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# RESEARCH ON THE CONTINUOUS POSITIONING CONTROL TO SERVO-PNEUMATIC SYSTEM*

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**Abstract:** Pneumatic driven system has been widely used in industrial automation mainly for relatively simply tasks with open-loop control. With regard to closed-loop controlled axes in robotics, servo-electric driven systems have been dominant up to now. This paper introduces a new closed-loop control servo-pneumatic system that can do continuous positioning. The mathematical model of the servo-pneumatic system was developed accurately through analysis of the flow characteristics of the proportional flow valve and the friction of the cylinder. The optimum control strategy with friction compensation is presented in this paper. Experiments demonstrate that the servo-pneumatic system has excellent tracking characteristics and can rival the expensive servo-electric system in many areas of industrial control.

**Key words:** mathematical model, servo-pneumatic, robot, positioning
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## INTRODUCTION

Servo-pneumatic systems have been studied since the 1950's. Due to their natural characteristics of air compressibility, poor damping ability, significant mechanical friction, and strong nonlinearities, their application was restricted. With the rapid development in the field of electronics (especially digital electronics) and of modern control theory, many servo-pneumatic position control systems have been developed in the automation industry (Zhou., 1999).

This paper presents a mathematical model of a servo-pneumatic system developed accurately through analysis of the flow characteristics and the mechanical friction. As many previous servo-pneumatic systems generally function point-to-point and as our intensive research on the continuous path tracking by friction compensation showed that nonlinear mechanical friction causes a stick-slip response in the servo-pneumatic system (Yang et al., 1997; Tokashiki et al., 1999); we developed a servo-pneumatic robot consisting of three-degree-of-freedom servo-pneumatic axes. Experiment results showed that the robot could do well in continuous position tracking.

## MATHEMATICAL MODEL OF VALVE-CONTROLLED SERVO-PNEUMATIC CYLINDER

Fig.1 shows a valve-controlled cylinder. The mass of gas in chamber $A$ with volume $V_a$ is $m_a$ and that in chamber B with volume $V_b$ is $m_b$. $\rho_a$ is the mass density of the gas in the chamber A and $\rho_b$ is that in B.

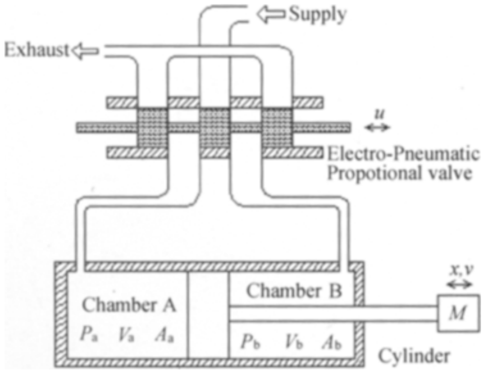

**Fig.1 Schematic diagram of valve-controlled cylinder**

The continuous flow equations are

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$$q_{ma} = \frac{dm_a}{dt} = \frac{d(\rho_a V_a)}{dt} \quad (1)$$

$$q_{mb} = \frac{dm_b}{dt} = \frac{d(\rho_b V_b)}{dt} \quad (2)$$

The pressure change in the cylinder chambers can be obtained from the following equations deduced from the energy equations of gas,

$$\frac{dp_a}{dt} = R\frac{C_p}{C_v}T_s\frac{q_{ma}}{V_a} - \frac{C_p}{C_v}\frac{p_a}{V_a}\frac{dV_a}{dt} \quad (3)$$

$$\frac{dp_b}{dt} = R\frac{C_p}{C_v}T_s\frac{q_{mb}}{V_b} - \frac{C_p}{C_v}\frac{p_b}{V_b}\frac{dV_b}{dt} \quad (4)$$

where, $T_s$, air supply temperature;

$C_p$, specific heat at constant pressure;

$C_v$, specific heat at constant volume;

