

## NUMERICAL INVESTIGATION ON THE PHASE CHANGE OF WATER-SATURATED POROUS MEDIA WITH THERMOSYPHON

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**Abstract:** Numerical investigations were carried out to determine the coupled heat transfer of water-saturated porous media with a two-phase closed thermosyphon for soil freezing, and to examine the characteristics of the freezing heat transfer in the water-saturated porous media. The whole control volume includes the thermosyphon and the porous media. The two-dimensional governing equations for the water-saturated porous media are used.

The conjugation of heat transfer between the thermosyphon and porous media is reflected through thermal balance between the thermosyphon and the porous media.

The finite-difference method was used to solve the two-dimensional governing equation for the water-saturated porous media and the heat transfer characteristics of the thermosyphon, obtain the flow fields and the temperature distributions in the soil. This paper deals mainly with the effect of some factors (such as soil properties, climate and thermosyphon dimensions) on the heat transfer rate of the thermosyphon and the growth of the freezing front. The predictions of the present study agree well with the measured data.

**Key words:** thermosyphon, water-saturated porous, soil freezing, coupled heat transfer

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### INTRODUCTION

In cold regions, soil expansion resulted from a soil freezing process often deforms and damages some construction foundations. One way to stabilize the foundation is use of two-phase closed thermosyphon in the cold regions. Due to its excellent heat transfer characteristics, the two-phase closed thermosyphon placed in the soil can change the soil temperature distributions and the water-saturated flow field, thicken the freezing front, and limit soil expansion.

Freezing heat transfer in water-saturated porous media was extensively investigated (Beckermann, 1988; Sasaki et al, 1989; 1990). The transient conjugate heat transfer of the thermosyphon with freezing of pure water was investigated experimentally and numerically by Wu and Gu (1989; 1993). However, there are no reports of study on the coupled heat transfer of a vertical two-phase closed thermosyphon with the soil, and on the water-saturated soil freezing resulted from the coupled heat transfer.

Based on the governing equations for the porous media(soil) and the heat transfer characteristics of the thermosyphon, the present work is

aimed to derive a numerical model to describe the coupled heat transfer of a vertical two-phase closed thermosyphon with water-saturated soil freezing; and is focused on the effect of the soil properties, the climate conditions and the thermosyphon characteristics on the coupled heat transfer and the soil freezing processes. This numerical model outputted predictions agreeing well with measured data and can be used to predict soil temperature distribution and soil freezing processes, and direct engineering applications of the thermosyphon for cold regions.

### ANALYSIS

The physical model established in this study is a two-dimensional cylinder coordinate system as depicted in Fig.1.

The heat flux in the soil is transferred and dissipated into the environment by a two-phase closed thermosyphon, with its evaporator  $L_e$  is placed in the water-saturated porous media(soil) and its condenser  $L_c$  opened to the air. The initial temperature of the porous media is above its freezing point. When the air temperature is be-

low the soil temperature, the condenser of the thermosyphon is cooled by the chill air, and the thermosyphon begins to work. The heat flux of the thermosyphon is influenced by the soil heat transfer, while the change of the soil temperature distribution and the freezing layer thickness are controlled by the heat flux of the thermosyphon. This process is a coupled heat transfer between the thermosyphon and the soil freezing.

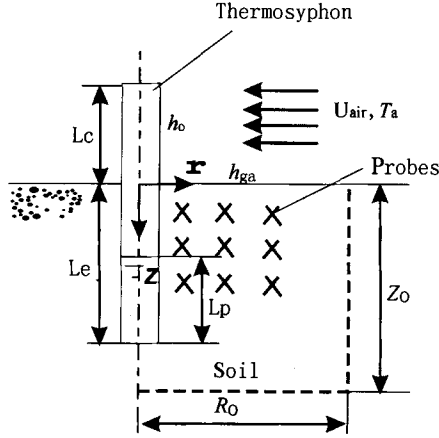


Fig.1 Schematic diagram

For the soil, the following assumptions are made:

- (1) The soil is a porous medium with isotropic and homogeneous properties,
- (2) The fluid flow in the unfrozen porous layer is laminar and incompressible,
- (3) Volume change resulted from the freezing expansion can be negligible and the particles of the porous medium are rigid,
- (4) Thermophysical properties of the water are constant except for its density.

