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Robust distributed model predictive consensus of discrete-time multi-agent systems: a self-triggered approach*

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Abstract: This paper investigates the consensus problem of a nonlinear discrete-time multi-agent system (MAS) under bounded additive disturbances. We propose a self-triggered robust distributed model predictive control (DMPC) consensus algorithm. A new cost function was constructed and the MAS was coupled through this function. Based on the proposed cost function, a self-triggered mechanism was adopted to reduce the communication load. Furthermore, to overcome additive disturbances, the model predictive controller of each agent iteratively solved a local min-max optimization problem under the worst-case scenario. Sufficient conditions were provided to guarantee the iterative feasibility of the algorithm and the consensus of the closed-loop MAS. For each agent, we provide concrete form of compatibility constraint and consensus error terminal region. Numerical examples are provided to illustrate the effectiveness and correctness of the proposed algorithm.

Key words: Consensus; Self-triggered control; Distributed model predictive control
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1 Introduction

As an advanced control method, model predictive control (MPC) has advantages of handling system constraints in an explicit form (Mayne et al., 2000) and implementing optimal control. Therefore, MPC has attracted extensive attention from researchers in the control field (Magni et al., 2003; Li and Shi, 2014; Rosolia et al., 2017). In addition, some large-scale MASs have emerged, such as multi-region power systems (Mohamed et al., 2011) and, wireless sensor networks (Xi et al., 2010). Inspired by the distributed control system (DCS) with MPC, there are several studies of DMPC in the literatures, for example, (Summers and Lygeros, 2012; Al-Gherwi et al., 2013; Zheng et al., 2013). In contrast with the traditional centralized MPC through a single controller, DMPC is more appealing due to a flexible control structure and efficient performance.

Recently, DMPC for MAS has become a hot research area, and one common research direction is the consensus of MAS using DMPC schemes. A wide variety of solution strategies have been proposed to ensure consensus (Müller et al., 2012; Zhan and Li, 2013; Cheng et al., 2015; Li and Yan, 2015). The consensus problem of MAS is required to design a distributed control protocol, which utilizes neighboring information to reach a state agreement with respect to each agent. For linear MAS, Li and Yan (2015) developed a new distributed receding horizon control (RHC) protocol, which first explicitly expressed neighbor information versus implicit de-

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scription (Zhan and Li, 2013). In addition, more detailed results to ensure consensus were developed in Li and Yan (2015) for MAS with linear-time invariant and one-dimensional dynamics. Li et al. (2016) focused on how to design an information exchange mechanism for consensus. In particular, the authors analyzed detailed consensus conditions for finite horizon cases and infinite horizon case. They also indicate that consensus performance is related to the network topology. For nonlinear MAS, a relatively general cooperative control DMPC framework was reported in Müller et al. (2012), where each agent adopted a non-iterative solution method, that is, optimization problems are solved only once at each sampling instant, which results in less communication requirements. In Gao et al. (2017), consensus for second-order nonlinear systems with a dynamic reference were considered, where three critical components, including the terminal cost, terminal region and auxiliary controller, were denoted in an easily understandable way. Moreover, the authors proposed a time-varying compatibility constraint to ensure the convergence of the closed-loop system. Note that external disturbances were not considered in the aforementioned papers (Müller et al., 2012; Zhan and Li, 2013; Cheng et al., 2015; Li and Yan, 2015; Li et al., 2016; Gao et al., 2017). In a practical environment, systems are inevitably affected by ubiquitous uncertainties. Therefore, considering bounded additive disturbances, this paper proposes a robust DMPC consensus strategy for a class of nonlinear MAS.

It is worth noting that the vast majority of existing DMPC algorithms require control execution at each sampling instant (Su et al., 2019a), and inevitably, a great deal of computing and communication consumption are generated in this process. Simultaneously, due to the limitations of the actual network, a large amount of communication may induce a certain degree of deterioration on the controlled system. Event-triggered control is an effective energy-saving strategy that can achieve aperiodic control for a small average sampling rate (Zou et al., 2017). Currently, event-triggered control is widely applied in various fields. Some theoretical and practical results can be seen in previous publications (Ferrara et al., 2012; Lehmann et al., 2013; Zou and Xiang, 2019). Recently, Zou et al. (2020a) and Zou et al. (2020b) applied the event-triggered scheme to consensus-tracking control and containment control of MAS, and Zeno behavior was avoided by designing appropriate triggered rules. However, an event-triggered mechanism needs an additional detection part to continuously obtain the current state of the actual system, which is undesirable for some systems with high sampling costs. Thus, self-triggered control was proposed using previously predicted states to pre-determine the next triggering instant (Heemels et al., 2012.), which solved the disadvantage of the high-frequency sampling problem of event-triggered control. A self-triggered MPC algorithm for a single nonlinear system was explored in several previous studies (Hashimoto et al., 2017; Liu et al., 2018; Su et al., 2019), which implemented a self-triggered control to MPC and stabilized the closed-loop system. In Liu et al. (2018), Hashimoto et al. (2017), and Su et al. (2019b), the control input and self-triggered strategy were designed via an optimization problem, whereas obtained corresponding results were asynchronously determined. In Li et al. (2018), an independent variable reflecting the cost of communication was incorporated into the system's cost function to synchronously trade off the desired triggering and control behavior, which may require less conservative design parameters compared with several other studies (Hashimoto et al., 2017; Liu et al., 2018; Su et al., 2019). For a linear MAS, Zhan et al. (2019) and Mi et al. (2020) co-designed a self-triggered mechanism and DMPC to achieve a coordinated value, which efficiently reduced the communication load. However, few studies combined the self-triggered mechanism and DMPC algorithm to solve the consensus problem of nonlinear uncertain MAS. Thus, we propose a self-triggered robust DMPC consensus algorithm, which considers both control costs and communication load. Our study partially extends the results of Liu et al. (2018) to the case of distributed control of MAS.

The main contributions of this paper are twofold:

1. We introduce a self-triggered strategy via optimization, which relieves the heavy communication burden. The maximal triggering interval is user-defined and no more than the predicted horizon. Considering bounded additive disturbances, for each discrete-time nonlinear agent, we use a min-max robust DMPC that explicitly includes uncertainty

realizations as optimized decision variables in the entire optimization control problem, which is more intuitive compared to robust DMPC using a nominal model (Zhan et al., 2019).

2. Sufficient conditions are presented to guarantee the feasibility of the optimization algorithm and the consensus over the considered MAS, where we employ invariant set theory to implement input-to-state practical stable (ISpS) with respect to consensus error. Moreover, based on the latest neighboring information, we provide a specific form of compatibility constraints and consensus error terminal regions.

The rest of this paper is organized as follows. The preliminaries and problem formation are described in Section 2. In Section 3, a robust self-triggered consensus algorithm based on DMPC is proposed. The corresponding feasibility and consistency analysis are performed in Section 4. Section 5 conducts numerical examples on a group of cart-damper-spring systems to verify the effectiveness of the proposed algorithm. Section 6 summarizes the full paper.

