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Dynamic aspects of domination networks*

Yu-xian LIU[‡1,2], Ronald ROUSSEAU[3,4]

[1]*Tongji University Library, Tongji University, Shanghai 200092, China*
[2]*School of History and Archives, Yunnan University, Kunming 650091, China*
[3]*Faculty of Social Sciences, University of Antwerp, Antwerpen 2020, Belgium*
[4]*ECOOM, KU Leuven, Leuven 3000, Belgium*
E-mail: yxliu@tongji.edu.cn; ronald.rousseau@kuleuven.be

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Abstract: A dynamic quantitative theory and measurement of power or dominance structures are proposed. Such power structures are represented as directed networks. A graph somewhat similar to the Lorenz curve for inequality measurement is introduced. The changes in the graph resulting from network dynamics are studied. Dynamics are operationalized in terms of added nodes and links. Study of dynamic aspects of networks is essential for potential applications in many fields such as business management, politics, and social interactions. As such, we provide examples of a dominance structure in a directed, acyclic network. We calculate the change in the D-measure, which is a measure expressing the degree of dominance in a network when nodes are added to an existing simple network.

Key words: Domination; Power structure; Digraphs; Network dynamics
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1 Introduction

The universe is often viewed through the dualities of high and low, left and right, yin and yang, or dominance and subordination. Yet, each of these two opposing elements is not usually present in equal amounts. We explore how these notions can be expressed in mathematical terms using zero-sum arrays (Liu et al., 2017). Specifically, these zero-sum arrays will be used to study dominance and subordination.

Network theory is an essential part of contemporary science. Biological networks, such as protein-protein networks, computer networks for resource sharing or providing connectivity, social networks, business networks, and scientific collaboration networks are among the best known networks. Our point of focus is the study of dominance, defined in detail further on in the paper. Scientific disciplines benefit if networks representing (part of) the discipline are studied from many angles, with dominance and subordination being two of many possible aspects. Because science is a formal and an informal structure, it also includes dominance structures, which may change, for instance, when top scientists change affiliations. An interesting example of a power structure was provided in a criminal or dark network (Toth et al., 2013).

Power structures are ubiquitous. Blogs and information systems are modern forms of social power (Wei, 2009; Liu et al., 2013; Lu et al., 2015; Nord et al., 2016). Politics often revolves around having or not having the power to change society. Sales (1991), for instance, studied the relation between the state and the society to which it is linked in terms of power structures. Universities and systems of education also have been described as power structures (van de Graaff et al., 1978; Clark, 1987). Power structures

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‡ Corresponding author
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ORCID: Yu-xian LIU, https://orcid.org/0000-0001-9294-919X; Ronald ROUSSEAU, https://orcid.org/0000-0002-3252-2538

among academics may influence scholarly impact in research (Truex et al., 2009). Empowerment of women is another topic that has lately received special attention (Gutiérrez et al., 2000; Pacardo-Mercado, 2013; Oreglia and Srinivasan, 2016). Yet, the majority of these studies are qualitative in nature. Hence, a quantitative framework for measuring power or dominance may be of general interest. This has been provided in Liu et al. (2017). In this earlier work we discussed local and global dominance as special network structures.

In this new investigation we discuss dynamic aspects of dominance structures by adding nodes and links to a network. Studying dynamic aspects of networks is essential for potential applications across various fields. In business management, for instance, employees get promoted or receive new responsibilities, possibly leading to a changed power structure in the organization. This happens on a much larger scale when two companies merge, where tensions may also emerge as new boundaries are formed. These aspects are not studied in this paper, but we refer to Montgomery and Oliver (2007) where, among other things, boundary-spanning activities were studied in relation to team-based structures, network organizations, inter-organizational alliances, and professional–organizational integration. Dominance relations also abound in the animal kingdom. The formation of such relations (pecking orders) among chickens was studied in Chase (1982), while Shizuka and McDonald (2012), in a clearly quantitative study, provided a mathematical/statistical approach to the organization of dominance relations in a network context. In our earlier work (Liu et al., 2017), we have already made the following observation related to a changing situation: adding one extra subordinate node, i.e., a node with a negative flow, to a maximum D-graph leads to a network with a higher dominance, globally as well as locally. We reiterate that in the previous study and also here the focus of our attention is directed at structures, not elements or single nodes. This work is an extension of Liu and Rousseau (2017).

2 Zero-sum arrays and D-curves

In this section we briefly recall the definitions and main results in Liu et al. (2017) as these will be necessary for understanding the developments presented in this paper.

2.1 Definition of arrays

If $\boldsymbol{X}$ is a (finite) array, i.e., an N-tuple, then the i^{th} element of $\boldsymbol{X}$ is denoted as $(\boldsymbol{X})_i=x_i$, where x_i is a real number. Components of any array used in this work are assumed to be ranked in decreasing order. If $\boldsymbol{X}$ is an array then $-\boldsymbol{X}$, referred to as the opposite array, denotes the array where every component x_i is replaced by its opposite, namely $-x_i$. Also, the components of $-X$ are ranked in decreasing order. After re-ranking, we obtain the following relation: $(-\boldsymbol{X})_i=(\boldsymbol{X})_{N-i}=x_{N-i}$.

Example 1 If $\boldsymbol{X}$=(5, 1, −7), then $-\boldsymbol{X}$=(7, −1, −5).

2.2 Definition of a zero-sum array

If $\boldsymbol{X}=(x_1, x_2, \ldots, x_N)$ is a real-valued array such that $\sum_{i=1}^{N} x_i = 0$, then $\boldsymbol{X}$ is called a zero-sum array. The set of all zero-sum arrays is denoted as $\boldsymbol{Z}$; its subset of arrays of length N, namely $\boldsymbol{Z}\cap\mathbb{R}^N$, is denoted as $\boldsymbol{Z}_N$.

2.3 Construction of a pseudo-Lorenz curve for zero-sum arrays

Supposing that $\boldsymbol{X}$ is a zero-sum array, we set

$$\begin{cases} I_+(\boldsymbol{X}) = \{i \in \{1,2,\ldots,N\} \text{ such that } x_i > 0\}, \\ I_0(\boldsymbol{X}) = \{i \in \{1,2,\ldots,N\} \text{ such that } x_i = 0\}, \\ I_-(\boldsymbol{X}) = \{i \in \{1,2,\ldots,N\} \text{ such that } x_i < 0\}. \end{cases}$$

As in Liu et al. (2017), we assume that $\boldsymbol{X}$ is not the trivial zero array; hence, $I_0(\boldsymbol{X})$ is not equal to the set of all natural numbers from 1 to N. This requirement implies that sets $I_+(\boldsymbol{X})$ and $I_-(\boldsymbol{X})$ are always non-empty, but they may have different numbers of elements. It also follows that N>1. We simply write I_+, I_0, or I_- when it is clear about which array we are talking.

