ISSN 1009 - 3095 *Journal of Zhejiang University(SCIENCE)* V.2, No.1, P.46 - 50, Jan. - Mar., 2001
http://www.chinainfo.gov.cn/periodical; http://www.zju.edu.cn;
http://lib.zju.edu.cn/English; http://www.zjupress.com

A MODEL BASED ALGORITHM FOR FAST DPIV COMPUTING

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Received Dec.28, 1999; revision accepted Aug.29, 2000

Abstract: Traditional DPIV (Digital Particle Image Velocimetry) methods are mostly based on area-correlation (Willert, C.E., 1991). Though proven to be very time-consuming and very much error prone, they are widely adopted because of they are conceptually simple and easily implemented, and also because there are few alternatives. This paper proposes a non-correlation, conceptually new, fast and efficient approach for DPIV, which takes the nature of flow into consideration. An Incompressible Affined Flow Model (IAFM) is introduced to describe a flow that incorporates rational restraints into the computation. This IAFM, combined with a modified optical flow method-named Total Optical Flow Computation (TOFC), provides a linear system solution to DPIV. Experimental results on real images showed our method to be a very promising approach for DPIV.

Key words: IAFM, flow visualization, optical flow, computer vision, DPIV
Document code: A **CLC number**: TN911.73

INTRODUCTION

Fluid dynamics research relies on the experimental observation and visualization of fluid flow phenomena (Kawashima, G. et al., 1995; Ohba, K. et al., 1995). Advances in computer technology allow for more accurate simulation and computation for this phenomenon. Digital Particle Image Velocimetry (Willert, C. E., 1991) (DPIV) is a well-known approach in this domain and still an active research area.

Conventional algorithms of DPIV are mostly correlation-based methods (Ohba, K. et al., 1995; Yamamoto, F. et al., 1996; Fuyuki, M. et al., 1995), and others such as, the template match method, SSDA fast correlation method, FFT fast correlation method etc. Athough effective, these methods are very time-consuming and prone to errors inevitably due to the properties of the correlation. So many researchers then focused on how to correct these erroneous vectors afterward (Fujita, I. et al., 1995; Kimura, I. et al., 1993). A DPIV method using feature tracking (Kaga, A. et al., 1993) can produce more exact vectors but is too time consuming. (Ohba, K. et al., 1993) proposed an approach based on optical flow computation (OFC) using a finite element method. The authors updated the velocities estimation by means of a Kalman Filter for a sequence of images to be processed, which may not be available in some experiments.

The above two methods regard two snapshot images as common images, and do not emphasize FLOW images. So they cannot utilize the nature (say, dynamics law) of flow in the computing. This paper provides a faster and efficient solution to DPIV that combines an Incompressible Affined Flow Model (IAFM) with Total Optical Flow Computation (TOFC). In order to satisfy the conditions required by traditional OFC, three new ideas are introduced. They are Total Brightening Constancy Constraint, Multiresolution Processing and Gauss Filtering respectively. By using the proposed methods, the nature of flow is considered easily as an integral part of the computation. No further postprocessing is needed in producing a smooth and plausible flow field.

IAFM-INCOMPRESSIBLE AFFINED FLOW MODEL

The mathematical models employed in this paper arise in the study of the geometric theory of differential equations and dynamic system analysis.

Let $\mathbf{Z}(x, y)$ denote a two-dimensional (2D) flow vector field,

$$Z(x,y) = p(x,y) \cdot \boldsymbol{i} + q(x,y) \cdot \boldsymbol{j} \quad (1)$$

Where $p(x,y), q(x,y)$ are components of $\boldsymbol{Z}(x,y)$ along the direction of the x-axis and y-axis respectively, $\boldsymbol{i}, \boldsymbol{j}$ and are the unit vectors. In differential equation form, it becomes,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = p(x,y) \quad (2)$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = q(x,y) \quad (3)$$

According to fluid dynamics, p and q are the solutions of the 2-D Navier-Stokes Equations

$$\mathrm{div}(\boldsymbol{Z}) = \frac{\partial p}{\partial x} + \frac{\partial q}{\partial y} = 0 \quad (4)$$

$$\rho \mathrm{d}\boldsymbol{Z}/\mathrm{d}t = \nabla N + \mu \cdot \nabla \boldsymbol{Z} + \rho \cdot F \quad (5)$$

Where N, ρ, μ, F denote the pressure, density and coefficient of viscosity and external force respectively. Eq. (4) describes the law of incompressible fluids, and Eq. (5) is derived from Newton's Law.