Notations: $\mathbb{R}$ and $\mathbb{N}$ stand for the set of real numbers and non-negative integers, respectively. $\mathbb{R}^n$ denotes the set of n-dimensional real column vectors. Given an arbitrary column vector, $\|\cdot\|$ represents its Euclidean norm. Let $\mathbb{R}_{\geq c_1}, \mathbb{R}_{\geq(c_1,c_2]}, \mathbb{N}_{\geq c_1}$, and $\mathbb{N}_{\geq(c_1,c_2]}$ denote $\{t\in\mathbb{R} \mid t\geq c_1\}, \{t\in\mathbb{R} \mid c_1<t\leq c_2\}, \{t\in\mathbb{N} \mid t\geq c_1\}$, and $\{t\in\mathbb{N} \mid c_1<t\leq c_2\}$, respectively. A scalar continuous function $\alpha:\mathbb{R}_{\geq 0}\to\mathbb{R}_{\geq 0}$ is a class of K if it satisfies strictly increasing and $\alpha(0)=0$. If a function belongs to a class K and meets $\alpha(s)\to+\infty$ as $s\to+\infty$, it is called a class of K_∞. If for every fixed $t\in\mathbb{R}_{\geq 0}$, $\beta(\cdot, t)$ is a class of K; for every fixed $s\in\mathbb{R}_{\geq 0}$, $\beta(s, \cdot)$ is decreasing and $\lim_{t\to\infty}\beta(s, t)=0$, and then $\beta(\cdot,\cdot)$ is a class of KL.

2 Preliminaries and problem formulation

This paper studies a group of perturbed nonlinear MAS consisting of M agents. The communication topology can be represented by a directed graph $G=\{V, E, A\}$, where the vertex set is $V=\{1, 2, \ldots, M\}$, the edge set is $E\subseteq\{(i, j): i, j\in V, i\neq j\}$, and the adjacency matrix is $A=[a_{ij}\in\mathbb{R}^{M\times M}]$. In particular, if an agent i can receive the information from an agent j, then edge $(j, i)\in E$ and $a_{ij}=1$; otherwise, $a_{ij}=0$. The neighboring index set of agent i is denoted by $N_i=\{j\in V: (j, i)\in E\}$. $|N_i|$ represents the number of neighbors of agent i. Suppose that each vertex has no self-loop, i.e., $a_{ii}=0$, and the communication network over the MAS is directed, that is, $a_{ij}\neq a_{ji}$. Furthermore, we require that each system has at least one neighbor of information and each time instant can be measured.

To achieve consensus over MAS, each agent is modeled as

$$x_i(t+1) = f(x_i(t), u_i(t), w_i(t)), \quad (1)$$

for $i=1, 2, \ldots, M$, where $x_i(t)\in\mathbb{R}^n, u_i(t)\in\mathbb{R}^m$, and $w_i(t)\in W_i\subset\mathbb{R}^w$ represents the state, control inputs and additive disturbances of agent i. Here, $f:\mathbb{R}^n\times\mathbb{R}^m\times\mathbb{R}^w\to\mathbb{R}^n$ is an arbitrary nonlinear function with $f(0, 0, 0)=0$. For each agent i, define $e_i(t)=x_i(t)-\hat{x}_{-i}(t)$, where $e_i(t)$ is the state consensus error, and $\hat{x}_{-i}(t)$ represents the average state of its neighbors. The specific expression on $\hat{x}_{-i}(t)$ is given later. Let $\rho_i \triangleq \sup_{w_i(t)\in W_i}\|w_i(t)\|$ denote a known disturbance bound. Assume that consensus error and control input constraints are limited by $e_i(t)\in E_i$, $u_i(t)\in U_i$. Furthermore, E_i, U_i, and W_i are compact sets containing the origin in their interiors.

Definition 1 (Su et al., 2019a) (RPI set) For the established system model (1), a set $E_i\subseteq\mathbb{R}^n$ is called a robust positively invariant (RPI) set if for all $e_i(t)\in E_i$, then $e_i(t+l)\in E_i$, $l\in\mathbb{R}_{\geq 0}$ exists for all $w_i(t)\in W_i$.

Definition 2 (Regional ISpS) If there exists a RPI set $E_i\subseteq\mathbb{R}^n$ including the origin, a KL function β_i, a K function α_i and a constant $d_i\in\mathbb{R}_{\geq 0}$ satisfying

$$\|e_i(t)\| \leq \beta_i(\|e_i(0)\|, t) + \alpha_i(\sup_{\tau\in[0,t-1]}\|w_i(\tau)\|) + d_i, \quad (2)$$

where each $e_i(0)\in E_i$, $w_i(t)\in W_i$, then state consensus error dynamics of system (1) is said to be ISpS in E_i with respect to w_i. $e_i(0)$ is the initial state consensus error, and w_i is the disturbance.

Lemma 1 For every agent $i\in M$, let $\sigma_i(\cdot)\in K$, $\alpha_1(\cdot)$, $\alpha_2(\cdot)$, $\alpha_3(\cdot)\in K_\infty$ and $\tau_1,\tau_2\in\mathbb{R}_{>0}$. Given a set E_i as defined in Definition 2 and a function $V_i(e_i)$ $\mathbb{R}^n\to\mathbb{R}_{\geq 0}$ to satisfy the following two conditions:

(1) $\alpha_1\left(\|e_i\|\right)\leq V_i(e_i)\leq \alpha_2\left(\|e_i\|\right)+\tau_1$,

(2) $V_i(e_i(t+1))-V_i(e_i(t))\leq -\alpha_3\left(\|e_i\|\right)+\sigma_i\left(\|w_i\|\right)+\tau_2$,

for all $e_i\in E_i$, $w_i\in W_i$, the state consensus error dynamics of system (1) is ISpS in E_i with respect to w, where the function $V_i(\cdot)$ is called an ISpS-Lyapunov function. When $\tau_1=\tau_2=0$, $V_i(\cdot)$ is an ISS-Lyapunov function. The specific proof can be found in Lazar et al. (2008).

We designed a self-triggered robust DMPC consensus algorithm to obtain when and how to select control inputs for system (1) so that all agents can decrease the communication and computing resources and achieve consensus. We also assume that there is no delay in the transmission. The whole system operation procedure can be stated as follows: at the triggering instant t_k^i, each agent first deduces the self-triggered conditions according to system stability, and then determines the optimal control input sequence $\boldsymbol{u}_i^*(t_k^i)=\{u_i^*(t_k^i\,|\,t_k^i),u_i^*(t_k^i+1\,|\,t_k^i),\cdots,u_i^*(t_k^i+T-1\,|\,t_k^i)\}$ by solving an optimization problem. Before the next triggering instant t_{k+1}^i, each agent receives a predicted state sequence from every neighbor $j\in N_i$. Meanwhile, agent i stores its own predicted state sequence into a buffer area, and waits for other agents who demand it. The next calculation begins at t_{k+1}^i and repeats the above procedure.

Remark 1 Although this paper discusses the discrete-time nonlinear MAS, we can apply a nearly equal process to solve the consensus problem of the continuous-time nonlinear MAS with periodic sampling.