Note that $\sum_{i\in I_+} x_i = -\sum_{i\in I_-} x_i$. We next set $\Sigma_+ = \sum_{i\in I_+} x_i$ and $\forall i$=1, 2, …, N: $a_i=x_i/\Sigma_+$. With each zero-sum array $\boldsymbol{X}$, we associate a corresponding A-array, denoted as $\boldsymbol{A_X}$, with $\boldsymbol{A_X}=(a_1, a_2, \ldots, a_N)$. Clearly, $\boldsymbol{A_X}$ is also a zero-sum array. Furthermore, in our study we need the array $\boldsymbol{Q_X}$, with $(\boldsymbol{Q}_X)_j = q_j = \sum_{k=1}^{j}|a_k|$. Clearly, q_N, the last element in $\boldsymbol{Q_X}$, is equal to two.

2.4 Construction of a D-curve

D-curves of a zero-sum array $\boldsymbol{X}$, where symbol D stands for dominance, were introduced in Liu et al. (2017). Yet, for the sake of the reader we recall its construction: a D-curve of a zero-sum array $\boldsymbol{X}$ is defined as the polygonal line connecting the points:

$$(0,0)\to\left(\frac{1}{N},q_1\right)\to\ldots\to\left(\frac{i}{N},q_i\right)\to\ldots\to\left(\frac{|I_+|}{N},1\right)\to\left(\frac{N-|I_-|}{N},1\right)\to\ldots\to\left(\frac{k}{N},q_k\right)\to\ldots\to(1,2),$$

where $i\in I_+$ and $k\in I_-$.

The above definition is actually the description of a graph. Described as a function, this graph is denoted as $D_{\boldsymbol{X}}(t)$, $t\in[0, 1]$. We see that a D-curve is partly concave (the first part), and partly convex (the last part), as illustrated in Fig. 1.

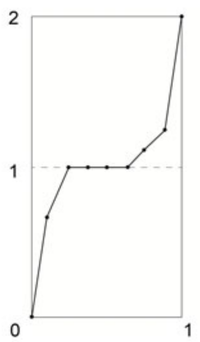

Fig. 1 D-curve of array (4, 2, 0, 0, 0, −1, −1, −4)

If $|I_+|\neq N-|I_-|$, where $|\cdot|$ denotes the number of elements in a set, then the D-curve has a horizontal part in the middle, at a vertical value equal to one.

Example 2 The D-curve of (4, 2, 0, 0, 0, −1, −1, −4), N=8, has an A-array (array of a-values) equal to $\boldsymbol{A}$=(4/6, 2/6, 0, 0, 0, −1/6, −1/6, −4/6). Hence, it connects points with ordinates (0, 4/6, 6/6, 6/6, 6/6, 6/6, 7/6, 8/6, 12/6=2). Fig. 1 shows this D-curve.

If $n_0=|I_+|$, then it follows that $D_{\boldsymbol{X}}(n_0/N)=1$. In the example of Fig. 1, n_0=2 and we see that the point with abscissa 2/8 indeed has an ordinate equal to 1.

2.5 Definition of equivalent zero-sum arrays

Zero-sum arrays that have the same D-curve are said to be equivalent. Arrays (4, 2, 0, 0, −1, −5), (8, 4, 0, 0, −2, −10), and (4/6, 2/6, 0, 0, −1/6, −5/6) are examples of equivalent arrays. Equivalent zero-sum arrays of length N all have the same A-array. Additionally, arrays such as (3, 2, 0, −5) and (3, 3, 2, 2, 0, 0, −5, −5) are equivalent zero-sum arrays, but with different lengths.

2.6 Partial orders for zero-sum arrays

Definition 1 (Dominance relation $\leq_D$ in $\boldsymbol{Z}$) Let $\boldsymbol{X}$ and $\boldsymbol{Y}$ be zero-sum arrays, where the length of the arrays can be different. Then we say that $\boldsymbol{X}$ is D-smaller than $\boldsymbol{Y}$, denoted as $\boldsymbol{X}\leq_D\boldsymbol{Y}$ (or $\boldsymbol{Y}\geq_D\boldsymbol{X}$) if, for each $t\in[0, 1]$, $D_{\boldsymbol{X}}(t)\leq D_{\boldsymbol{Y}}(t)$. $\boldsymbol{X}$ is strictly D-smaller than $\boldsymbol{Y}$, denoted as $\boldsymbol{X}<_D\boldsymbol{Y}$ if, for each $t\in[0, 1]$, $D_{\boldsymbol{X}}(t)\leq D_{\boldsymbol{Y}}(t)$ and there is at least one point t_0 (and hence infinitely many) where $D_{\boldsymbol{X}}(t_0)<D_{\boldsymbol{Y}}(t_0)$.

When $\boldsymbol{X}\leq_D\boldsymbol{Y}$, it is clear that the D-curve of $\boldsymbol{X}$ lies completely below the D-curve of $\boldsymbol{Y}$. It is now obvious that the relation $\leq_D$ determines a partial order in the set of all equivalence classes of zero-sum arrays. This observation can be written formally as

$$\boldsymbol{X}\leq_D\boldsymbol{Y}\ \text{if and only if}\ \forall t\in[0,1]: D_{\boldsymbol{X}}(t)\leq D_{\boldsymbol{Y}}(t),$$
$$\boldsymbol{X}=\boldsymbol{Y}\ \text{if and only if}\ \forall t\in[0,1]: D_{\boldsymbol{X}}(t)= D_{\boldsymbol{Y}}(t).$$

As the dominance relation $\leq_D$ is only a partial order, some arrays cannot be compared. For this reason they are said to be intrinsically incomparable. When discussing concrete dominance structures in this paper, we always mean the partially ordered set determined by the relation $\leq_D$. Next, we recall the following result, which will be needed further on:

Proposition 1 (Liu et al., 2017) If $\boldsymbol{X}\leq_D\boldsymbol{Y}$, then $-\boldsymbol{Y}\leq_D-\boldsymbol{X}$.