Under these assumptions, the governing equations are as follows.

In the porous media, the continuity, momentum, and energy equations are

$$\frac{\partial(ru)}{\partial z} + \frac{\partial(rv)}{\partial r} = 0 \quad (1)$$

$$\begin{aligned} & \left(\frac{\rho_l}{\epsilon}\right) \frac{\partial u}{\partial \tau} + \left(\frac{\rho_l}{\epsilon^2}\right) \left(u \frac{\partial u}{\partial z} + v \frac{\partial u}{\partial r}\right) \\ &= -\frac{\partial p}{\partial z} + \left(\frac{\mu_l}{\epsilon}\right) \left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial r^2}\right) \\ & - \left(A + \frac{\mu_L}{K} + \frac{\rho_L C |u|}{K}\right) u - \rho_m g \Omega \cdot \\ & (|T_f - T_m|^q - |T - T_m|^q) \end{aligned} \quad (2)$$

where the values of  $T_m$ ,  $\rho_m$ ,  $\Omega$  and  $q$  in Eq. 2 were taken from Gebhart and Mollendorf (1977).

$$\begin{aligned} & \left(\frac{\rho_l}{\epsilon}\right) \frac{\partial u}{\partial \tau} + \left(\frac{\rho_l}{\epsilon^2}\right) \left(u \frac{\partial v}{\partial z} + v \frac{\partial v}{\partial r}\right) \\ &= -\frac{\partial p}{\partial z} + \left(\frac{\mu_l}{\epsilon}\right) \left(\frac{\partial v^2}{\partial z^2} + \frac{\partial v}{\partial r^2}\right) \\ & - \left(A + \frac{\mu_L}{K} + \frac{\rho_L C |v|}{K}\right) v \end{aligned} \quad (3)$$

$$\begin{aligned} & \rho c \frac{\partial T}{\partial \tau} + \rho c_l \left(u \frac{\partial T}{\partial z} + v \frac{\partial T}{\partial r}\right) \\ &= k_{\text{eff}} \left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2}\right) + \epsilon \rho_l \Delta h \frac{\partial f}{\partial \tau} \end{aligned} \quad (4)$$

where

$$K = \frac{d^2 \epsilon^2}{150(1 - \epsilon)^2} \quad (5a)$$

$$C = \frac{1.75 d}{(150 \epsilon^2)^{0.5}} \quad (5b)$$

$$\rho c = \epsilon [f \rho c_l + (1 - f) \rho_s c_s] + [1 - \epsilon] \rho_p c_p \quad (5c)$$

$$k_{\text{eff}} = \epsilon [f k_l + (1 - f) k_s] + (1 - \epsilon) k_p \quad (5d)$$

where

- $A$  constant in Eq. 2 and 3
- $C$  defined in Eq. 5a
- $d$  particle diameter of porous medium
- $g$  gravitational acceleration
- $\Delta h$  latent heat of solidification
- $K$  permeability of porous medium
- $k_l$  thermal conductivity of liquid
- $k_s$  thermal conductivity of solid
- $k_p$  thermal conductivity of porous medium
- $k_{\text{eff}}$  effective conductivity
- $p$  pressure
- $T_f$  freezing temperature
- $\tau$  time
- $\epsilon$  porosity
- $v$  kinematic viscosity
- $c_p$  specific heat of porous medium
- $f$  local liquid fraction
- $r$  radial coordinate
- $u$  velocity in  $z$  direction
- $z$  axial coordinate
- $\rho_l$  density of liquid
- $\mu_l$  dynamic viscosity of liquid

For the thermosyphon, the following assump-

tions may be made;

(1) The liquid film inside the thermosyphon is laminar and the effect of the vapor flow can be negligible;

(2) The vapor in the thermosyphon is saturated and uniform in temperature;

(3) The evaporator section of the thermosyphon consists of two parts: the liquid film and the boiling pool. The temperature of the liquid film equals the vapor temperature. The boiling pool's saturation temperature varies with height due to the hydrostatic pressure.