3 Robust self-triggered DMPC consensus algorithm

Let t_k^i denote the k^{th} triggering instant of agent i with $k\geq 0$, the cost function of each agent at triggering instant can be defined as

$$
\begin{aligned}
&J_i^{H_k^i}(x_i(t_k^i),\hat{\boldsymbol{x}}_{-i}(t_k^i),\boldsymbol{u}_i(t_k^i),\boldsymbol{w}_i(t_k^i),T)\\
&=\sum_{l=0}^{H_k^i-1}\gamma L_i(x_i(t_k^i+l\,|\,t_k^i),\hat{x}_{-i}(t_k^i+l\,|\,t_k^i),u_i(t_k^i+l\,|\,t_k^i),w_i(t_k^i+l\,|\,t_k^i))\\
&+\sum_{l=H_k^i}^{T-1}L_i(x_i(t_k^i+l\,|\,t_k^i),\hat{x}_{-i}(t_k^i+l\,|\,t_k^i),u_i(t_k^i+l\,|\,t_k^i),w_i(t_k^i+l\,|\,t_k^i))\\
&+F_i(x_i(t_k^i+T\,|\,t_k^i),\hat{x}_{-i}(t_k^i+T\,|\,t_k^i)),
\end{aligned}\tag{3}
$$

where $T\in\mathbb{N}_{\geq 1}$ is the prediction horizon, H_k^i is the triggering interval calculated by $H_k^i=t_{k+1}^i-t_k^i$, $H_k^i\in\mathbb{N}_{[1,H_{\max}]}$. $H_{\max}$ is the maximal triggering interval with $H_{\max}\in\mathbb{N}_{[1,T]}$. $x_i(t_k^i+l\,|\,t_k^i)$ is the state prediction of agent i and regarding the future step t_k^i+l at time step t_k^i. L_i is the stage cost and its concrete form is a continuous function. $L_i(x_i(t_k^i+l\,|\,t_k^i),\hat{x}_{-i}(t_k^i+l\,|\,t_k^i),u_i(t_k^i+l\,|\,t_k^i),w_i(t_k^i+l\,|\,t_k^i))=\left\|x_i(t_k^i+l\,|\,t_k^i)-\hat{x}_{-i}(t_k^i+l\,|\,t_k^i)\right\|+\lambda\left\|u_i(t_k^i+l\,|\,t_k^i)\right\|-\psi\left\|w_i(t_k^i+l\,|\,t_k^i)\right\|$. Both λ and ψ are given weighing scalars and generally adopt $\lambda\in\mathbb{R}_{>0},\psi\in\mathbb{R}_{>0}$. F_i is the terminal cost, and its concrete form is also a continuous function $F_i(x_i(t_k^i+T\,|\,t_k^i),\hat{x}_{-i}(t_k^i+T\,|\,t_k^i))=\beta_i\left\|x_i(t_k^i+T\,|\,t_k^i)-\hat{x}_{-i}(t_k^i+T\,|\,t_k^i)\right\|$, and $\beta_i>0$ is a weighing scalar. Let $L_i(0,0,0)=0$ and $F_i(0,0)=0$. $\gamma\in(0,1)$ is a control parameter reflecting triggering or a communication effect. $\boldsymbol{u}_i(t_k^i)=\{u_i(t_k^i\,|\,t_k^i),u_i(t_k^i+1\,|\,t_k^i),\cdots,u_i(t_k^i+T-1\,|\,t_k^i)\}$ represents the future control inputs to obtain; $\boldsymbol{w}_i(t_k^i)=\{w_i(t_k^i\,|\,t_k^i),w_i(t_k^i+1\,|\,t_k^i),\cdots,w_i(t_k^i+T-1\,|\,t_k^i)\}$ and represents the additive disturbance sequence. The averaged state trajectory of the neighbors of agent i is $\hat{\boldsymbol{x}}_{-i}(t_k^i)=\{\hat{x}_{-i}(t_k^i\,|\,t_k^i),\hat{x}_{-i}(t_k^i+1\,|\,t_k^i),\cdots,\hat{x}_{-i}(t_k^i+2T\,|\,t_k^i)\}$ with

$$
\hat{x}_{-i}(t_k^i+l\,|\,t_k^i)=\sum_{j\in N_i}\frac{\hat{x}_j(t_k^i+l\,|\,t_k^i)}{|N_i|},
$$

where $\hat{x}_j(t_k^i+l\,|\,t_k^i)$ denotes the assumed state trajectory of agent j at t_k^i, which is obtained based on the received information of agent j at triggering instant

$\Gamma_j(t_k^i)$. $\Gamma_j(t_k^i)$ stands for the triggering instant that is closest and occurs before t_k^i of agent *j*. In Gao et al. (2017) and Zhan et al. (2019), the assumed state trajectory of agent *j* can be expressed as:

$$\hat{x}_j(t_k^i + l \mid t_k^i) = \begin{cases} x_j^*(t_k^i + l \mid \Gamma_j(t_k^i)), & l \in \mathbb{N}_{[0,T)}, \\ \mu \hat{x}_j(t_k^i + l \mid \Gamma_j(t_k^i)), & l \in \mathbb{N}_{[T,2T]}, \end{cases} \quad (4)$$

where $\hat{x}_j(\Gamma_j(t_k^i)+T \mid \Gamma_j(t_k^i)) = x_j^*(\Gamma_j(t_k^i)+T \mid \Gamma_j(t_k^i))$. To relieve the calculation burden, a fixed parameter μ, $\mu \in \mathbb{R}_{>0}$ was chosen to determine which information from neighbor agents was useful for cooperation.

Remark 2 In the cost function, we used the term $\left\| x_i(t_k^i + l \mid t_k^i) - \hat{x}_{-i}(t_k^i + l \mid t_k^i) \right\|$ instead of the term $\sum_{j \in N_i} a_{ij} \left\| x_i(t_k^i + l \mid t_k^i) - \hat{x}_j(t_k^i) \right\|_Q^2$ (Li and Yan, 2015), where the setting of $\hat{x}_{-i}(t_k^i + l \mid t_k^i)$ can reduce part of the calculation. Meanwhile, we used the Euclidean norm instead of the usual quadratic function. Note that for every agent i, we assume some predicted trajectories for its neighbors in Eq. (4) before the next state update because the current actual predicted trajectories $x_j(t_k^i + l \mid t_k^i)$, $j \in N_i$ are unknown as the self-triggered communication mechanism.

3.1 Min–max optimization

According to the defined cost function (3), each agent solves the following optimization problem SP_i:

$$V_i^{H_k^i}(e_i(t_k^i), T) \triangleq \min_{\boldsymbol{u_i}(t_k^i)} \max_{\boldsymbol{w_i}(t_k^i)} J_i^{H_k^i}(x_i(t_k^i), \hat{\boldsymbol{x}}_{\boldsymbol{-i}}(t_k^i), \boldsymbol{u_i}(t_k^i), \boldsymbol{w_i}(t_k^i), T) \quad (5a)$$

subject to

$$u_i(t_k^i + l \mid t_k^i) \in U_i, \quad (5b)$$

$$w_i(t_k^i + l \mid t_k^i) \in W_i, \quad (5c)$$

$$x_i(t_k^i + l + 1 \mid t_k^i) = f(x_i(t_k^i + l \mid t_k^i), u_i(t_k^i + l \mid t_k^i), w_i(t_k^i + l \mid t_k^i)), \quad (5d)$$

$$\left\| x_i(t_k^i + l \mid t_k^i) - \hat{x}_i(t_k^i + l \mid t_k^i) \right\| \le \frac{\alpha}{(T - H_k^i)} \min_{j \in N_i} \left\| x_j(\Gamma_j(t_k^i)) - \hat{x}_{-j}(\Gamma_j(t_k^i) \mid \Gamma_j(t_k^i)) \right\|, \quad (5e)$$

$$x_i(t_k^i + T \mid t_k^i) - \hat{x}_{-i}(t_k^i + T \mid t_k^i) \in E_i^f, \quad (5f)$$

where $x_i(t_k^i \mid t_k^i) = x_i(t_k^i), e_i(t_k^i) = x_i(t_k^i) - \hat{x}_{-i}(t_k^i)$, $\alpha \in \mathbb{R}_{(0,1)}$ is a constant. E_i^f is the state consensus error terminal region including the origin. In Zhan et al. (2019), the compatibility constraint in Eq. (5e) ensures a certain degree of consensus, which implies that the predicted trajectory can not be far away from the assumed one.

