vehicle are in a safe range. The major problem of the articulated vehicle is its maneuverability along sharp curves. The corner swept path width (SPW90) is the standard test condition to observe the maneuverability of the tractor-trailer at a low speed (Kusters, 1995). The test condition is shown in Fig. 12. The SPW of the vehicle while traversing a 90° corner is defined as the maximum width of the path swept by the vehicle as it travels around the corner.

Fig. 13 shows the active steering angles of the tractor's rear axle, and the trailer's front, middle, and rear axles. Fig. 14 shows the corner SPW of the tractor-trailer without control and with active steering control. It can be seen that the corner SPW of the tractor-trailer is greatly improved. The SPW without control is 4.8 m, while it is reduced to 4.2 m when fitted with active steering control. The test results show that the maneuverability of the tractor-trailer at a low speed has been improved. Thus, the active steering controller can improve the maneuverability of the articulated vehicle along sharp curves (rectangular curves).

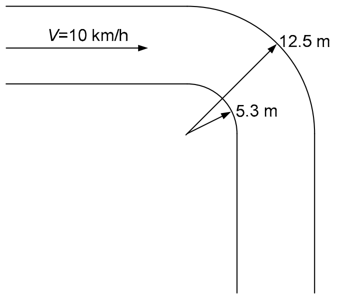

Fig. 12 Simulation road conditions

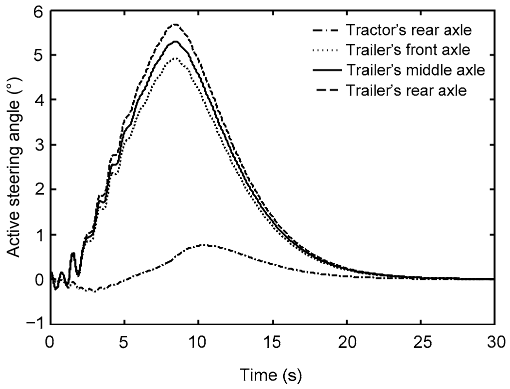

Fig. 13 Active steering angles

4.2 Simulation test at a high speed

The single lane change test shown in Fig. 15 is simulated to test the performance with active steering control. The simulation time step is 0.01 s.

The simulation results at a high speed are shown in Figs. 16–25. The vehicle speed is 100 km/h. Fig. 16 shows the steering wheel angle input from the driver, and Fig. 17 shows the active steering angle of each axle. Figs. 18 and 19 show the yaw rates of the tractor and trailer, respectively. The conventional vehicle without control is not stable, since the yaw rate curves fluctuate. When the vehicle is fitted with the active steering system, the path errors of both the tractor and the trailer have been reduced (Figs. 20 and 21), which indicates that the path following has been improved. Figs. 22 and 23 show the side-slip angles of the tractor and trailer, respectively, from which it can be seen

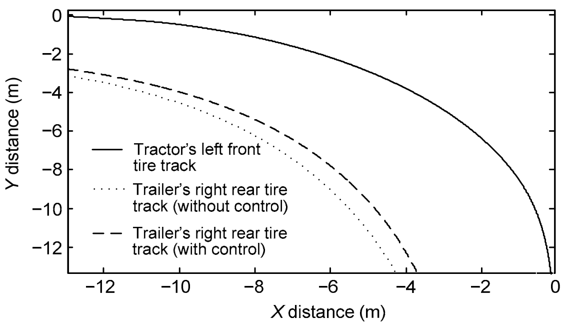

Fig. 14 Corner swept path width (SPW90)

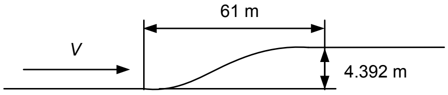

Fig. 15 Single lane change test (simulation time step is 0.01 s)

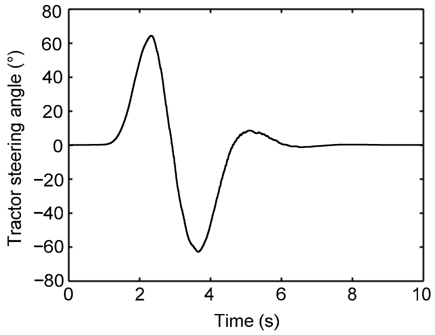

Fig. 16 Steering wheel angle input

that the side-slip angle has been greatly reduced with control. The handling behavior has been improved by active steering control.

Figs. 24 and 25 show the lateral accelerations of the tractor and trailer before and after the control, respectively. It can be seen that the lateral accelerations of the tractor and trailer without control fluctuate. Both of them have been greatly reduced and smoothened with active steering control, which shows that the rollover tendency of the tractor-trailer is decreased.