Generally speaking, these equations are not easy to solve because $p(x,y)$ and $q(x,y)$ are very complex, high order nonlinear functions of the image coordinates x and y. The nature of fluid flow, especially when turbulence is present is still not well understood. Many problems are unanswered. In this work we limited our research to simple steady Stokes flow, and think this simplification is justified because many flow phenomena can be described by such a simple model (Rao, A, R., 1992) Steady Stokes flow is flow satisfying two conditions : (1) The flow is steady or never changes in relation to time; (2) The Reynolds number is small enough.

In order to provide a suitable model of such kind of flow for processing, we need to liberalize by decomposing Eq. (4) and Eq. (5) into their Taylor series at point (x_0, y_0) up to the first ordered components, which result in

$$p(x,y) = p(x_0,y_0) + a(x - x_0) + b(y - y_0) \quad (6)$$

$$q(x,y) = q(x_0,y_0) + c(x - x_0) + d(y - y_0) \quad (7)$$

Where

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \frac{\partial p}{\partial x} & \frac{\partial p}{\partial y} \\ \frac{\partial q}{\partial x} & \frac{\partial q}{\partial y} \end{bmatrix}_{x = x_0, y = y_0}$$

Thus, the flow is described by ordinary differential equations whose coefficients are equivalent to the partial derivative and constant terms. Eqs. (6)(7) are called Affined Flow Models (AFM) in flow research.

The AFM has many applications in fluid dynamics research, in Flow Visualization (Helman, J., 1989), for example, which proved to be an acceptable simplified model for fluid flow. Similar models, with other names such as linear phase portraits (Rao, A, R., 1992), were introduced by other researchers for analyzing oriented texture (Rao, A, R., 1992; Shu, C., 1993) or flow pattern classification (Zhang, J. et al., 1994; Nogawa, H. et al., 1997). But most of their works mainly (in some senses) focused on how to smooth, describe, or estimate a given flow field, which differ from ours, say, by focusing on how to obtain flow vector data from observation and measurement.

In this paper, we adopt the AFM model because the nature of the underlying flow can be embedded in a natural way in the computation of velocity vectors. Substituting Eq. (4) into Eqs. (6) and (7), yielded the following new model:

$$p(x,y) = p(x_0,y_0) + a(x - x_0) + b(y - y_0) \quad (8)$$

$$q(x,y) = q(x_0,y_0) + c(x - x_0) - a(y - y_0) \quad (9)$$

Please note that the difference between Eqs. (6)(7) and Eqs. (8)(9), of the new model has, the constraint of incompressibility [Eq. (4)] embedded in it. That is why we called it IAFM (Incompressible Affined Flow Model). By introducing another constraint of DPIV imaging, say, the TOFC (total brightening constancy equation), we then relate this flow model with the observation equation.

TOFC-TOTAL OPTICAL FLOW COMPUTATION

A basic assumption of DPIV is that the exposure time (the interval between two successive frames) is short enough to consider the images as instantaneous snapshots of the velocity field. Based on this assumption, we regard the graylevel value at every point of every particle in the flow does not change. In other words, the perceived changes in image brightening must be entirely due to motion. This is called brightening constancy constraint in traditional optical flow computation (OFC) (Horn, B, K, P., 1986), and is written as

$$d\,G(x,\ y,\ t\ ) = 0 \tag{10}$$

Where $G(x,\ y,\ t)$ represents the image function of the flow field and x, y, t the space and time parameters, and d denotes differential operator.