We designed the state consensus error terminal region E_i^f to satisfy

$$E_i^f \triangleq \Big\{ x \in X_i \mid \left\| x - \hat{x}_i(t_k^i + l \mid t_k^i) \right\| \le \alpha/T \min_{j \in N_i} \left\| x_j(\Gamma_j(t_k^i)) - \hat{x}_{-j}(\Gamma_j(t_k^i) \mid \Gamma_j(t_k^i)) \right\|, \quad l > T - (H_k^i)^* \Big\}. \quad (6)$$

Assumption 1 At any triggering instant in system (1), an auxiliary local feedback control law $\bar{u}_i = k_i(x_i, \hat{x}_{-i})$ exists, where $k_i: \mathbb{R}^n \to \mathbb{R}^m$, $k_i(0, 0)=0$, such that the state consensus error terminal region E_i^f is an RPI set. Meanwhile, $\bar{u}_i \in U_i$ holds for all $(x_i - \hat{x}_{-i}) \in E_i^f$, i.e., $e_i \in E_i^f$.

Remark 3 For the auxiliary local feedback control law, we designed a set of fixed gains k_i offline (Lazar et al., 2008) to satisfy E_i^f as an RPI set. In addition, both control behavior and communication behavior are involved in Eq. (3). To prove the considered MAS can reach a consensus, for each agent, we denote the cost function as $J_i(x_i(t_k^i), \hat{\boldsymbol{x}}_{\boldsymbol{-i}}(t_k^i), \boldsymbol{u_i}(t_k^i), \boldsymbol{w_i}(t_k^i), T)$ when γ=1 and $H_k^i = 1$, and then its optimal cost is $V_i(e_i(t_k^i), T)$. This is the so-called time-driven DMPC without considering communication cost.

3.2 Self-triggering in optimization

Between any two successive triggering instants, the control input is in the form of

$$u_i^{ST}(x_i(t_k^i+l\mid t_k^i))=\boldsymbol{u}_i^*(t_k^i),\ l\in\mathbb{N}_{[0,t_{k+1}^i-t_k^i-1]},\tag{7}$$

where $\boldsymbol{u}_i^*(t_k^i)$ is a set of the optimal control sequence obtained at t_k^i by solving the optimization problem SP_i. The triggering instant is defined as follows:

$$\begin{aligned}&t_{k+1}^i=t_k^i+(H_k^i)^*\\&(H_k^i)^*\triangleq\max\{H_k^i\in\mathbb{N}_{[1,H_{\max}]}\mid\\&\qquad V_i^{H_k^i}(e_i(t_k^i),T)\le V_i^1(e_i(t_k^i),T)\}\end{aligned}\tag{8}$$

Remark 4 Note that for each agent, the control input from t_k^i+1 to t_{k+1}^i-1 is derived from an open-loop min-max optimization problem, which depends on the previous sampling instant t_k^i. Because of the self-triggered mechanism, communication resources can be saved as the communication period increases. We obtained the optimal triggering interval $(H_k^i)^*$ by checking whether the optimal cost was dropping and choosing a satisfied maximal triggering interval.

The self-triggered robust DMPC consensus algorithm for Eq. (1) is summarized in Algorithm 1.

Remark 5 In Algorithm 1, the initial state trajectory $\hat{\boldsymbol{x}}_{-i}(t_0^i)$ of the neighbors of agent i is assumed by applying zero control without constraints (5e) and (5f). Eqs. (5e) and (5f) are adopted to solve problem SP_i only when $k\ge1$. Meanwhile, the control actions switch from solving problem SP_i to the auxiliary feedback control law as long as all subsystems' state consensus error trajectories entered the error terminal region, which further saves computation resources. The above control idea is called a dual-mode strategy (Dunbar, 2005). In addition, we must point out that although the self-triggered robust consensus algorithm effectively eases the communication load in this paper, it also adds optimization computation due to the trigger mechanism. The quantization technique is an effective tool to save control costs and communication resources, and many interesting results have been reported such as those from Wan et al. (2019), Xu et al. (2018) Yang et al. (2018), and Feng et al. (2018). Therefore, it is important to utilize quantized self-triggered control to optimize control and communication costs.

Algorithm 1 Self-triggered robust DMPC consensus algorithm

Off-line:
Require: $\lambda,\psi,\mu,\gamma,\alpha,\beta_i,k_i,T,H_{\max}$
On-line:
1. **Initialization**: For each agent $i\in M$,
1.1 set $t_0^i=0$ as the first triggering instant with k=0;
1.2 transmit its state sequence $\hat{x}_i(\tau\mid 0)$ to its neighbors and receive $\hat{x}_j(\tau\mid 0)$ from every number $j\in N_i$, $\tau\in[0, T)$; and
1.3 solve problem SP_i to obtain $(H_k^i)^*$ and $u_i^*(0)$.
2. **While** each $e_i(t_k^i+l\mid t_k^i)\notin E_i^f$ **do**
2.1 **while** $t_k^i, k\ge1$ is not triggered **do**
2.2 apply $u_i^{ST}(t_{k-1}^i)$ at $t\in[t_{k-1}^i,t_{k-1}^i+T)$;
2.3 obtain $\hat{x}_i(\tau\mid t_k^i),\tau\in[t_k^i,t_{k+1}^i]$ based on Eqs. (3) and (4);
2.4 store $u_i^*(t_k^i\mid t_{k-1}^i)$ and $x_i^*(t_k^i\mid t_{k-1}^i)$;
2.5 **end while**
2.6 measure the current state $x(t_k^i)$;
2.7 solve SP_i in Eqs. (5) and (8) to obtain $u_i^*(\tau\mid t_k^i)$ and $H_i^*(t_k^i),\tau\in[t_k^i,t_k^i+T)$;
2.8 set k=k+1; and
end while
3. apply the auxiliary feedback control law $\bar{u}_i=k_i(x_i,\hat{x}_{-i})$ to the corresponding subsystems.

4 Feasibility and stability analysis

The optimization independence of MPC in adjacent time shows that the optimization feasibility of SP_i at the current moment does not guarantee that the next moment is feasible. Thus, we must provide conditions to ensure that Algorithm 1 has iterative feasibility. Moreover, the property of iterative feasibility ensures that the optimization problem (5) is solvable.

Assumption 2 E_i^f satisfies $E_i^f\subseteq E_i$, $0\in E_i^f$.

Assumption 3 The existence of α_l, α_f, $\alpha_F\in K_\infty$, α_w, $\sigma\in K$, such that

(1) $L_i(x_i,\hat{x}_{-i},u_i,w_i)\ge\alpha_l\left(\|x_i-\hat{x}_{-i}\|\right)-\alpha_w\left(\|w_i\|\right)\ge0$ for $\forall(x_i-\hat{x}_{-i})\in E_i$, $\forall u_i\in U_i$, $\forall w_i\in W_i$;

(2) $\alpha_f\left(\|x_i-\hat{x}_{-i}\|\right)\le F_i(x_i,\hat{x}_{-i})\le\alpha_F\left(\|x_i-\hat{x}_{-i}\|\right)$ for $\forall(x_i-\hat{x}_{-i})\in E_i^f$;

(3)
$F_i(x_i(t_k^i+T+1\mid t_k^i),\hat{x}_{-i}(t_k^i+T+1\mid t_{k+1}^i))-F_i(x_i(t_k^i+T\mid t_k^i),\hat{x}_{-i}(t_k^i+T\mid t_{k+1}^i))\le -L_i(x_i(t_k^i+T\mid t_k^i),\hat{x}_{-i}(t_k^i+T\mid t_{k+1}^i),u_i(t_k^i+T\mid t_k^i),w_i(t_k^i+T\mid t_k^i))+\sigma\left(\left\|w_i(t_k^i+T)\right\|\right)$
for $\forall(x_i-\hat{x}_{-i})\in E_i^f,\forall w_i\in W_i$.