The heat transfer coefficients for the condensation, film evaporator and boiling pool are written respectively as follows (Shirail, 1982, He and Ma, 1992a, 1992b).

$$h_c \frac{(v_l^2/g)^{1/3}}{k_l} = (4/3)^{4/3} Re_c^{-1/3} \quad (6)$$

where

$$Re_c = \frac{4q_e L_c}{\lambda \mu_l}$$

$$h_f \frac{(v_l^2/g)^{1/3}}{k_l} = (4/3)^{4/3} Re_f(z)^{-1/3} \quad (7)$$

where

$$Re_f(z) = \frac{4q_e(L_e - z)}{\lambda \mu_l}$$

$$\frac{h_p d_i}{k_i} = 14.45 \left( \frac{d_i q_e}{\lambda \mu_l} \right)^{0.39} Pr^{0.75} \left( \frac{\rho_l}{\rho_v} \right)^{0.2} \left( \frac{H_p^* L_e}{d_i} \right)^{0.12} \quad (8)$$

where the dimensionless boiling pool height is

$$H_p^* = \frac{H_p}{H_l} = (1.59 F^{0.95} Re_f^{-0.35} H_l^{*0.37} + 1) L_e \quad (9a)$$

$$H_l^* = L_p / L_e \quad (9b)$$

$$F = \frac{4q_e L_e}{\rho_v \lambda} / \{ 1.53 [ \sigma g [ \rho_l - \rho_v ] / \rho^2 v ]^{0.25} \} d_i \quad (9c)$$

$$Re_f = \frac{4q_e L_e}{\lambda \mu_l} \quad (9d)$$

where

$h_c$  heat transfer coefficient of condensation

$h_f$  heat transfer coefficient of liquid film

$h_p$  heat transfer coefficient of liquid pool

$Re_c$  Reynolds number of condensation

$Re_f$  Reynolds number of liquid film

$\lambda$  latent heat of vaporization

$\sigma$  surface tension

$\nu_l$  kinematic viscosity of liquid

$H_p$  height of liquid pool

$H_l$  height of liquid

$H_p^*$  dimensionless height of liquid pool

$H_l^*$  dimensionless height of liquid

$L_p$  length of liquid pool

$L_e$  length of the evaporator section

$l_c$  length of condensation

$q_c$  heat flux of condensation

$q_e$  heat flux of the evaporator section

$Pr$  Prandtl number

$\rho_v$  density of vapor

$d_i$  inner diameter

$k_i$  inner conductivity

The conjugation of heat transfer between the thermosyphon and porous media is reflected through thermal balance between the thermosyphon and the porous media.

$$k_{\text{eff}} \frac{\partial T}{\partial r} \Big|_{r=R_i} = h_e (T_s - T_{w_e}) \quad (10)$$

where

$h_e$  heat transfer coefficient of the evaporator section

$T_s$  temperature of saturated

$T_{w_e}$  wall temperature of east nodal

However, one of the important problems for the coupled heat transfer is to find the wall temperature distribution of the thermosyphon since the wall of the thermosyphon is an interface of the coupled heat transfer and the boundary conditions for the soil heat transfer.

Where

$$T_s(z) = \begin{cases} T_v & \text{for liquid film} \\ f[P(T_v) + \rho_l g(z - L_e + H_p)] & \text{for liquid pool} \end{cases} \quad (11)$$

$$h_e = \begin{cases} h_f & \text{for liquid film} \\ h_p & \text{for liquid pool} \end{cases} \quad (12)$$

The initial and boundary conditions for the entire porous domain are

$$\tau = 0: T = T(z), u = 0, v = 0$$

$$z = 0: k_{\text{eff}} \frac{\partial T}{\partial z} = h_{\text{ga}} (T - T_a), u = 0, v = 0$$

$$z = \infty: T = T(\tau), u = 0$$

$$z > L_c, r = 0: \frac{\partial T}{\partial r} = 0, v = 0$$

$$z \leq L_c, r = R_0: T = Tw_e(z), u = 0, v = 0$$

$$r = \infty: \frac{\partial T}{\partial r} = 0, v = 0$$

where  $h_{ga}$  is the heat transfer coefficient between ground surface and air.

## THE METHODOLOGY

A general form can be obtained for the governing equations from Eqs. 1 through 4

$$B_1 \frac{\partial \Phi}{\partial \tau} + B_2 \nabla \cdot (V\Phi) = \nabla \cdot (B_3 \nabla \Phi) + S \quad (13)$$

Where  $\Phi$  is a general dependent variable,  $V$  is the velocity factor. The values from  $B_1$  through  $B_3$  are the coefficients of the relevant equations. These equations are discretized with the control volume based on the finite-difference formulation and solved by the SIMPLE algorithm (Patankar et al., 1980).