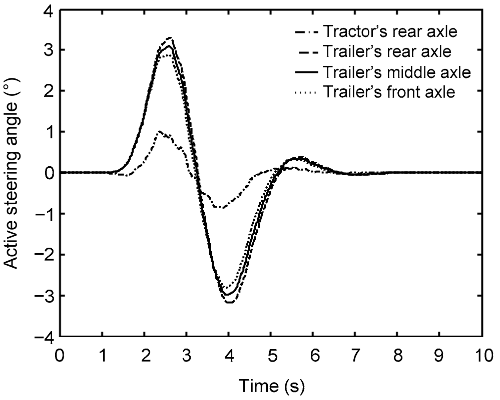

Fig. 17 Active steering angle

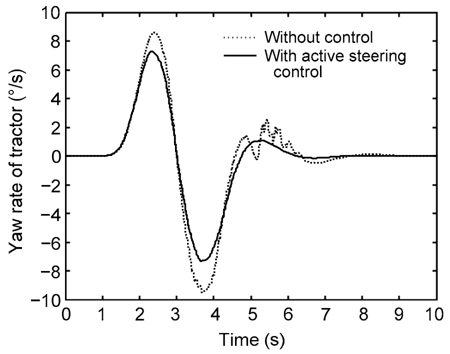

Fig. 18 Yaw rate of the tractor

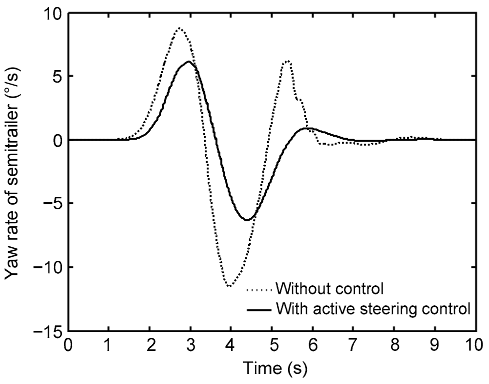

Fig. 19 Yaw rate of the trailer

Rearward amplification (RA) indicates the magnitude of lateral acceleration experienced by the trailer when the tractor performs a sudden evasive maneuver. It is defined as the ratio of the peak lateral acceleration of trailer's CG to that of the tractor's front axle. RA refers to the degree to which the trailing units amplify or exaggerate the lateral motions of the tractor unit. It occurs mainly in vehicles with one or more articulation points after they perform a rapid evasive maneuver such as a lane change. In our study, RA is evaluated using a standard Society of Automotive Engineers (SAE) J2179 lane change maneuver (Truck and Bus Powertrain Steering Committee, 1993). Since lateral acceleration is measured at the positions of the tractor's CG and trailer's CG, RA can