It can be observed that $F_i(\cdot,\cdot)$ is an ISS-Lyapunov function in E_i^f.

Lemma 2 If Assumption 3 is satisfied for any state consensus error $(x_i(t_0)-\hat{x}_{-i}(t_0))\in E_i^f$ and admissible additive disturbance $w_i\in W_i$, then

$$F_i(x_i(t_m),\hat{x}_{-i}(t_m))-F_i(x_i(t_0),\hat{x}_{-i}(t_0))\le -\sum_{p=0}^{m-1}\Big(L_i(x_i(t_p),\hat{x}_{-i}(t_p),\bar{u}_i(t_p),w_i(t_p))-\sigma\left(\left\|w_i(t_p)\right\|\right)\Big), \tag{9}$$

where $x_i(t_m)$ is calculated by applying the auxiliary feedback control law $\bar{u}_i$, $m\in\mathbb{N}_{[1,T]}$.

Proof According to Assumption 3-3,

$$\begin{aligned}F_i(x_i(l+1\mid t_k),\hat{x}_{-i}(l+1\mid t_{k+1}))-F_i(x_i(l\mid t_k),\hat{x}_{-i}(l\mid t_{k+1}))\\ \le -(L_i(x_i(l\mid t_k),\hat{x}_{-i}(l\mid t_{k+1}),\bar{u}_i(l\mid t_k),w_i(l\mid t_k))\\ -\sigma\left(\left\|w_i(l\mid t_k)\right\|\right),\end{aligned} \tag{10}$$

for all $(x_i-\hat{x}_{-i})\in E_i^f$. Because E_i^f is an RPI set, by summing Eq. (10) from l=0 to l=m−1 the inequality (9) can be proven for agent i.

Theorem 1 (Feasibility and stability) For each agent i under Assumption 3, if SP_i is feasible at the initial triggering instant t_0^i, then Algorithm 1 is iterative feasible. Furthermore, the state consensus error dynamics of system (1) satisfies ISpS with respect to additive disturbances and it follows that as $t\to\infty$, $\left\|x_i-x_j\right\|=0$ for all i, j=1, 2, …, M, then MAS with self-triggered robust DMPC (7) and (8) can reach consensus.

The proof of Theorem 1 involves two parts: feasibility analysis and consensus analysis.

4.1 Feasibility analysis

Definition 2 (Initial feasible set) For every agent i, the set $E_i^{\text{MPC}}(T)\in E_i$ is called the initial feasible set of Algorithm 1, which indicates that the set of state consensus errors can be robustly controlled into E_i^f in T steps for all $e_i(t_0)\in E_i^{\text{MPC}}(T)$ and all $w_i\in W_i$.

Suppose that we obtain a feasible solution $\boldsymbol{u}_i^*(t_k^i)$ of problem SP_i and $(H_k^i)^*$ at t_k^i, then construct a feasible solution at triggering instant t_{k+1}^i. This can be expressed as

$$\begin{aligned}\tilde{\boldsymbol{u}}_i(t_{k+1}^i)=\Big\{&u_i^*(t_{k+1}^i\mid t_k^i),\cdots,u_i^*(t_{k+1}^i+T-(H_k^i)^*-1\mid t_k^i),\\ &\bar{u}_i(t_{k+1}^i+T-(H_k^i)^*\mid t_{k+1}^i),\cdots,\bar{u}_i(t_{k+1}^i+T-1\mid t_{k+1}^i)\Big\},\end{aligned} \tag{11}$$

with $\bar{u}_i(t_{k+1}^i+q\mid t_{k+1}^i)=k_i(x_i(t_{k+1}^i+q\mid t_{k+1}^i),\hat{x}_{-i}(t_{k+1}^i+q\mid t_{k+1}^i))$ and $q\in\mathbb{N}_{[T-(H_k^i)^*,T-1]}$.

When $t_{k+1}^i\le t<t_{k+1}^i+T-(H_k^i)^*$, Eqs. (5b)–(5f) can be easily satisfied since $\tilde{u}_i(t\mid t_{k+1}^i)=u_i^*(t\mid t_k^i)$ during this period. When $t_{k+1}^i+T-(H_k^i)^*\le t<t_{k+1}^i+T$ and by using Assumption 1, Eqs. (5b)–(5d) and (5f) can be satisfied. In addition, according to the definition of the terminal region of the state consensus error in Eq. (6), we can obtain compatibility constraint (5e) when $t_{k+1}^i+T-(H_k^i)^*\le t<t_{k+1}^i+T$. Thus, all constraints in problem SP_i are satisfied, and $\tilde{\boldsymbol{u}}_i(t_{k+1}^i)$ is indeed a feasible solution at t_{k+1}^i. In summary, as long as problem SP_i admits a feasible solution at the initial instant t_0^i, then from the induction principle we can obtain feasible solutions for all $k\ge 0$.

4.2 Consensus analysis

Due to the iterative feasibility, we know the optimization calculation between two successive triggering instants in Algorithm 1 is relevant, and the value of the cost function defined in Eq. (2) is relevant and comparable.

Lemma 3 For the optimal cost defined in Eq. (5), $V_i^1(e_i(t_k^i),T)\le V_i(e_i(t_k^i),T)$ exists.

Proof Suppose that $\boldsymbol{u}_i^*(t_k^i)=\{u_i^*(t_k^i\mid t_k^i),\cdots,u_i^*(t_k^i+T-1\mid t_k^i)\}$ and $\boldsymbol{w}_i^*(t_k^i)=\{w_i^*(t_k^i\mid t_k^i),\cdots,w_i^*(t_k^i+T-1\mid t_k^i)\}$ are the solutions by $V_i(e_i(t_k^i),T)$, then according to optimality, we obtain

$$
\begin{aligned}
&V_i^1(e_i(t_k^i),T) \le \max_{\boldsymbol{w}_i(t_k^i)} J_i^1(x_i(t_k^i),\hat{\boldsymbol{x}}_{\text{-}i}(t_k^i),\boldsymbol{u}_i^*(t_k^i),\boldsymbol{w}_i(t_k^i),T)\\
&= \max_{\boldsymbol{w}_i(t_k^i)} J_i(x_i(t_k^i),\hat{\boldsymbol{x}}_{\text{-}i}(t_k^i),\boldsymbol{u}_i^*(t_k^i),\boldsymbol{w}_i(t_k^i),T)\\
&+(\gamma-1)L_i(x_i(t_k^i),\hat{x}_{\text{-}i}(t_k^i),u_i^*(t_k^i),w_i(t_k^i))\\
&= V_i(e_i(t_k^i),T)+(\gamma-1)L_i(x_i(t_k^i),\hat{x}_{\text{-}i}(t_k^i),u_i^*(t_k^i),w_i(t_k^i)).
\end{aligned}
$$

According to Assumption 3-1, $L_i(x_i(t_k^i),\hat{x}_{-i}(t_k^i),$ $u_i^*(t_k^i),w_i(t_k^i))\ge 0,$ and $\gamma\in(0,1)$, Lemma 3 holds.