2.7 Maximum and minimum D-curves

2.7.1 Maximum D-curves

For a fixed N, the maximum D-curve occurs when the origin (0, 0) is connected to the point with coordinates (1/N, 1), and then further linearly connected to the endpoint (1, 2). This D-curve corresponds to all zero-sum arrays of the form $\boldsymbol{X}$=(s, −t, …,

$-t$), with s, t>0 and s=(N−1)t. Clearly, considering N as a variable, the line y=x+1, connecting points (0, 1) and (1, 2), is an upper bound for all these D-maximum N-curves.

2.7.2 Minimum D-curves

For a fixed N, a minimum D-curve is obtained by linearly connecting the origin (0, 0) to the point with coordinates ((N−1)/N, 1), and then further to point (1, 2). This minimum D-curve corresponds to all arrays of the form $\boldsymbol{Y}$=(u, …, u, −v), with u, v>0 and v=(N−1)u. If $\boldsymbol{X}$ is a maximum N-array, then −$\boldsymbol{X}$ is a minimum one. The line y=x, connecting the origin and point (1, 1), is a lower bound for all these minimum D-curves.

In Proposition 2, we note the first dynamic aspect:

Proposition 2 If N increases, then the maximum D-curve also becomes larger in the partial order of D-curves. Similarly, the minimum D-curves become smaller.

2.8 A measure respecting the dominance relation $\leq_D$ in $\boldsymbol{Z}$

From the previous constructions and definitions we note that the area between the D-curve and the line y=x respects the D partial order. This area is denoted as $\mathrm{AR_D}(\boldsymbol{X})$. For any zero-sum array this area takes values on the interval [0, 1]. We refer to the corresponding numerical value as the D-measure: it is denoted as $\mathrm{AR_D}$ and is calculated by the following formula:

$$\mathrm{AR_D}(\boldsymbol{X})=\frac{1}{N}\sum_{i=1}^{N}q_i-\frac{N+2}{2N},$$

where the q-values are the components of the array $\boldsymbol{Q}_X$ defined earlier.

Example 3 For $\boldsymbol{X}$=(5, 2, 0, −3, −4), the D-measure is obtained as follows:

$$\mathrm{AR_D}(\boldsymbol{X})=\frac{1}{5}\left(\frac{5}{7}+\frac{7}{7}+\frac{7}{7}+\frac{10}{7}+\frac{14}{7}\right)-\frac{7}{10}=\frac{37}{70}.$$

We already know that the maximum D-curves correspond to arrays of the form (s, −t, …, −t), with s, t>0. They have D-measures equal to (N−1)/N. The minimum D-curves correspond to arrays such as $\boldsymbol{Y}$=(u, …, u, −v), u, v>0. They have D-measures equal to 1/N. For a fixed N, the minimum D-curves are opposites of the maximum D-curves. Clearly, their D-measures sum to 1.

2.9 Transfer property for D-curves

Recall that Dalton's transfer property (Dalton, 1920) states that if one takes from a poorer item (person or household) and gives to a richer one, inequality increases. Obviously, the transfer principle does not hold for D-curves. If one takes a positive amount from a negative item and gives to one that is less negative, but still stays non-positive, this operation decreases the D-curve and hence the D-measure decreases too. We refer to the result of such a transfer as an opposite transfer principle.

2.10 Applications to directed networks

We will use D-curves to measure the dominance power in an acyclic or loopless graph.

The number of edges in digraph G with node j as their initial (terminal) node is called the out-degree (in-degree) of node j. These numbers are denoted as $\alpha_j^+(\alpha_j^-)$. Now we put $\alpha_j=\alpha_j^+-\alpha_j^-$ (Egghe and Rousseau, 2004). Parameter α_j characterizes the flow through node j. More precisely, if it is positive, there are more edges leaving node j than reaching it. The number of edges in G, denoted as ε, is related to the degrees of its nodes by the following equation:

$$\varepsilon=\sum\nolimits_j\alpha_j^+=\sum\nolimits_j\alpha_j^- \text{ or } \sum\nolimits_j\alpha_j=0,$$

where the summation is over all nodes of graph G. Because sequence $(\alpha_j)_j$ is a zero-sum array, we can apply D-theory to it. When using sequence $(\alpha_j)_j$, this theory will be referred to as a local dominance theory, or LDT for short. The number α_j is called a local flow number or simply the local flow, and the corresponding zero-sum array is called a local flow array. In this study, we will use the terminology of local flow to contrast it to the global flow.

We further consider a global dominance theory, or GDT for short, where we use arrays of the form $\boldsymbol{\Sigma}$=(σ_1, σ_2, …, σ_N), defined as follows:

$$\sigma_i=\sigma_i^+-\sigma_i^-.$$

Here, σ_i^+ denotes the sum of the lengths of the chains starting from node i, and σ_i^- denotes the sum of the lengths of all chains ending at node i.

Definition 2 A local source of a digraph is a node having in-degree zero, and a strictly positive out-degree.

If a local source can reach any other node in a digraph, it is called a network source.

Definition 3 A local sink of a digraph is a node having out-degree zero, and a strictly positive in-degree.

Before continuing our discussion we recall the following definitions:

Definition 4 (Dominance nodes) A node with the highest global flow in a D-graph is called a global dominance node; a node with the highest local flow in a D-graph is called a local dominance node.

Next we study the graphs for the maximum and minimum D-curves in LDT and GDT.

Proposition 3 For a fixed N, the graph shown in Fig. 2 yields the only graph corresponding to a maximum global D-array.

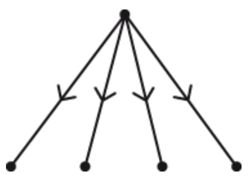

Fig. 2 An example of an N-node graph corresponding to a maximum D-curve (N=5)

Proposition 4 The maximum D-graph for LDT is the same as that for GDT.

Proposition 5 For a fixed N, the graph shown in Fig. 3 yields the only graph corresponding to a minimum D-array.

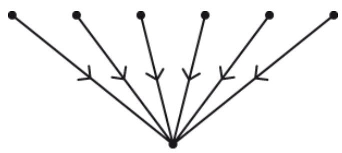

Fig. 3 A graph corresponding to the minimum (local and global) D-curve (N=7)

Note that the minimum curves can be obtained by reversing the direction of the arrows of the maximum curves. Of course, the maximum and minimum curves are extreme cases and situations in between are more common.