Unfortunately, because most DPIVs are illumined by a flash lamp whose intensity cannot be controlled precisely, Eq. (10) should not hold if we consider the change of illumination. So, let the change of illumination be denoted by $\mathrm{d}i$, and introduce a total brightening constancy constraint as

$$\mathrm{d}G(x,\ y,\ t\ ,\ i\ ) = 0 \tag{11}$$

By substituting p and q in Eq.(11), we get

$$G_x \cdot p + G_y \cdot q + G_t + G_i \frac{\mathrm{d}i}{\mathrm{d}t} = 0 \tag{12}$$

Based on differential concept, let $\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{\Delta i}{\Delta t}$, $k = \frac{1}{\frac{\Delta i}{\Delta t} + 1}$, so (13) replaces (12)

$$G_x \cdot p + G_y \cdot q + G_t + \frac{1-k}{k} G_i = 0 \tag{13}$$

Where $k = illumination\,(t)/illumination\ (t + \Delta t)$, means that the change of illumination is linear. We call Eq. (11) TOFC (Total Optical Flow Computation).

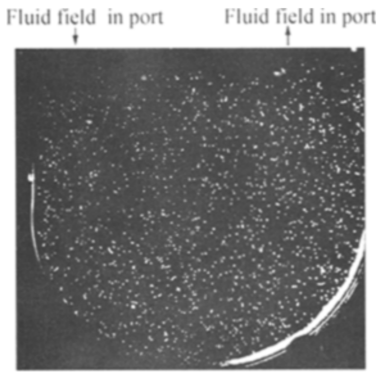
Two conditions required to ensure the validity of the TOFC. are:

(i) The estimation is valid as long as the slope of the brightening function of the pattern does not exceed the pattern displacement. Multiresolution Processing according to our method can assure this, by smoothing and sub-sampling of the source image, such as a Gaussian Pyramid; (ii) the intensity profile of image G $(x,\ y)$ must be differentiable, otherwise gradients in Eq.(13) cannot be calculated. We satisfied this condition by smoothing the sub-sampled image by Gaussian Filtering.

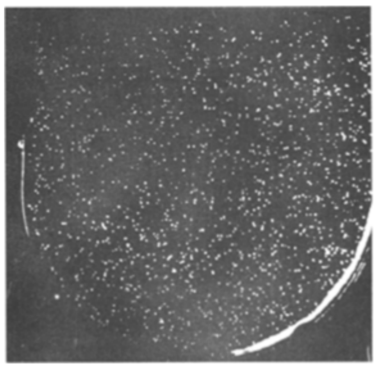
Eq. (13) is an equation in three unknowns p, q and k. We showed in Eq. (8)(9) that $(p,\ q)$ can be decomposed into finite linear terms. By combining Eq. (8)(9) with Eq. (13) and taking measurements at many points, an over-determined linear system solution of these unknowns are obtained. A Least Square method (LS) is then used to solve this problem.

ALGORITHM AND IMPLEMENTATION

This section gives a brief description of the algorithm using IAFM and some experiments to show the efficiency of the provided approach. The DPIV images shown in Fig.1 were captured in a real experiment on fluid dynamics. The size of each raw image was 1024 × 1024.



(a)



(b)

Fig.1 DPIV images used in our experiment
(a) image Ⅰ; (b) image Ⅱ

1. Preprocessing

According to the first condition of TOFC, we must first smooth and sub-sample the source images obtained from DPIV because the aim and function of DPIV is to record a very short time exposure photograph of the particles in the flow. If the average flow velocity is much greater than the average particle diameter, then Eq. (11) does not hold. A classical multi-resolution technique-Gaussian Pyramid was applied. The ratio of sub-sample is due to the maximum velocity in the flow. In our experiment, we re-sampled the size 256×256 raw images. A Guassian filter of ($\sigma = 3.0$ was applied to assure the validity of the gradient computation of the images.

2. Divide into patches

An IAFM is a good approximation of the flow field within small local patches. According to this, we divided the images into overlapped square patches. That is to say that only the velocity at each point inside a small patch has IAFM property. The size of each patch is determined according to the velocity the field resolution. Here we wanted to obtain a 32×32 velocity field, so the size of 16×16 or 24×24 of each patch are preferred.