By using the definition of SP_i in Eq. (5), for all $e_i(t_k^i)\in E_i^{\text{MPC}}(T)$ we obtain

$$
\begin{aligned}
V_i^{(H_k^i)^*}(e_i(t_k^i),T) &= J_i^{(H_k^i)^*}(x_i(t_k^i),\hat{\boldsymbol{x}}_{-i}(t_k^i),\boldsymbol{u}_i^*(t_k^i),\boldsymbol{w}_i^*(t_k^i),T)\\
&\ge \min_{\boldsymbol{u}_i(t_k^i)} J_i^{(H_k^i)^*}(x_i(t_k^i),\hat{\boldsymbol{x}}_{-i}(t_k^i),\boldsymbol{u}_i(t_k^i),0,T)\\
&\ge \gamma\alpha_l\left(\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i)\right\|\right).
\end{aligned} \tag{12}
$$

To get the upper bound of the class K_∞ function of the value function, define a field of origin $O_r=\left\{(x_i-\hat{x}_{\text{-}i})\in E_i \mid \left\|x_i-\hat{x}_{\text{-}i}\right\|<r\right\}$ and satisfy $O_r\subseteq E_i^f$. O_r exists as E_i^f including the origin. Then, consider the following two situations:

Case (1): For all $e_i(t_k^i)\in E_i^f$, then

$$
\begin{aligned}
&J_i^1(x_i(t_k^i),\hat{\boldsymbol{x}}_{\text{-}i}(t_k^i),\tilde{\boldsymbol{u}}_i(t_k^i),\boldsymbol{w}_i(t_k^i),T+1)\\
&= J_i^1(x_i(t_k^i),\ \hat{\boldsymbol{x}}_{\text{-}i}(t_k^i),\boldsymbol{u}_i^*(t_k^i),\boldsymbol{w}_i(t_k^i),T)\\
&+F_i(x_i(t_k^i+T+1\mid t_k^i),\hat{x}_{\text{-}i}(t_k^i+T+1\mid t_k^i))\\
&-F_i(x_i(t_k^i+T\mid t_k^i),\hat{x}_{\text{-}i}(t_k^i+T\mid t_k^i))+L_i(x_i(t_k^i+T\mid t_k^i),\\
&\hat{x}_{\text{-}i}(t_k^i+T\mid t_k^i),\bar{u}_i(t_k^i+T\mid t_k^i),w_i(t_k^i+T\mid t_k^i)),
\end{aligned} \tag{13}
$$

where $\tilde{\boldsymbol{u}}_i(t_k^i)=[\boldsymbol{u}_i^*(t_k^i),\bar{u}_i(t_k^i+T\mid t_k^i)]$ and $\bar{u}_i(t_k^i+T\mid t_k^i)=k_i(x_i(t_k^i+T\mid t_k^i),\hat{x}_{-i}(t_k^i+T\mid t_k^i))$. Under Assumption 3-3 and the sub-optimality of the input control signals $\tilde{\boldsymbol{u}}_i(t_k^i)$, then

$$
\begin{aligned}
V_i^1(e_i(t_k^i),T+1) &\le \max_{\boldsymbol{w}_i(t_k^i)} J_i^1(x_i(t_k^i),\hat{\boldsymbol{x}}_{\text{-}i}(t_k^i),\tilde{\boldsymbol{u}}_i(t_k^i),\boldsymbol{w}_i(t_k^i),\\
&T+1)\le V_i^1(e_i(t_k^i),T)+\sigma(\rho_i).
\end{aligned} \tag{14}
$$

Similarly, according to the triggering mechanism (8) and the above idea, then

$$
\begin{aligned}
&V_i^{H_k^i}(e_i(t_k^i),T)\le V_i^1(e_i(t_k^i),T)\\
&\le V_i^1(e_i(t_k^i),1)+(T-1)\sigma(\rho_i)\\
&\le F_i(x_i(t_k^i),\hat{x}_{-i}(t_k^i))+T\sigma(\rho_i)\\
&+(\gamma-1)L_i(x_i(t_k^i),\hat{x}_{-i}(t_k^i),\bar{u}_i(t_k^i),w_i(t_k^i))\\
&\le \alpha_F(\| x_i(t_k^i)-\hat{x}_{-i}(t_k^i)\|)+T\sigma(\rho_i).
\end{aligned} \tag{15}
$$

Case (2): For all $e_i(t_k^i)\in E_i^{MPC}(T)\notin E_i^f$, then $e_i(t_k^i)\notin O_r$ and $\left\|x_i-\hat{x}_{-i}\right\|\ge r$. Since the iterative feasibility of the optimization problem has previously been proven, a group of feasible control solutions exists that can satisfy all constraints of the optimization problem; meanwhile, the optimal cost is bounded. Therefore, for the finite prediction horizon T, a large positive number $D<+\infty$ is admitted such that $V_i^{H_k^i}(e_i(t_k^i),T)<D$ for all time instants. Define $\theta=\max(1,D/\alpha_F(r))$ and a class of $K_\infty:\bar{\alpha}(s)=\theta\alpha_F(s)$. Apparently, $\bar{\alpha}(s)\ge\alpha_F(s)$ for all $s\in\mathbb{R}_{>0}$. The result is represented by

$$
\begin{aligned}
V_i^{H_k^i}(e_i(t_k^i),T) &\le D\frac{\alpha_F\left(\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i)\right\|\right)}{\alpha_F\left(\left\|r\right\|\right)}+T\sigma(\rho_i)\\
&\le \bar{\alpha}\left(\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i)\right\|\right)+T\sigma(\rho_i).
\end{aligned} \tag{16}
$$

Combined with inequalities (15) and (16), we can conclude

$$
V_i^{H_k^i}(e_i(t_k^i),T)\le\bar{\alpha}\left(\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i)\right\|\right)+T\sigma(\rho_i). \tag{17}
$$

In accordance with the triggering mechanism and Lemma 3, the result is Eq. (18).

For $l\in\mathbb{N}_{[0,T-(H_k^i)^*-1]}$ and by using the triangle inequality and Eq. (5f), it holds that

$$\begin{aligned}&\left\|x_i^*(t_{k+1}^i+l\,|\,t_k^i)-\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_{k+1}^i)\right\|\\&\quad-\left\|x_i^*(t_{k+1}^i+l\,|\,t_k^i)-\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_k^i)\right\|\\&\leq \left\|\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_{k+1}^i)-\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_k^i)\right\|\\&=\left\|\sum_{j\in N_i}\frac{\hat{x}_j(t_{k+1}^i+l\,|\,t_{k+1}^i)-\hat{x}_j(t_{k+1}^i+l\,|\,t_k^i)}{|N_i|}\right\|\\&\leq\frac{\alpha}{(T-H_k^i)}\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i\,|\,t_k^i)\right\|.\end{aligned}\tag{19}$$

Substituting Eq. (19) into Eq. (18) and considering Lemma 2, we obtain Eq. (20).

The last two terms in Eq. (20) can be treated as a constant τ_2. According to the sufficient conditions of ISpS in Lemma 1, we show that state consensus error dynamics of system (1) is ISpS at triggering instants in $E_i^{\mathrm{MPC}}(T)$ with respect to w_i, and it follows that $\lim_{t\to\infty} x_i(t)-\hat{x}_{-i}(t)=0$, which implies the considered MAS can reach consensus.

5 Simulations

Consider a four-agent cart-damper-spring system, the dynamics of each agent is in the form of Eq. (21).

$$\begin{aligned}x_{i,1}(t_{k+1}^i)&=x_{i,1}(t_k^i)+x_{i,2}(t_k^i)T_i,\\x_{i,2}(t_{k+1}^i)&=-\frac{k_iT_i}{M_i}\mathrm{e}^{-x_{i,1}(t_k^i)}x_{i,1}(t_k^i)+\frac{M_i-h_iT_i}{M_i}x_{i,2}(t_k^i)\\&\quad+\frac{T_i}{M_i}u_i(t_k^i)+\frac{T_i}{M_i}w_i(t_k^i),\end{aligned}\tag{21}$$

where $x_{i,1}$ and $x_{i,2}$ express the displacement of the cart and its velocity, respectively, and k_i=0.25 N/m is the linear spring factor, h_i=1.20 Ns/m is the damper factor, M_i=1 kg is the mass of the cart, T_i=0.4 s is the sampling period. For simplicity, each agent has the same system parameters. The input control force is u_i, which is limited to $-2\leq u_i\leq 2$. The additive disturbance constraint is set as $-0.2\leq w_i\leq 0.4$. In addition, each agent can communicate with its neighbor agents.