Sanvill's flow rate equation (Sanvill et al., 1971) is normally used in the mathematical model,

$$q_m = \begin{cases} \dfrac{A(u)}{\sqrt{T}}\sqrt{\dfrac{k}{R}\dfrac{2}{k-1}}p_s\sqrt{\left(\dfrac{p}{p_s}\right)^{2/k} - \left(\dfrac{p}{p_s}\right)^{k+1/k}} \\ \qquad \cdots\cdots \dfrac{p}{p_s} > C_t \\ \dfrac{A(u)}{\sqrt{T}}p_s\sqrt{\dfrac{k}{R}\left(\dfrac{2}{k+1}\right)^{k+1/k-1}} \cdots\cdots \dfrac{p}{p_s} \leqslant C_t \end{cases} \quad (5)$$

where, $C_t = \left(\frac{2}{k+1}\right)^{k/k-1}$

Bobrow's represented that the Sanvill's flow rate equation was for theoretical situation, and presented a flow rate equation (6) obtained by experiments and trials ( Bobrow et al., 1998).

$$q_m = C_f A(u)(p_s - p)^{1/2} \quad (6)$$

When $V =$ constant,

$$\frac{dp}{dt} = R\frac{C_p}{C_v} = R\frac{C_p}{C_v}T_s\frac{q_m}{V}.$$

Please check with original if this shonld be transposed as shown. We can obtain $q_m$ from $\frac{dp}{dt}$. After many experiments, we found that differentiating the pressure after the experimental pressure data were processed digitally would lead to large error in $q_m$. So we differentiated the original pressure data to get the flow rate trend curve which reflects the real flow rate curve to a certain extent. By this means, we found that Sanvill's flow rate equation and Bobrow's flow rate equation could not fit perfectly with the flow rate trend curve (Fig.2 and Fig.3). So we derived a new flow equation which can fit the flow rate trend curve very well (Fig.4).

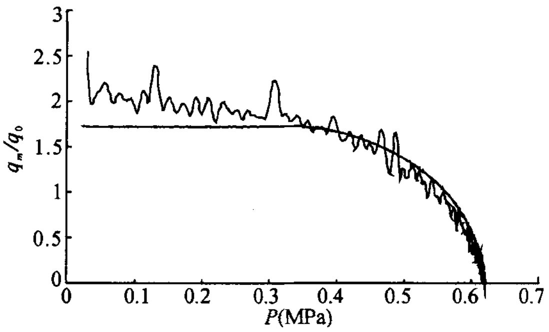

**Fig.2 Sanville's flow curve and experimental flow trend**

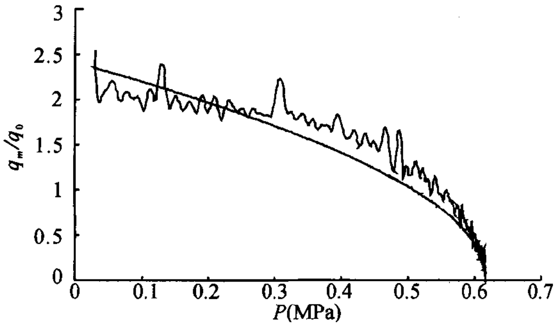

**Fig.3 Bobrow's flow curve and experimental flow trend**

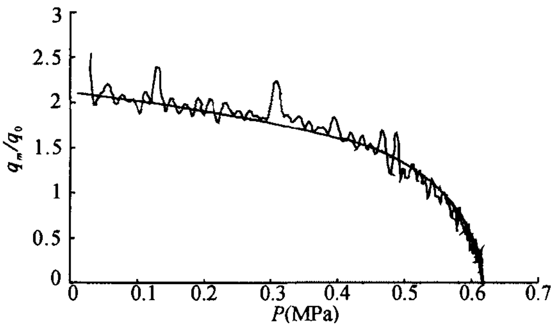

**Fig.4 New flow curve and experimental flow trend**

Where, $C_{f1}$ and $C_{f2}$ are experimental coefficients, $\lambda = 0.25$.

In order to reduce cylinder friction, we used a compact cylinder with linear guide. The friction model was optimized.

$$F_f = \pm\ (F_j - k_{v1} \cdot v),\ \text{when } v < v_d$$

$$F_f = \pm\ (F_j - k_{v1} \cdot v_d + k_{v2} \cdot (v - v_d)),\ \text{when } v \geqslant v_d \quad (8)$$

where, $F_j$, still friction force;

$k_{v1}$, $k_{v2}$, dynamic friction coefficients;

$v_d$, critical velocity;

Equation of motion,

$$(p_a A_1 - p_b A_2) - F_f = M \frac{d^2 x}{dt} \tag{9}$$

where, $A_1$, ram area in chamber A;

$A_2$, ram area in chamber B;

Eqs. (1), (2), (3), (4), (7), (8), (9) comprise the mathematical model describing the dynamic behavior of the valve-controlled cylinder.