2.11 Terminology and meaning: hierarchies versus power (dominance)

In this short subsection we want to explain our terminology. Consider, for instance, the digraph shown in Fig. 2. There is not much hierarchical structure here, but the digraph reflects a very strong power structure: that of one ruler and many equally powerless subordinates.

In applications of D-curves to institutes, research groups, or scientists as nodes, we want to gauge the extant power structure. The greater the inequality among the positive nodes, the more powerful the order relation. However, it is also true that the more even the negative nodes (in the sense of evenness as defined in Nijssen et al. (1998)), the more powerful the extant order structure.

3 Dynamic aspects of networks and properties of D-curves

3.1 Modeling: the need for examples and case studies

When a new measure is proposed, one usually derives theoretical properties and explains the possible benefits of using such a measure. This was done in Liu et al. (2017). Studying dynamic aspects of networks and their corresponding dominance measures is essential for potential applications in fields such as business management, politics, and social interactions. We already mentioned examples in politics (Sales, 1991), and universities and systems of education (van de Graaff et al., 1978; Clark, 1987). We further recalled the important case of changing power structures when two companies merge.

3.2 Dynamic series of networks

We already know that adding a node in a digraph which dominates the network source makes this new node the network source; hence, it becomes a global dominance node. If the network is a linear structure, then the new node also becomes the local dominance node.

Linear structures are clear hierarchies, but they are not interesting in the context of power structures:

they are always intrinsically incomparable, locally as well as globally, and because of their symmetry, their D-measures are always equal to 0.5.

Next we consider the dynamic series of networks as illustrated in Fig. 4.

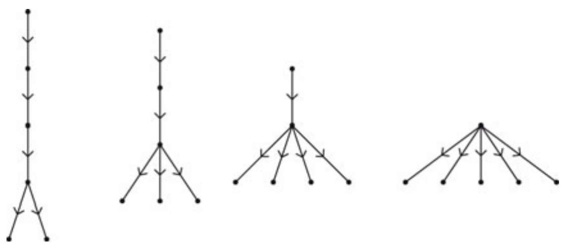

Fig. 4 Four graphs with increasing D-measures

Except for the last graph, there are always two nodes with a strictly positive flow. For a given N, such local arrays have the following structure, $\left(K,1,\underbrace{0,\dots,0}_{(N-K-3)\text{ times}},\underbrace{-1,\dots,-1}_{(K+1)\text{ times}}\right)$ for K=1 to N−3. In the example shown in Fig. 4, N=6 and these arrays are (1, 1, 0, 0, −1, −1), (2, 1, 0, −1, −1, −1), and (3, 1, −1, −1, −1, −1). To these, we add the array (5, −1, −1, −1, −1, −1). The D-measures for these arrays are 0.5, 0.611, 0.708, and the largest one has a D-value equal to 0.833 (namely 5/6). In all cases, these arrays clearly reflect an increasing dominance structure for K=1 to N−3. The last array in the row, being a maximum D-array, is the largest in the dominance structure.

Adding an arrow from a node with positive flow to another node with strictly positive flow, but with a flow value which is at most equal to that of the first one, increases the dominance structure. This result holds, however, only for the local theory. It follows from the fact that the transfer principle holds among nodes with a strictly positive flow. Indeed, adding a link leads to a local increase of one for the node from which the links start, and reduces the flow value by one for the node in which the link terminates. An example is provided in Figs. 5 and 6.

The network shown in Fig. 5 has a local D-array (3, 3, 2, −1, −1, −1, −1, −1, −1, −2), with a D-measure equal to 0.675. If we add an arrow from node b to node d, we obtain the local D-array (4, 3, 1, −1, −1, −1, −1, −1, −1, −2), with a D-measure equal to 0.7. Local D-curves for the original situation as well as for the new one are shown in Fig. 6.

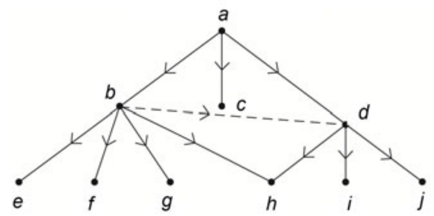

Fig. 5 Network with local D-array (3, 3, 2, −1, −1, −1, −1, −1, −1, −2)

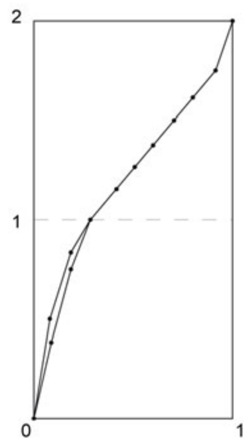

Fig. 6 D-curves corresponding to Fig. 5 and a positive transfer

In general, however, adding an arrow changes the global D-curve in an unpredictable way. The global D-array of the network shown in Fig. 5 is (17, 3, 2, −1, −3, −3, −3, −3, −3, −6), with a D-measure of 0.7. Node b has a global flow equal to 3 and node d has a global flow equal to 2. When we add a link from node b to node d, the new D-array becomes (28, 10, −1, −1, −3, −3, −3, −8, −8, −11), with a D-measure of 0.666, where node b now has a global flow equal to 10 and node d has a global flow equal to −1. The corresponding global D-curves intersect and hence these two D-arrays are intrinsically incomparable (Fig. 7).

If we add an arrow, and the positive global flow of a node remains positive, then it is possible that the corresponding D-curves are comparable. If, however, a node's positive global flow becomes negative, then the corresponding D-curves cannot be comparable.

Adding an arrow from a node which is non-positive to a node that does not have a larger flow value decreases the local dominance structure. This again follows from the fact that the opposite transfer principle holds among nodes with non-positive flow values. For example, if, in Fig. 5, we add an arrow from node c to node h, we obtain the local D-array (3, 3, 2, 0, −1, −1, −1, −1, −1, −3) with D-measure equal to 0.6, leading to the local D-curve shown in Fig. 8.