In the energy conservation equation, the local liquid fraction of this time step in each control volume must be updated by that of the last time step (Lacroix et al., 1990).

$$f = f^0 + \left( \sum a_{nb} T_{nb} + a_p^0 T_p^0 - a_p T_p \right) [\epsilon c_l \Delta \tau / (\rho_l \Delta h \Delta V)] \quad (14)$$

where  $a_{nb}$  and  $T_{nb}$  are coefficients and temperature of the near nodes, respectively.  $a_p^0 T_p^0$  and  $a_p T_p$  are the old values and present values of the node, respectively.

If  $f < 0$  then set  $f = 0$

If  $f > 1$  then set  $f = 1$

If  $0 < f < 1$  then set  $T = T$

Calculations begin from the thermosyphon in the present study. At first, according to Eqs. 5 to 13, the wall temperature of the evaporator of the thermosyphon is evaluated from the heat rate transferred by the thermosyphon which is assumed or calculated at the last time step. Next, in the light of the boundary conditions and the wall temperature of the evaporator, the temperature field and the stream function are obtained by Eqs. 1 to 3. And the heat rate of the thermosyphon is renewed. Then the iterations are repeat-

ed until the following criterion is satisfied at all the grid points:

$$\left| \frac{Q_r - Q_b}{Q_r} \right| < 10^{-3} \quad (15)$$

Where

$$Q_r = \int_0^{z_0} \int_0^{R_0} [\rho c (T - T^0) + \epsilon \rho \Delta h \cdot (f - f^0)] r dr dz$$

$$Q_b = Q_w + Q_e + Q_s + Q_n$$

Eq. 16 is the error range of the energy balance equation.  $Q_r$  denotes the enthalpy increase of the control volume caused by temperature, local liquid fraction and phase change.  $Q_w, Q_e, Q_s$  and  $Q_n$  denotes the energy transferred through the west, east, south and north nodal, respectively.  $Q_b$  denotes the total energy transferred through all boundary surfaces.  $Q_r$  must equal  $Q_b$ , which is the energy balance equation.

## RESULTS AND DISCUSSIONS

In this paper, the values of some parameters, such as porosity of soil ( $\epsilon$ ), air temperature ( $T_a$ ), air velocity ( $U_{air}$ ) and coefficient ( $h_{ga}$ ) of heat transfer between ground surface and air that considered the effect of solar radiation are those of the cold region of northeast China. The effect of  $\epsilon, T_a$  and the diameter of the thermosyphon ( $d_0$ ) on heat transfer of the entire domain is considered. Some numerical results, such as streamline, isotherm and freezing front are analyzed. Fig. 2 compares predicted and measured temperature variation with time for the local point under the soil. Each radial point temperature and the temperature variation with time at the same depth of the soil ( $z$ ) are different under the action of the thermosyphon. The change of temperature for some points near the thermosyphon is more rapid. Fig. 3 shows the predicted and measured change of the freezing front with time. Fig. 2 and Fig. 3 show good agreement between modeled results and experimental data.

The soil porosity  $\epsilon$  and the environment air temperature  $T_a$  have relevant effect on the heat transfer rate of the thermosyphon and the freez-