## CONTROL STRATEGY

Status control has proved extremely useful for the pneumatic servo-position system. There are three status variables normally: displacement '$x$', velocity '$v$' and acceleration '$a$'. In this paper, the friction status variable '$F_f$' is used to reduce the stick-slip response in the pneumatic servo-position system. Differentiating '$x$' and filtering the resulting data reduces the noise effect and yield the status variable '$v$'. Status variables '$a$' and '$F_f$' are obtained from status observers (Eq. 10 and Eq. 11). Assume, $x_1 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$, $x_2 = \ddot{x}$ the acceleration status observer $\hat{a} = \hat{x}_2$

$$w = \hat{x}_2 - K_1 x_1$$

$$\dot{w} = (A_{22} - K_1 A_{12}) w + (B_2 - K_1 B_1) u + [K_1 (A_{22} - K_1 A_{12}) + (A_{21} - K_1 A_{11})] x_1 \tag{10}$$

where, $K_1$, $A_{11}$, $A_{12}$, $A_{21}$, $A_{22}$, $B_1$, $B_2$ are the acceleration state-space equation coefficients.

Assume,

$$x_1 = \dot{x}$$

$$x_2 = F_f$$

$$u_p{}' = p_a - \frac{A_2}{A_1} p_b$$

the friction force status observer $\hat{F}_f = \hat{x}_2$

$$w = \hat{x}_2 - K_f x_1$$

$$\dot{w} = (A_{f22} - K_f A_{f22}) w + (B_{f2} - K_f B_{f1}) u_p + [K_f (A_{f22} - K_f A_{f12}) + (A_{f21} - K_f A_{f11})] x_1 \tag{11}$$

where, $K_f$, $A_{f11}$, $A_{f12}$, $A_{f21}$, $A_{f22}$, $B_{f1}$, $B_{f2}$ are the friction state-space equation coefficients.

Fig. 5 shows the pneumatic circuit of a single-freedom system. Fig. 6 shows the control strategy. In order to reduce the friction effect, let $G_f(s) = -\frac{1}{K_0 G(s)}$.