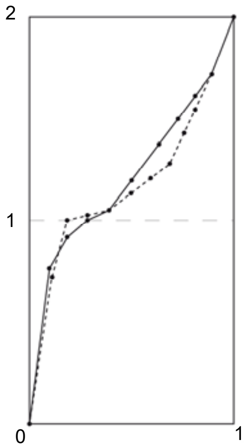

Fig. 7 An example of two global D-arrays resulting from adding an arrow from a node with a positive flow to one with a lower positive flow

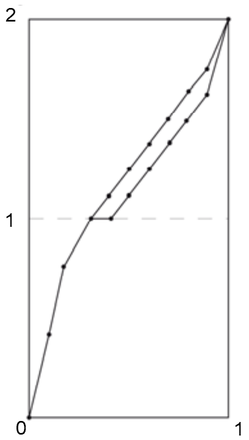

Fig. 8 Local D-curves illustrating the opposite transfer principle

Proposition 6 Given a fixed number of nodes with positive values (and a fixed number of nodes with negative values), the higher the dominance structure is, the more concentrated the positive values are (concentrated in the usual Lorenz sense). Moreover, the more even nodes with a negative flow are, the more concentrated the structure is as a whole. Here the term "even" is used in the sense of evenness for Lorenz curves (Lorenz, 1905; Nijssen et al., 1998; Marshall et al., 2011; Rousseau, 2011).

Proposition 6 again follows from the transfer principle.

What happens if we add an arrow from a node with a positive flow value to a node with a negative flow value? Before trying to answer this question, we first show an interesting example (Fig. 9). The figure on the left has a local array $\boldsymbol{X}$=(3, 0, −1, −1, −1) with a D-measure equal to 0.7, and a global array $\boldsymbol{X}'$=(5, 0, −1, −1, −3) with a D-measure equal to 0.62. Adding an arrow from the dominance node a to the most subordinate node e yields arrays $\boldsymbol{Y}$=(4, 0, −1, −1, −2) with a D-measure equal to 0.65 and $\boldsymbol{Y}'$=(6, 0, −1, −1, −4) with a D-measure equal to 0.6.

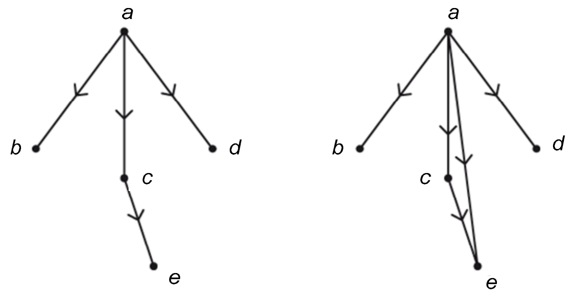

Fig. 9 Adding an arrow from a dominance node to the most subordinate node

We see from Fig. 10 that $\boldsymbol{Y}<_{\mathrm{D}}\boldsymbol{X}$. Similarly, one finds that $\boldsymbol{Y}'<_{\mathrm{D}}\boldsymbol{X}'$. The reason is that while nothing has changed to the other nodes, node e came "closer" to the dominating node, and hence became less subordinate. Values of the D-measure confirm this observation.

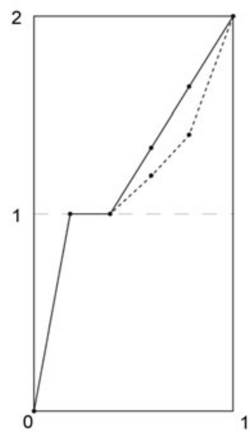

Fig. 10 Local D-curves for the graph in Fig. 9

The previous example is an illustration of the following proposition, where we make a distinction between the local case and the global case.

Proposition 7

1. Local case. Assume that a network has only one node with a strictly positive local flow; hence, this node is the network source. Consider a node with a strictly negative flow, which is moreover not directly linked to this source. Then, adding a link from the source to this node, one obtains a local D-curve which is strictly lower than the original one.

2. Global case. Assume that a network has only one node with a strictly positive global flow; hence, this node is the network source. Consider a node with a strictly negative global flow, which is moreover not directly linked to this source. Then, adding a link from the source to this node, one obtains a global D-curve which is strictly lower than the original one.

Proof First let us prove the local case. Assume there are N nodes in the network and we consider the D-array $(x_1, x_2, \ldots, x_N)$. By assumption the sum of all positive degrees is x_1. Adding an arrow from the global source to another node leads to a local array of the form $(x_1+1, x_2, \ldots, x_j-1, \ldots, x_N)$ (possibly after re-ranking), with j equal to one of the numbers $\{2, 3, \ldots, N\}$.

As the second node, after re-ranking, is at most equal to zero and as there are at least three nodes (if there were two nodes they had to be directly connected), we see that this operation lowers the negative part of the graph. Such a case is illustrated in Fig. 11. This figure shows a new link (dotted line) added from node a to node f. Originally, the local array was (2, 0, 0, 0, 0, 0, −1, −1) with a D-measure equal to 0.563. After the addition of a new link, the new local array is (3, 0, 0, 0, 0, 0, −1, −2) with a D-measure equal to 0.542.

Next we consider the global case. Consider a node n which is neither the network source nor a sink. We know that n is not directly linked to the source. By assumption it is indirectly linked to the network source. Then, because n is not a sink, at least one intermediate node has a strictly positive global flow. Yet, because it is assumed that there is only one node with a strictly positive global flow, this case is excluded. Consequently, a node different from the source and not directly linked to it must be a sink. Then all nodes except the network source and the sinks must have a flow value equal to zero. Assuming there are (k_1+k_2) sinks, namely k_1 directly connected to the source and k_2 indirectly connected, then we are dealing with global arrays of the form

$$\left(x, \underbrace{0, \ldots 0}_{k_2 \text{ times}}, \underbrace{-1, \ldots, -1}_{k_1 \text{ times}}, \underbrace{-3, \ldots, -3}_{k_2 \text{ times}}\right).$$

Here, $x=k_1+3k_2$. Adding a link from the source to an unconnected sink leads to the array $\left(x+1, \underbrace{0, \ldots 0}_{k_2 \text{ times}}, \underbrace{-1, \ldots, -1}_{k_1 \text{ times}}, \underbrace{-3, \ldots, -3}_{(k_2-1) \text{ times}}, -4\right)$. Its D-curve is situated under the original one. Fig. 12 illustrates this part of Proposition 7. In this figure a new link is added from node a to node f. Originally, the global array is (10, 0, 0, 0, −1, −3, −3, −3) with a D-measure equal to 0.65. After the addition of a new link, the new global array is (11, 0, 0, 0, −1, −3, −3, −4) with a D-measure equal to 0.636.