For the four agents, their neighboring sets are N_1={2}, N_2={1, 3}, N_3={2, 4}, N_4={3}. Some parameters obtained offline are selected as follows: T=5, λ=0.01, $H_{\max}$=4, ψ=2, α=0.2, $E_i^f=2$, and β_i=3 for all i. The auxiliary local feedback control gain and the initial state of four agents are designed as k=[−0.6, −0.4; −0.6, −0.4; −0.5, −0.3; −0.5, −0.4] and x=[3.4, −1.5; 0.6, 0.5; −1.2, 2; 2.5, −1.2], respectively. Using MATLAB fminimax modular, the proposed self-triggered DMPC consensus algorithm is

$$\begin{aligned}&V_i^{(H_{k+1}^i)^*}(x_i(t_{k+1}^i),T)-V_i^{(H_k^i)^*}(x_i(t_k^i),T)\\&\leq V_i^1(x_i(t_{k+1}^i),T)-V_i^{(H_k^i)^*}(x_i(t_k^i),T)\\&\leq\sum_{l=0}^{T-(H_k^i)^*-1}\left(\left\|x_i^*(t_{k+1}^i+l\,|\,t_k^i)-\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_{k+1}^i)\right\|+\left\|x_i^*(t_{k+1}^i+l\,|\,t_k^i)-\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_k^i)\right\|\right)\\&\quad+\sum_{l=T-(H_k^i)^*}^{T-1}\left(\left\|x_i(t_{k+1}^i+l\,|\,t_{k+1}^i)-\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_{k+1}^i)\right\|+\lambda\left\|\bar{u}_i(t_{k+1}^i+l\,|\,t_{k+1}^i)\right\|-\psi\left\|w_i(t_k^i+l\,|\,t_k^i)\right\|\right)\\&\quad-\sum_{l=0}^{(H_k^i)^*-1}\gamma\left(\left\|x_i^*(t_k^i+l\,|\,t_k^i)-\hat{x}_{-i}(t_k^i+l\,|\,t_k^i)\right\|+\lambda\left\|u_i^*(t_k^i+l\,|\,t_k^i)\right\|-\psi\left\|w_i(t_k^i+l\,|\,t_k^i)\right\|\right)\\&\quad+F_i(x_i(t_{k+1}^i+T\,|\,t_{k+1}^i),\hat{x}_{-i}(t_{k+1}^i+T\,|\,t_{k+1}^i))-F_i(x_i(t_k^i+T\,|\,t_k^i),\hat{x}_{-i}(t_k^i+T\,|\,t_k^i))\end{aligned}\tag{18}$$

$$\begin{aligned}&V_i^{(H_{k+1}^i)^*}(x_i(t_{k+1}^i),T)-V_i^{(H_k^i)^*}(x_i(t_k^i),T)\\&\leq\sum_{l=0}^{T-(H_k^i)^*-1}\frac{\alpha}{(T-H_k^i)}\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i\,|\,t_k^i)\right\|+(H_k^i)^*\sigma(\rho_i)\\&\quad-\sum_{l=0}^{(H_k^i)^*-1}\gamma\left(\left\|x_i^*(t_k^i+l\,|\,t_k^i)-\hat{x}_{-i}(t_k^i+l\,|\,t_k^i)\right\|+\lambda\left\|u_i^*(t_k^i+l\,|\,t_k^i)\right\|-\psi\left\|w_i(t_k^i+l\,|\,t_k^i)\right\|\right)\\&\leq-\gamma\alpha_l\left(\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i\,|\,t_k^i)\right\|\right)+\gamma\alpha_w\left(\left\|w_i(t_k^i+l\,|\,t_k^i)\right\|\right)+\alpha\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i\,|\,t_k^i)\right\|+(H_k^i)^*\sigma(\rho_i)\end{aligned}\tag{20}$$

executed. To show the performance level under the proposed Algorithm 1, we consider two configurations of γ=0.85 and γ=0.5 on the constrained min–max optimization problems. The parameter k represents the number of samples. The performance results are shown in Figs. 1–5. For each agent, Figs. 1 and 2 display the state trajectories of each agent, while Fig. 3 displays the control inputs. Figs. 4 and 5 separately display the corresponding triggering instants of each agent, which shows that all triggering intervals converge to $H_{\max}$=4. It can be observed that a smaller γ has a lower triggering frequency, which suggests that the burden of communication can be reduced. For further comparisons, we use time-driven

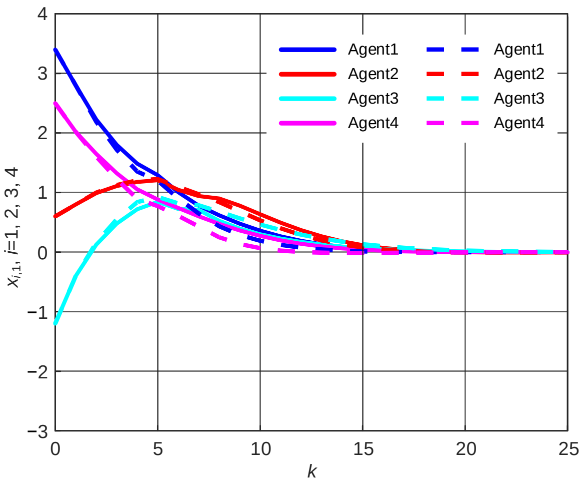

Fig. 1 Trajectories of the system state $x_{i,1}$ with γ=0.85 (solid lines) and γ=0.5 (dashed lines)

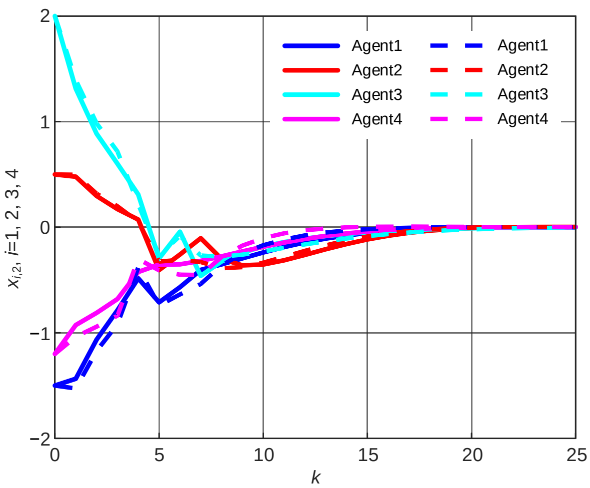

Fig. 2 Trajectories of the system state $x_{i,2}$ with γ=0.85 (solid lines) and γ=0.5 (dashed lines)