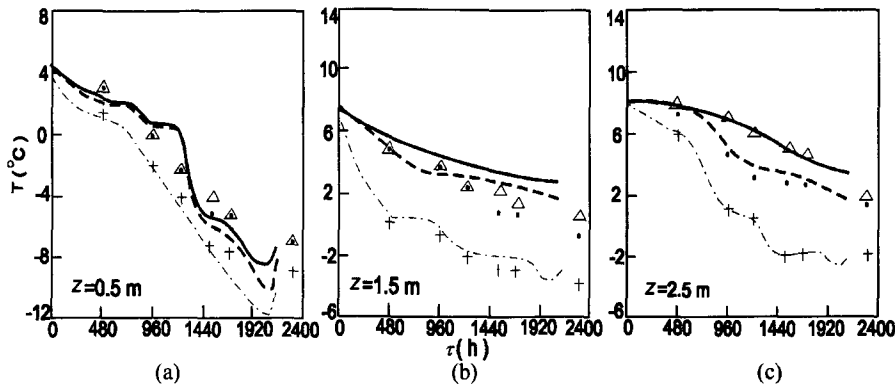


Fig. 2 Variation of local soil temperature with time

(a)  $z = 0.5\text{ m}$ ; (b)  $z = 1.5\text{ m}$ ; (c)  $z = 2.5\text{ m}$

Radius	$r = 0.25\text{ m}$	$r = 0.75\text{ m}$	$r = 1.2\text{ m}$
Predicted	— · —	— · —	— · —
Measured	+	·	△

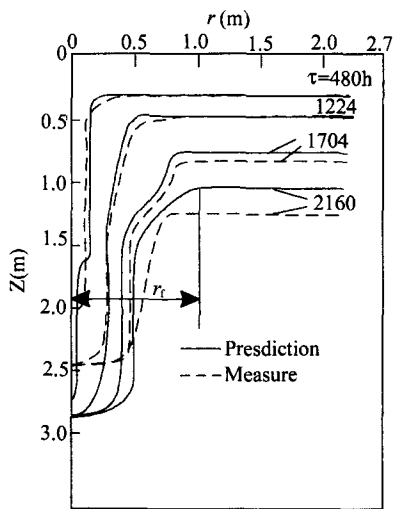


Fig. 3 Change of the freezing front with time

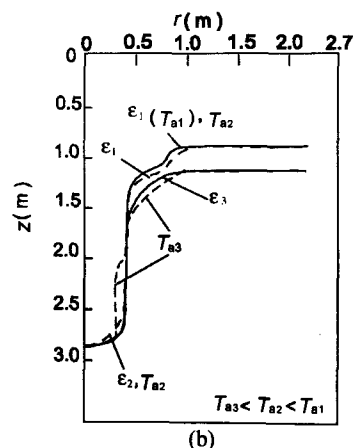
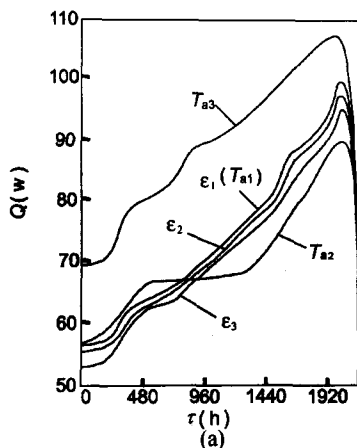


Fig. 4 Numerical results for  $\epsilon_1 = 0.511, \epsilon_2 = 0.5, \epsilon_3 = 0.48$  and different  $T_a$

(a) heat transfer; (b) freezing front

ing, respectively, as shown in Fig. 4. Obviously, the lower the air temperature, the higher is the heat transfer rate of the thermosyphon, since the temperature difference between the air and the condenser of the thermosyphon is increased. According to the average air temperature of four months in the winter season and air temperature variation of each day in the cold region,  $T_a$  is treated linearly. The heat transfer rate of the thermosyphon increases with increase of soil porosity  $\epsilon$ , but the frozen region of lower soil porosity is larger than the frozen region of higher porosity (Fig. 4). For lower porosity soil, the water content in the soil is less, the freezing scope is wider at the same heat transfer rate. The diameter ( $d_0$ ) of the thermosyphon affects the heat transfer rate and the freezing front (Fig. 5). The rate

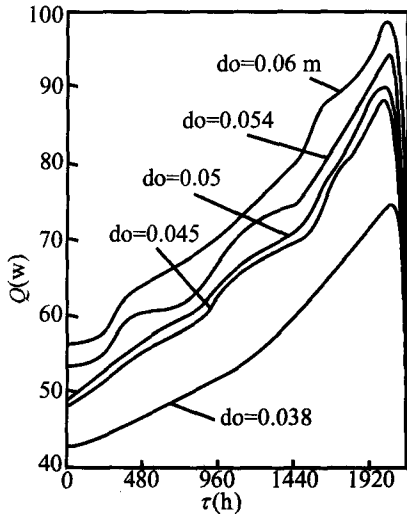


Fig.5 Variation of heat transfer rate with time for different thermosyphon diameters

of the heat transfer inside the thermosyphon increases with increase of thermosyphon diameter.