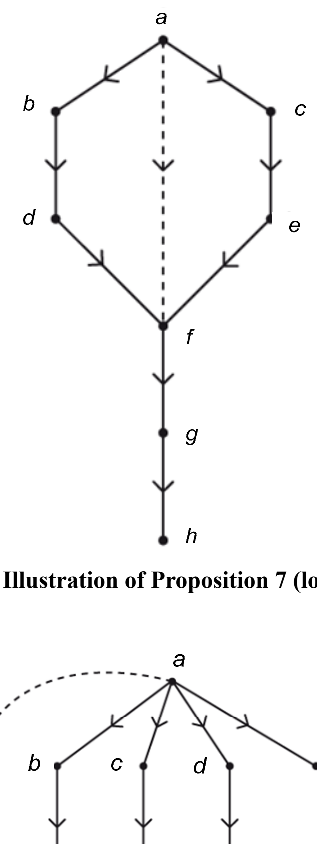

Fig. 11 Illustration of Proposition 7 (local case)

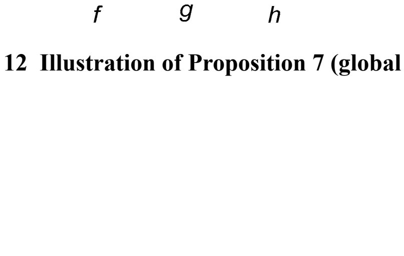

Fig. 12 Illustration of Proposition 7 (global case)

4 Adding a node that becomes the network source or sink

Next we study the following operations:

(O1) Adding a node to a maximum D-graph which links only to the network source

This implies that the new node is the new network source. The resulting network, however, is not a maximum one.

(O2) Adding a node to a maximum D-graph which links to every other node

In this constellation the new node is the new network source and the resulting network is not a maximum.

(O3) Adding a node to a minimum D-graph in such a way that the network sink links to it

This implies that the new node is the new network sink and that the resulting network is not a minimum one anymore.

(O4) Adding a node to a minimum D-graph such that all other nodes link to it

This operation implies that the new node is the network sink and the network is not a minimum one anymore.

We already note that operations (O1) and (O2) applied to a maximum D-curve never turn a node with a negative flow into one with a positive flow.

Example 4 Operations (O1) and (O2) are illustrated in Fig. 13.

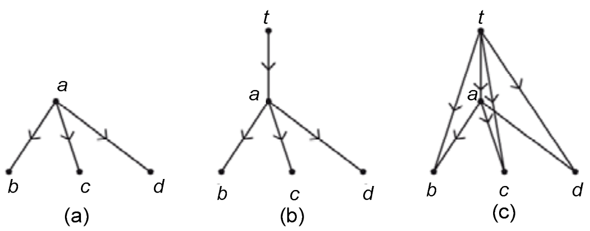

Fig. 13 Three cases: (a) a maximum dominance graph; (b) the graph created by adding one node, *t*, linked to the network source; (c) the graph created by adding one node, *t*, directly linked to each other node

Fig. 13a has the same local and global array, namely (3, −1, −1, −1) with a D-measure of 0.75. The local array of Fig. 13b is (2, 1, −1, −1, −1) with a D-measure of 0.633, while its global array is (7, 2, −3, −3, −3) with a D-measure of 0.656. Fig. 13c leads to the local array (4, 2, −2, −2, −2) with a D-measure of 0.633, and the corresponding global array (10, 2, −4, −4, −4) with a D-measure of 0.667. These D-values are brought together in Table 1.

Table 1 D-values resulting from Fig. 13

Item	D-value		
	Fig. 13a	Fig. 13b	Fig. 13c
Local	0.75	0.633	0.633
Global	0.75	0.656	0.667

Though we add a source to a graph corresponding to a maximum D-curve, the dominance in this new graph is not higher than that in the original one. Moreover, though the local D-curves of Figs. 13b and 13c coincide, we notice that the network source in Fig. 13b is the new node *t*. In Fig. 13b, node *a* is a dominance node in the local sense.

Focusing for the moment on operations (O1) and (O2) for which the new node becomes the network source, we consider the case of adding a node to a maximum D-curve. In this way we distinguish four cases (Table 2). Recall that we start from a maximum D-curve with *N* nodes. Its local and global arrays are the same, i.e., $\mathbf{MX}=\left(N-1,\underbrace{-1,-1,\ldots,-1}_{(N-1)\ \text{times}}\right)$. The corresponding D-curve consists of two segments. The first connects (0, 0) with (1/*N*, 1), while the second connects (1/*N*, 1) to (1, 2). These segments have slopes *N* and *N*/(*N*−1), respectively.

The four cases indicated in Table 2 always lead to D-curves consisting of three segments. In each case the slope of the final segment is the same and equal to (*N*+1)/(*N*−1), which is always strictly larger than *N*/(*N*−1). Clearly, this line segment is situated under the original one as it is steeper and ends in the same point. This already shows that the new graph can never be situated above the original one. The original and the new graphs can be incomparable or the new one can be smaller than the old one. The resulting four curves are never the maximum D-curves. Exact results are formulated in Propositions 8–11.

Proposition 8 If a node is linked only to the source of a maximum D-curve with *N* nodes, then the resulting local D-curve is always smaller in the D-order.

Proof The slope of the first segment resulting from this operation is (*N*+1)(*N*−2)/(*N*−1). As (*N*+1)(*N*−2)/(*N*−1)<*N* the new D-curve starts under the original one, and because it also ends under the original one, the new local D-curve is smaller than the original one.

Table 2 Arrays resulting from operations (O1) and (O2), applied to a maximum D-curve

Type	Adding nodes and links to only the source (O1)	Adding nodes and links to all existing nodes (O2)
LDT	$\mathbf{MX}_{L1}=\left(N-2,1,\underbrace{-1,-1,\ldots,-1}_{(N-1)\text{ times}}\right)$	$\mathbf{MX}_{L2}=\left(N,N-2,\underbrace{-2,-2,\ldots,-2}_{(N-1)\text{ times}}\right)$
GDT	$\mathbf{MX}_{G1}=\left(2N-1,N-2,\underbrace{-3,-3,\ldots,-3}_{(N-1)\text{ times}}\right)$	$\mathbf{MX}_{G2}=\left(3N-2,N-2,\underbrace{-4,-4,\ldots,-4}_{(N-1)\text{ times}}\right)$

Proposition 9 If a node linked only to the source is added to a maximum D-curve with N nodes then, for $N>3$, the resulting global D-curve is always smaller in the D-order. If $N=2$ or 3, the original curve and the new one intersect; hence, the corresponding arrays are incomparable.