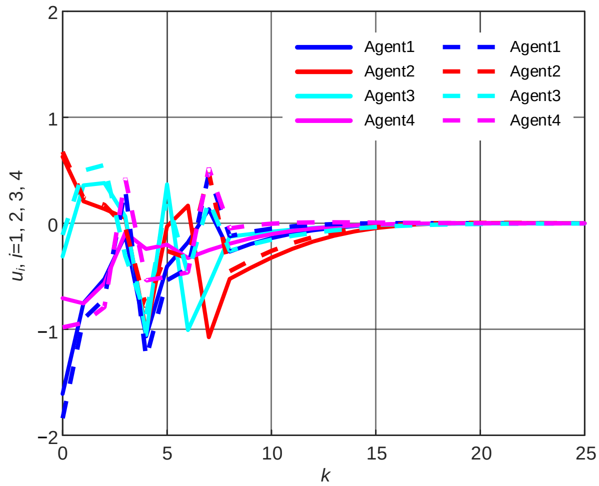

Fig. 3 Trajectories of the control input u_i with γ=0.85 (solid lines) and γ=0.5 (dashed lines)

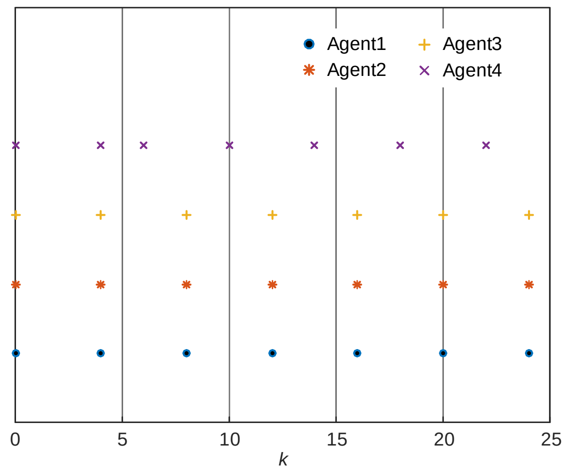

Fig. 4 Triggering instants with γ=0.5

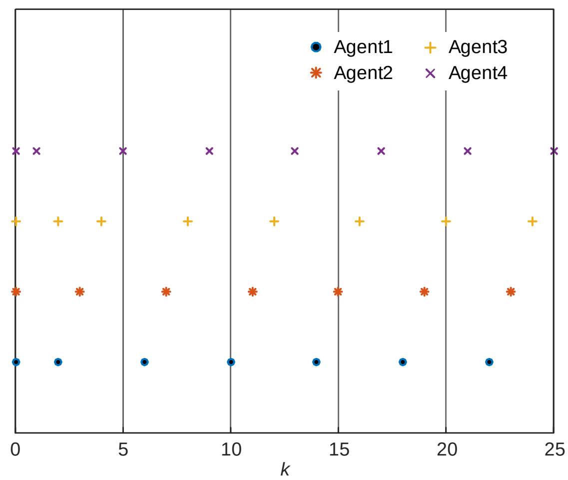

Fig. 5 Triggering instants with γ=0.85

obtain results. The comparison of triggering numbers regarding different γ and time-driven is presented in Table 1, which shows that the self-triggered approach significantly reduces the communication cost. Figs. 6 and 7 plot the evolution of system states by self-triggered and time-driven, respectively. Note that the performance of self-triggered control is comparable with that of time-driven control, where the considered MAS reach consensus in two strategies. Fig. 8 provides the control inputs of two strategies.

Table 1 Comparison of total triggering numbers

Control parameter	Average triggering numbers	Total triggering numbers
Time-driven	agent1=1.000; agent2=1.000 agent3=1.000; agent4=1.000	100
γ=0.85	agent1=3.571; agent2=3.571 agent3=3.125; agent4=3.125	30
γ=0.5	agent1=3.571; agent2=3.571 agent3=3.571; agent4=3.571	28

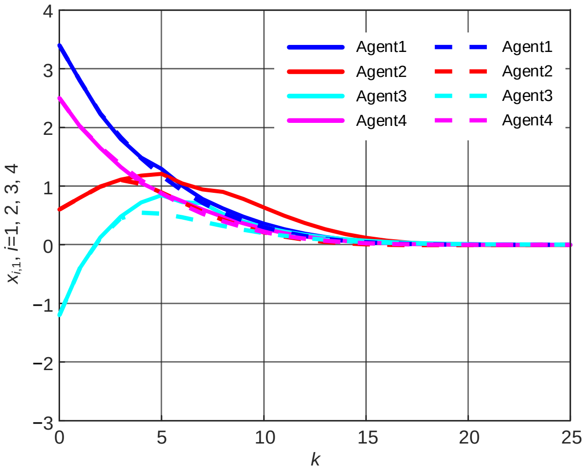

Fig. 6 State trajectories $x_{i,1}$ by self-triggered (solid lines) and time-driven (dashed lines)

6 Conclusion

In this paper, the robust DMPC method is utilized to study the consensus problem of discrete nonlinear MAS with additive disturbances. We propose a self-triggered control scheduler based on a min-max optimization problem to determine the control inputs and maximize the triggering interval. The control sequence only updates and transmits at triggering instants, which can significantly reduce communication costs. The conditions that guarantee the

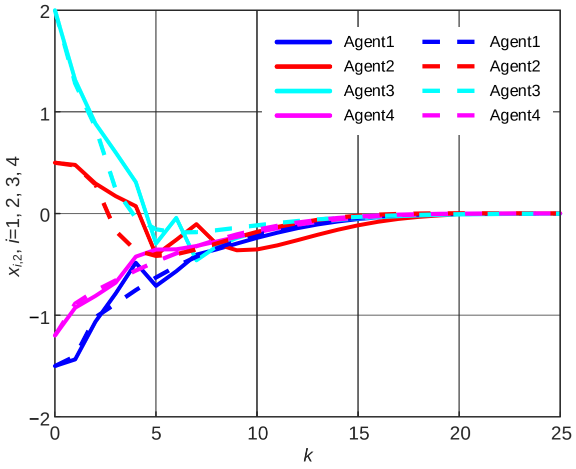

Fig. 7 State trajectories $x_{i,2}$ by self-triggered (solid lines) and time-driven (dashed lines)

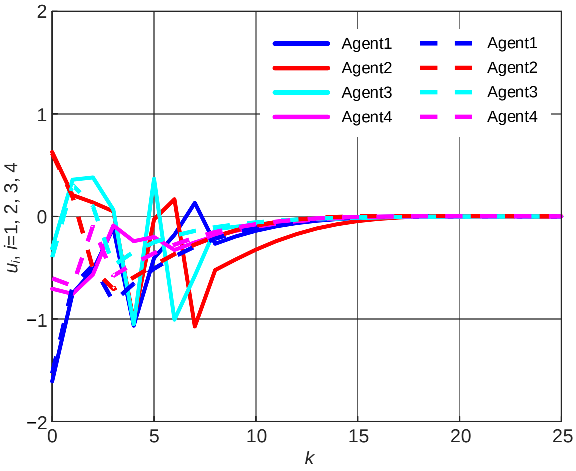

Fig. 8 Control input u_i by self-triggered (solid lines) and time-driven (dashed lines)

feasibility of the algorithm and the consensus over the perturbed nonlinear MAS are sufficient and practicable, and we utilize the invariant set theory to realize ISpS with respect to state consensus error to ensure the closed-loop MAS can achieve consensus. Furthermore, simulation examples show the effectiveness of the algorithm.

Contributors

Qing-ling WANG, Yan-xu SU, and Chang-yin SUN guided the research. Jia-qi LI performed the experiments and drafted the manuscript. Qing-ling WANG and Yan-xu SU helped organize the manuscript. Jia-qi LI revised and finalized the paper.

Compliance with ethics guidelines

Jia-qi LI, Qing-ling WANG, Yan-xu SU, and Chang-yin SUN declare that they have no conflict of interest.

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