For the case of  $\epsilon = 0.511$ ,  $d_0 = 0.06$  m,  $U_{air} = 2.98$  m/s,  $h_{ga} = 8.5$  w/m<sup>2</sup> · °C, and air temperature  $T_a$ , the patterns of streamlines and isotherms at  $\tau = 10$  days are as shown in Fig.6a and 6b, respectively. In Fig.6a, there are two counterrotating vortices due to the density inversion of water near 4 °C. One of the vortices is the counterclockwise vortex in the < 4 °C region and near the evaporator of the thermosyphon, and the other is the clockwise vortex in the higher temperature unfrozen region with larger area. With the increase of heat rate transferred from the soil, the freezing region is increased more and more, and the counterclockwise vortex is also increased, as depicted in Fig.7, showing that

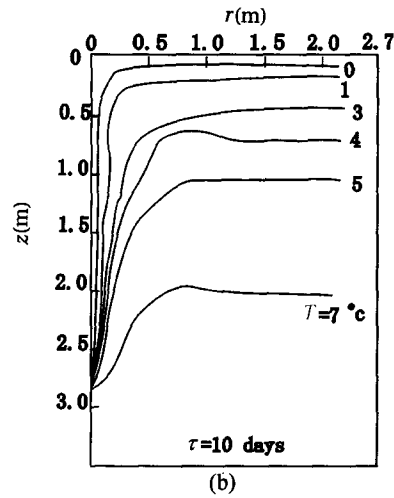
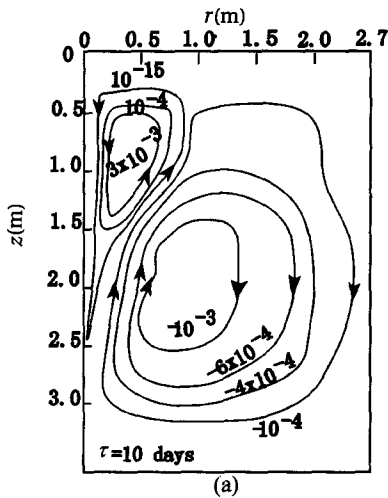


Fig.6 Numerical results for  $\tau = 10$  days  
(a) streamlines; (b) isotherms

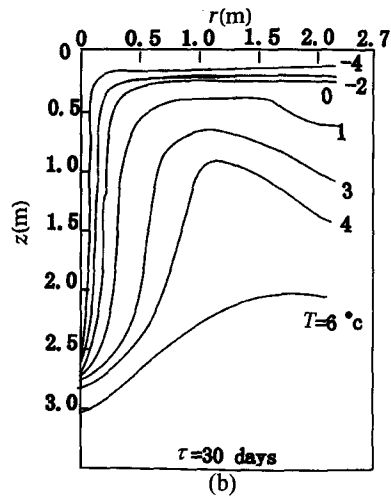
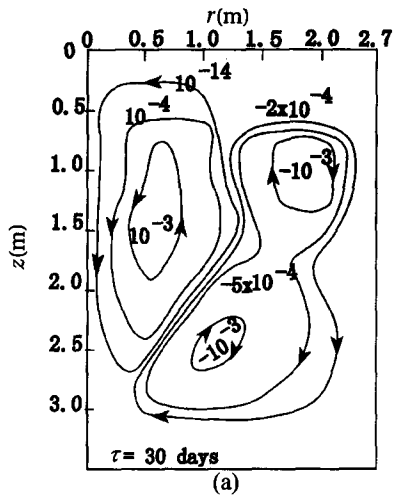


Fig.7 Numerical results for  $\tau = 30$  days  
(a) streamlines; (b) isotherms

there are two vortices of equal magnitude in the unfrozen region because the density inversion of water near 4 °C causes the change of the buoyancy force which divides the > 4 °C vortex region and the < 4 °C region.

## CONCLUSIONS

A vertical two-phase closed thermosyphon was used in this study on water-saturated soil freezing yielding numerical results agreeing well with experimental data and indicating that the thermosyphon could change the temperature distribution of the soil and the thickness of the freezing front. The heat transfer rate of the thermosyphon is affected by soil porosity, environment temperature and thermosyphon diameter.

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