Proof The slope of the first segment resulting from this operation is equal to $(N+1)(2N-1)/(3N-3)$. Because $(N+1)(2N-1)/(3N-3)<N$ for $N>3$, the new D-curve starts under the original one; as we already know that the new D-curve always ends under the original one, we conclude that, for $N>3$, the new global D-curve is smaller than the original one. For $N=2$ or 3, the new D-curve starts above the original one and ends below it; hence, the two curves are incomparable.

Proposition 10 If a node linked to all existing nodes in a maximum D-curve is added to this graph, then the resulting local D-curve is, for $N>2$, smaller in the D-order. For $N=2$, the two arrays are incomparable.

Proof The slope of the first segment of the new curve resulting from this operation is equal to $N(N+1)/(2N-2)$. Because $N(N+1)/(2N-2)<N$ for $N>3$, the new local D-curve is, in these cases, smaller than the original one. If $N=3$, the slope of the first segment of the new curve is 3, which coincides with that of the original curve. Because the new curve ends under the original one, this means that also for $N=3$, the new local D-curve is smaller than the original one. Finally for $N=2$, the slope of the first segment is 3, larger than 2, the slope of the original one. Hence, in this case the two curves are incomparable.

Proposition 11 If a node linked to all existing nodes in a maximum D-curve is added to this graph, then the resulting global D-curve is, for $N>4$, smaller in the D-order. For $N=2$, 3, or 4, the curves intersect and hence the arrays are incomparable.

Proof The slope of the first segment of the new curve resulting from this operation is equal to $(N+1)(3N-2)/(4N-4)$. Because $(N+1)(3N-2)/(4N-4)<N$ for $N>4$, the new local D-curve is, in these cases, smaller than the original one. For $N=2$, 3 or 4, the two curves are incomparable.

Next we consider operations (O3) and (O4) for which the new node becomes the network sink. We consider the case of adding a node to a minimum D-curve. In this way we again distinguish four cases (Table 3). For a minimum D-curve with N nodes, the local and global arrays are the same and are equal to

$$\mathbf{MN}=\left(\underbrace{1,1,\ldots,1}_{(N-1)\text{ times}},-(N-1)\right).$$

Table 3 Arrays resulting from operations (O3) and (O4), applied to a minimum D-curve

Type	Adding nodes and links to only the sink (O3)	Adding nodes and links to all existing nodes (O4)
LDT	$\mathbf{MN}_{L1}=\left(\underbrace{1,1,\ldots,1}_{(N-1)\text{ times}},-1,-(N-2)\right)$	$\mathbf{MN}_{L2}=\left(\underbrace{2,2,\ldots,2}_{(N-1)\text{ times}},-(N-2),-N\right)$
GDT	$\mathbf{MN}_{G1}=\left(\underbrace{3,3,\ldots,3}_{(N-1)\text{ times}},-(N-2),-(2N-1)\right)$	$\mathbf{MN}_{G2}=\left(\underbrace{4,4,\ldots,4}_{(N-1)\text{ times}},-(N-2),-(3N-2)\right)$

We observe that $\mathbf{MN}=-\mathbf{MX}$, $\mathbf{MN}_{L1}=-\mathbf{MX}_{L1}$, $\mathbf{MN}_{L2}=-\mathbf{MX}_{L2}$, $\mathbf{MN}_{G1}=-\mathbf{MX}_{G1}$, and $\mathbf{MN}_{G2}=-\mathbf{MX}_{G2}$. As $\boldsymbol{X}\leq_D\boldsymbol{Y}$, which implies $-\boldsymbol{Y}\leq_D-\boldsymbol{X}$, we have the following four results:

1. If a node linked only to the sink is added to a minimum D-curve with N nodes, then the resulting local D-curve is always larger in the D-order.

2. If a node linked only to the sink is added to a minimum D-curve with N nodes, then, for $N>3$, the resulting global D-curve is always larger in the D-order. If $N=2$ or 3, the original curve and the new one intersect; hence, the corresponding arrays are incomparable.

3. If a node linked to all existing nodes in a minimum D-curve is added to this graph, then the resulting local D-curve is, for $N>2$, larger in the D-order. For $N=2$, the two arrays are incomparable.

4. If a node linked to all existing nodes in a minimum D-curve is added to this graph, then the resulting global D-curve is, for $N>4$, larger in the D-order. For $N=2$, 3, or 4, the curves intersect and hence the arrays are incomparable.

Next, we compare the two new situations in the case where we start from a maximum D-curve.

Proposition 12 (Local case) Given a maximum D-curve, if $N<4$, the D-curve obtained by adding a node which dominates all (other) nodes is higher than the D-curve resulting from adding a node that dominates the source; for $N=4$, the D-curve resulting from adding a node dominating all (other) nodes coincides with the D-curve resulting from adding a node dominating the source; if $N>4$, the D-curve of adding a node which dominates all (other) nodes is lower than the D-curve resulting from adding a node which dominates the source.

Proof The local D-array resulting from adding a node which dominates all other nodes is $(N, N-2, \underbrace{-2,\ldots,-2}_{(N-1)\ \text{times}})$; the D-curve of this array is a curve that connects

$$(0,0)\rightarrow\left(\frac{1}{N+1},\frac{N}{2N-2}\right)\rightarrow\left(\frac{2}{N+1},1\right)\rightarrow\ldots\rightarrow(1,2).$$

The array resulting from adding a node which dominates the source is $(N-2,\ 1,\underbrace{-1,\ldots,-1}_{(N-1)\ \text{times}})$. The D-curve of this array is a curve connecting

$$(0,0)\rightarrow\left(\frac{1}{N+1},\frac{N-2}{N-1}\right)\rightarrow\left(\frac{2}{N+1},1\right)\rightarrow\ldots\rightarrow(1,2).$$

These two graphs consist of three segments with coinciding third segments. The initial points of the first segment of the two curves, namely the origin, and the end points of the second segment, namely the point with coordinates $(2/(N+1), 1)$ coincide. Hence, the curves are always comparable and the relation between the two cases is completely determined by the point with abscissa $1/(N+1)$. Denoting the difference of the ordinates of this point by $\varDelta$ yields $\varDelta=\frac{N}{2N-2}-\frac{N-2}{N-1}=\frac{4-N}{2(N-1)}$. If $N<4$, then $\varDelta>0$ and the D-curve resulting from adding a node dominating all other nodes is higher than the D-curve resulting from adding a node dominating the source. If $N=4$, then $\varDelta=0$ and the two curves completely coincide. If $N>4$, then $\varDelta<0$ and the D-curve resulting by adding a node dominating all other nodes is lower than the D-curve resulting from adding a node which dominates the source.