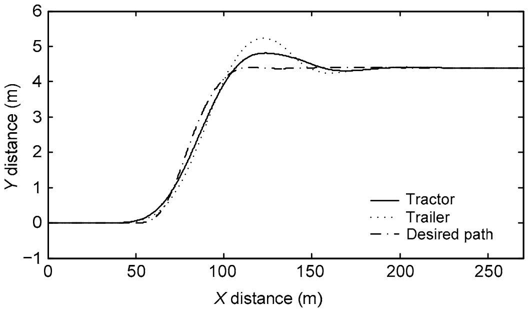

Fig. 20 The paths of the tractor and trailer without control

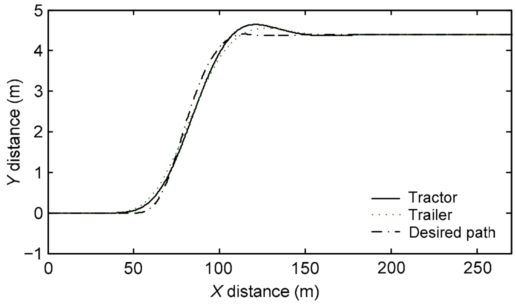

Fig. 21 The paths of the tractor and trailer with active steering control

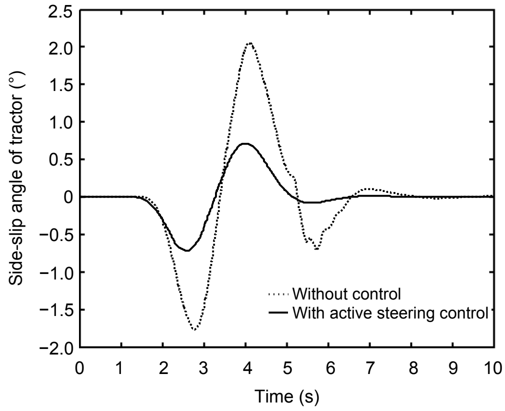

Fig. 22 Side-slip angle of the tractor

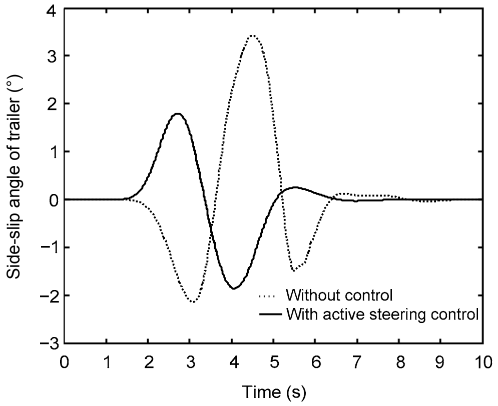

Fig. 23 Side-slip angle of the trailer

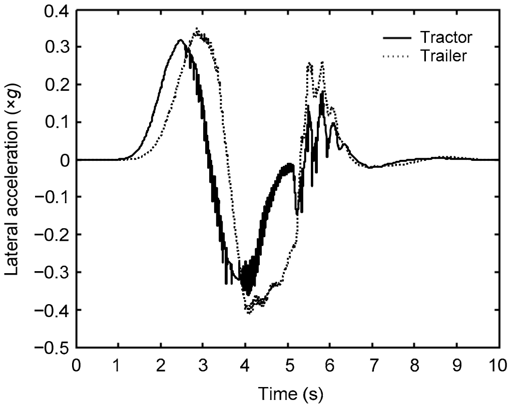

Fig. 24 Lateral acceleration without control

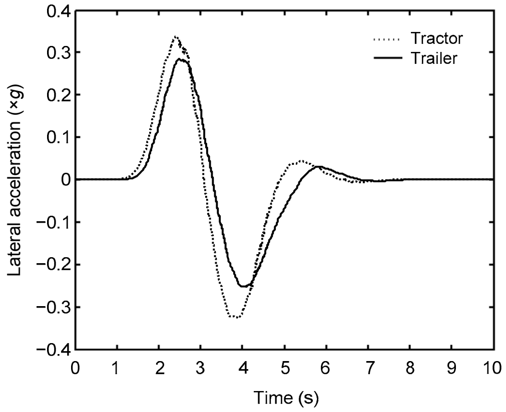

Fig. 25 Lateral acceleration with active steering control

be calculated as

$$\mathrm{RA} = \max\left(\left|a_{y2_{\mathrm{CG}}}\right|\right) \Big/ \max\left(\left|a_{y1_{\mathrm{CG}}}\right|\right), \tag{17}$$

where $a_{y1_{\mathrm{CG}}}$ (m/s^2) is the lateral acceleration of the tractor's CG and $a_{y2_{\mathrm{CG}}}$ (m/s^2) is the lateral acceleration of the trailer's CG.

This is slightly different from the RA defined above. However, the normalized values of the RA relative to the conventional vehicle are comparable. As shown in Figs. 24 and 25, the RA of the conventional vehicle without control is 1.144, while that of the vehicle with active steering control is reduced to 0.838. So, the active steering controller has good performance in improving the rollover of articulated vehicles.

It can be concluded from the simulation results that the lateral stability and the rollover of the articulated vehicle can benefit a lot from the active steering controller.

5 Conclusions

This paper focused on the active steering controller for articulated vehicles based on the LQR theory. The following conclusions can be drawn from this study:

1. The 3-DOF model of the tractor-trailer with steered trailer axles has been built. The SAPSO algorithm was applied to identify the key parameters of the model under specified vehicle speeds and steering wheel angles, and thus the key parameters of the simplified model can be obtained according to the vehicle condition by interpolation. The simulation results showed that vehicle parameter outputs of the simplified model and TruckSim agreed well, providing the ideal reference yaw rate for the controller.

2. The active steering controller for the tractor and trailer based on LQR has been proposed. The tractor and trailer active steering controller was designed to follow the desired yaw rates of the tractor and trailer and minimize the tractor and trailer's CG side-slip angles at the same time.

3. The test results at a low speed showed that the active steering controller can improve the maneuverability of the articulated vehicle along sharp curves (rectangular curves).

4. The results showed that the stability indexes of the tractor and the trailer at a high speed can be controlled in a low range simultaneously, which indicates the controller's significant effects in improving the lateral stability at a high speed for articulated vehicles.

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