1	0	-1	1	2	1
2	0	-2	0	0	0
1	0	-1	-1	-2	-1

Fig.2 Sobel masks

3. Select feature points

Referring to Verri and Poggio's work (Verri, A., 1989), the brightening constancy equation is more accurate at the point where the magnitude of G is large enough. So we first computed the gradient by a Sobel mask (Fig.2) at each points inside a patch, then sorted them by the magnitude of gradients: $|G| = \sqrt{G_x^2 + G_y^2}$, selected several points with largest $|G|$, and called them feature points. We selected the first 64 points that were enough for over-determining.

4. Least square estimations

Consider the linear system of equations (8) (9) and (13) at selected feature points inside each patch. Let the lower left corner of a patch be at (x_0, y_0) and substitute Eqs (8) and (9) in Eq. (13). Then we get six unknowns $X = \{p_0, q_0, k, a, b, c\}$ with one equation. If there exist 64 feature points, it will be 64 equations with 6 unknowns, then the problem can be solved by an LS (Least Square) method by means of a pseudo inverse operator

$$X = (A^T \cdot A)^{-1} \cdot A^T \cdot B \tag{14}$$

5. Experimental Results

Following the steps mentioned above, the global flow field of images of Fig.1 was reconstructed piecewise by an LS method. The resultant 2D flow field is shown in Fig.4. For comparison, we also show the resultant flow field (Fig.5) obtained from a conventional correlation-based method. It took 2.7 seconds using our method to produce a 32×32 flow field when our program was run on a Pentium/166MHz PC, as compared to 35.0 seconds by a correlation based method. Notice that Fig.5 has many erroneous vectors while Fig.4 is rather smooth, and has very delicate structures. For example, our observation in the experiment confirmed the existence of the small vertex denoted by an arrow in the left part of Fig.4. Furthermore, we could measure the local properties of the resultant flow in an analytic way, for example, by considering, the small vortex structure at the left of Fig.4 as the origin of the local coordinates system with the vortex as its center.

$$\begin{cases} p = 0.012x + 0.064y \\ q = -0.012x - 0.012y \end{cases} \tag{15}$$

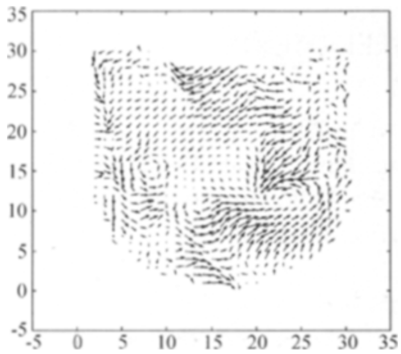


Fig.4 Flow field by our method

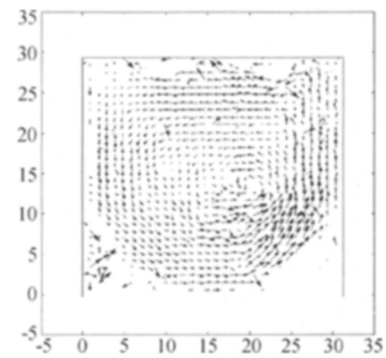


Fig.5 Flow field by correlation

CONCLUSIONS

Most conventional methods of DPIV follow the scheme of area-correlation, and often regard the DPIV images as two common images, and do not make use of the nature of FLOW, and as a result, are not only very time consuming but also very error prone.

In this paper, we aim to provide a unified approach that takes advantage of both an IAFM model and TOFC computation. By introducing three new preprocessing techniques (Total Optical Flow Computation, Multiresolution Processing and Gauss Filtering) we solved the problem of DPIV by solving an over-determined linear system. Experiments on real images showed the proposed algorithm to be a fast and efficient approach for DPIV.

Further researches will include hardware (e.g., DSP chips) implementation. We believe our algorithm is much promising for the realization of Real-time Flow Reconstruction and Visualization.

ACKNOWLEDGMENTS

We are grateful to Professor Affeld, K. of Technische Universität Berlin (Germany) for providing the DPIV images and reference papers and inspiring discussion.

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