Proposition 13 (Global case) For $N>2$, the D-curve resulting from adding a node dominating all (other) nodes in the maximum D-curve graph is higher than the D-curve resulting from adding a node dominating the dominance node. If $N=2$, then the two curves coincide.

Proof The global D-array for the first case is $(3N-2, N-2, -4, \ldots, -4)$ with $(N-1)$ times the value -4; in the second case, it is $(2N-1, N-2, -3, \ldots, -3)$, with $(N-1)$ times the value -3. These two D-curves are divided into three segments with coinciding third segments. Again, the end of the first segment is the key to determining the dominance order. The difference in ordinates, denoted as $\varDelta$, is $\frac{3N-2}{4N-4}-\frac{2N-1}{3N-3}=\frac{N-2}{12(N-1)}$. This expression is never negative. It is zero for $N=2$ and strictly positive for $N>2$. Hence, if $N>2$, the D-curve resulting from adding a node dominating all other nodes in the maximum D-curve graph is always situated higher than the D-curve resulting from adding a node dominating the dominance node. If $N=2$, the two D-curves coincide.

Similar results can be formulated for the minimum D-curves. We leave this to the reader.

5 Adding a node that is directly dominated by every other node

Proposition 14 The following statements hold for the local as well as for the global case. Adding one node that is linked directly to the source of a maximum D-curve graph increases the dominance degree of the graph; adding one node directly dominated by every other node leads to a D-curve that intersects the original one.

This proposition is illustrated in Fig. 14.

Proof Adding a node that is linked directly to the source of a graph corresponding to a maximum D-curve (Fig. 14b), increases the dominance degree of the graph, which follows from the proposition which states that if N increases, then the maximum D-curve becomes larger too (in partial order of D-curves) and this is correct for the local case as well as the global case.

Fig. 14c leads to a local array equal to $\left(N, \underbrace{0,...,0}_{(N-1)\ \text{times}}, -N\right)$ and a global array equal to $\left(3N-2, \underbrace{0,...,0}_{(N-1)\ \text{times}}, -(3N-2)\right)$, which have the same D-curves. In both cases they intersect the original graph, and hence the resulting networks are incomparable.

Note that the resulting graphs are the maximum graphs in the global hierarchy theory (Egghe, 2002). Such graphs are also maximum in the local hierarchy theory, but are not the only ones that are maximum (Egghe and Rousseau, 2004).

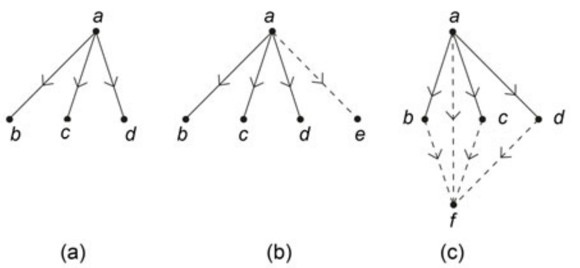

Fig. 14 Adding a node to the network (a), the case of a node dominated by the dominance node only (b), and the case of adding a node dominated by all other nodes (c)

6 Discussion

D-curves have properties making them suitable for measuring dominance: if N increases, then the maximum D-curve becomes larger, and the minimum D-curves become smaller, corresponding to the fact that dominance increases when there are more subordinates and decreases when there are fewer subordinates. In this investigation we formulate changes in terms of added nodes and links.

An important limitation of our approach is that if two nodes x and y are "equal" in the sense that in a network there is a link from x to y and vice versa, then our approach cannot be applied as this form of equality leads to a cycle so that our zero-sum theory is not applicable. We propose a study of this type of relation as a problem for further investigation.

When companies merge or a new member joins a system, re-arrangements take place. Then one may ask how the dominance structure of the system changes. Our investigation shows that only when the new node is linked to the source in the maximum dominance structure, will the dominance structure increase. Adding a node that dominates all others, even when this is the former source in the maximum dominance structure, will decrease the dominance structure.

In the introduction we hinted at possible applications in electronic networks and citation networks. With regard to the first, we think such applications are outside the scope of this contribution, and for the second application, we refer the reader to a recent publication (Liu and Rousseau, 2019).

This discussion leads to a sociological question about the relation between change in a mathematical structure and its relation to a corresponding emotional change. In practice, replacing a director by a new director does not change the mathematical structure, but may lead to some quite significant emotional changes. Similarly, putting a "super-director" in a business network, i.e., someone in a directly dominant position over all others, decreases the mathematical dominance structure (as we have shown), but may change the emotional structure of the business unit to a very large extent. Some follow-up questions would be: How long does it take before the resulting turmoil settles down and the emotional structure is stable again? To what extent does the answer to this question depend on the existing mathematical structure? Such reflections are of great interest but outside the scope of our work.

7 Conclusions

We have applied partial orders in zero-sum arrays to dominance structures in an acyclic, directed network. These arrays consist of positive and negative values. The D-curves we have constructed are partly concave and partly convex. The curves follow from their construction that other properties such as permutation and scale-invariance are satisfied.

In this paper, we have provided some further examples of a dominance structure in a directed, acyclic network. Then, we have calculated the D-measure when nodes are added to an existing simple network. We have demonstrated an interesting change in the dominance structure when a dominance interaction happens from dominating individuals (those who have positive flows in the D-array) to subordinate individuals (those with negative flows). The results show that when the system is monopolistic, i.e., just one individual has power, the dominance interaction decreases the dominance structure. Under other conditions, the dominance interaction leads to intersecting dominance curves, showing that the corresponding dominance structures are intrinsically incomparable.

Contributors

Ronald ROUSSEAU designed the research. Yu-xian LIU and Ronald ROUSSEAU performed calculations and proved the propositions. Yu-xian LIU drafted the manuscript. Ronald ROUSSEAU helped organize the manuscript.

Compliance with ethics guidelines

Yu-xian LIU and Ronald ROUSSEAU declare that they have no conflict of interest.

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