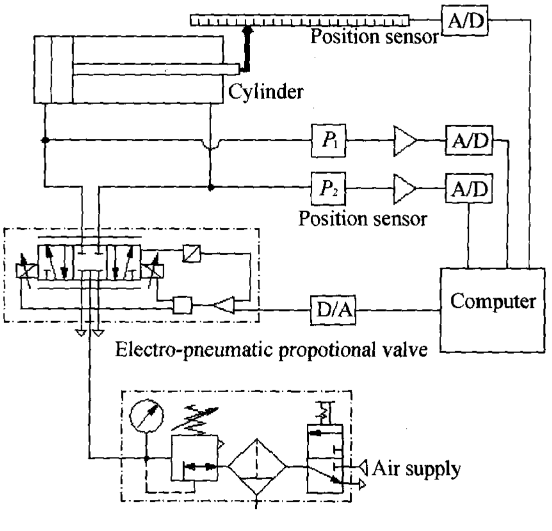

**Fig. 5 Pneumatic circuit of single-freedom system**

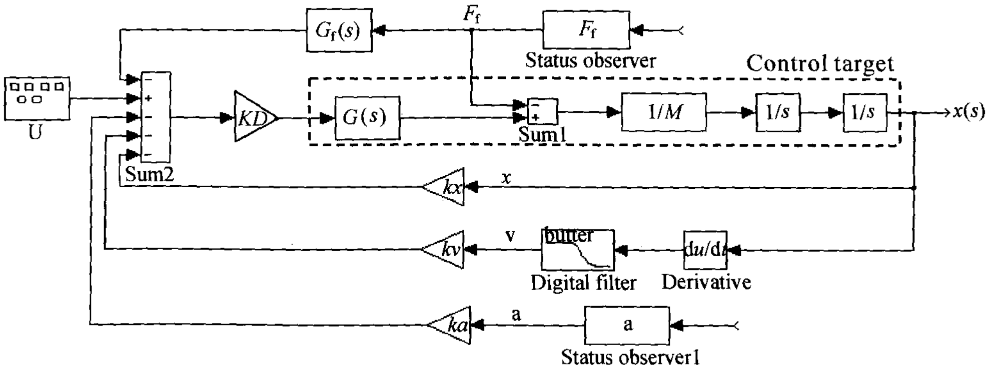

**Fig. 6 Control strategy**

## EXPERIMENT

A servo-pneumatic robot consisting of three-degree-of-freedom servo-pneumatic axes, is developed. When a trajectory is inputted, the computer will process the data and send data to every axis controller. Fig.7 shows the system structure.

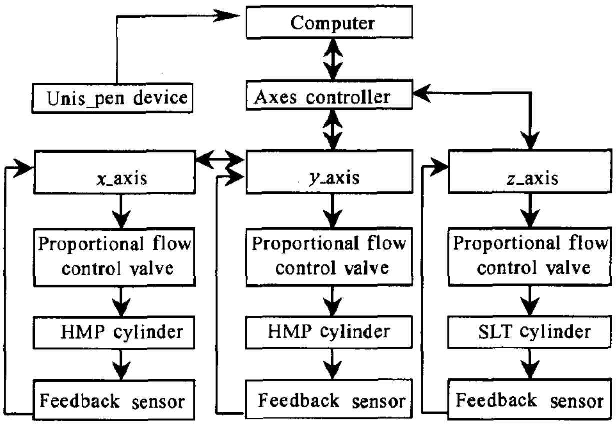

**Fig.7 System structure**

The trajectory-input device is an unis-pen that can record three-dimensional position to computer by handwriting. You can also input a trajectory by other way, such as circle generated by a mathematical function.

Fig.8 and Fig.9 show the results of trajectory position tracking in xyz space coordinates.

## CONCLUSIONS

The results (Fig.8) showed that the servo-pneumatic positioning system can do well in continuous trajectory position tracking. It is important for the designer of a servo-pneumatic positioning system to reduce the friction of cylinder, foid an accurate mathematical model, and optimize the control strategy. The experiment results showed that the servo-pneumatic robot has excellent tracking characteristics and can rival the expensive servo-electric robot in many areas of industrial automation.

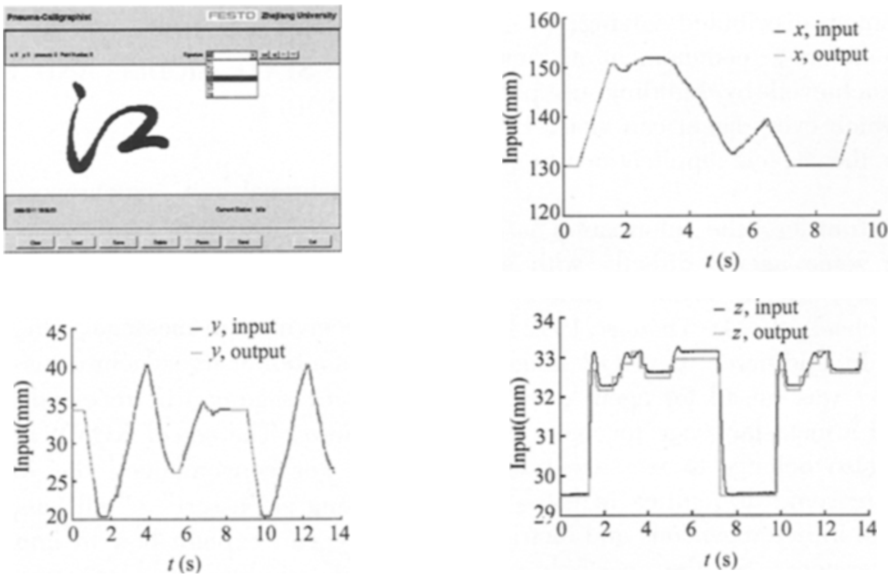

**Fig.8 Trajectory position tracking of Chinese word "江"**

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