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COMPUTERIZED KINEMATIC AND DYNAMIC ANALYSIS OF LARGE DEPLOYABLE STRUCTURES*

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Abstract: As Huston's form of Kane's equation cannot be easily applied to large deployable structures, what is needed is further development of Kane's equation as described in this paper. Fully-Cartesian-coordinate (FCC) method uses Cartesian coordinates of points and Cartesian components of unitary vectors as generalized coordinates to describe three-dimension mechanisms. This FCC method avoids the need to consider angular coordinates and the resulting solution is just the space position of the structures. The FCC form of Kane's equation derived in this study is suitable for solution by computer method and is a good base for further simulation research. A numerical example showed that it is effective.

Key words: deployable structure, dynamic, kinematic, Kane's equations
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INTRODUCTION

People did not pay enough attention to deployable structures the last 30 years until they were used in spaceflight and satellite communication as deployable antenna and solar array. Study of deployable structures requires thorough knowledge of the kinematic and dynamic characteristics of deployment process at first, and then find a proper method for describing large deployable structures. Kane presented a new dynamics equation and pointed out that it was the most suitable for dealing with complex spacecraft dynamics (Kane, 1961; 1985). Huston (1974) developed Kane's equation into a new form, which some people named Huston's form of Kane's equation. Singh (1985) took full advantage of the singular value decomposition (SVD) of the Jacobi matrix of the constraint equations to derive kinematic equations. The orthogonal complement array of constraint Jacobi can reduce differential algebraic equations (DAEs) to ordinary differential equations (ODEs) with elimination of undetermined multipliers using PUTD (Amirouche, 1987).

However, Huston's form of Kane's equation is based on an open (not closed) loop multibody system. When the system has close loops, it must be described as an open loop system with additional constraint equations. In fact, large deployable structures contain a good few closed loops (Langbecker, 1999). This raises some questions: Huston's form remains an effective method? Are there any other effective methods?

Jalón (1981) and Bayo (1991) employed FCC as generalized coordinate to describe three-dimension mechanisms. In the present paper FCC gives the analytical expression of the mass center velocity and acceleration, and the angular velocity of a typical body. Then the partial velocity and partial angular velocity are derived from them. After the FCC form of Kane's equation was formulated, a numerical example showed that this method is effective.

A TYPICAL BODY

Fig. 1 shows a rigid body B in multibody system A. A dextral set of mutually perpendicular vectors $\boldsymbol{O_0x}$, $\boldsymbol{O_0y}$, $\boldsymbol{O_0z}$, treated as reference frame (i.e. global base, denoted as $\boldsymbol{n}_{o1}$, $\boldsymbol{n}_{o2}$, $\boldsymbol{n}_{o3}$), are fixed in A. It is assumed that system A has N_1 points and N_2 unitary vectors. Then the generalized coordinates are defined as $x_{i,1}$, $x_{i,2}$, $x_{i,3}$

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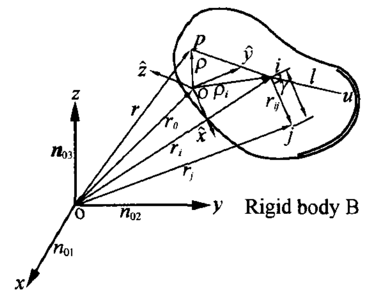

Fig.1 ***A typical body***

$1 \leqslant i \leqslant (N_1 + N_2)$

where $x_{i,m}$ ($m = 1,2,3$) are the Cartesian coordinates of point or the Cartesian components of unitary vector. The total number of generalized coordinates is $N_3 = 3(N_1 + N_2)$.

The generalized speeds are defined as

$$\begin{cases} y_{3i-2} = \dot{x}_{i,1} \\ y_{3i-1} = \dot{x}_{i,2} \\ y_{3i} = \dot{x}_{i,3} \end{cases} \tag{1}$$

The typical body is modeled with two points i, j and a unitary vector $\boldsymbol{u}$. γ ($\gamma \neq 0, 180$) is the radian measure of the angle between $\boldsymbol{r}_{ij}$ and $\boldsymbol{u}$. Point O is the mass center of the typical body. P is a random point in the typical body and O_0 is the origin point in the reference frame. Some vectors are defined as follows

$$\boldsymbol{r}_i = \boldsymbol{O_0 i},\ \boldsymbol{r}_j = \boldsymbol{O_0 j},\ \boldsymbol{r}_0 = \boldsymbol{O_0 O},\quad \boldsymbol{r} = \boldsymbol{O_0 P},\ \boldsymbol{\rho}_i = \boldsymbol{Oi},\ \boldsymbol{\rho} = \boldsymbol{OP} \tag{2}$$

One can also define α, β as $\alpha = \sin\gamma$. Now an auxiliary reference unitary vector $\boldsymbol{v}$ is introduced. $\boldsymbol{v} = (\alpha l)^{-1}(\boldsymbol{r}_{ij} \times \boldsymbol{u})$. Then one can create a local right-handed set mutually perpendicular coordinate system $O - \hat{x}\hat{y}\hat{z}$ fixed in B with $\boldsymbol{O\hat{x}}$ parallel to the vector $\boldsymbol{r}_{ij}$, $\boldsymbol{O\hat{z}}$ parallel to the vector $\boldsymbol{v}$, and $\boldsymbol{O\hat{y}}$ lay on $\boldsymbol{O\hat{x}}$ and $\boldsymbol{O\hat{z}}$ according to right-handed rule. Then right-handed set mutually perpendicular unitary vectors in local base are given by

$$\boldsymbol{e}_1 = \boldsymbol{r}_{ij}/l,\quad \boldsymbol{e}_2 = \boldsymbol{u}/\alpha - \beta\boldsymbol{r}_{ij}/\alpha l,\quad \boldsymbol{e}_3 = -\tilde{u}\boldsymbol{r}_{ij}/\alpha l \tag{3}$$

where $\tilde{u}$ is dual matrix of vector $\boldsymbol{u}$,

$$\tilde{u} = \begin{bmatrix} 0 & -x_{u,3} & x_{u,2} \\ x_{u,3} & 0 & -x_{u,1} \\ -x_{u,2} & x_{u,1} & 0 \end{bmatrix}$$

They form the transform matrix

$$R = [\boldsymbol{e}_1 \quad \boldsymbol{e}_2 \quad \boldsymbol{e}_3] \tag{4}$$

Project $\boldsymbol{iP}$ on $\boldsymbol{r}_{ij}, \boldsymbol{u}, \boldsymbol{v}$ with (c_1, c_2, c_3) function as coordinates.

$$\boldsymbol{iP} = \boldsymbol{r} - \boldsymbol{r}_i = \boldsymbol{\rho} - \boldsymbol{\rho}_i = c_1\boldsymbol{r}_{ij} + c_2\boldsymbol{u} + c_3\boldsymbol{v} \tag{5}$$

Then one can get a vector $\boldsymbol{r}$ expressed by points, (i, j) and unitary vector† (u)

$$\boldsymbol{r} \underset{(3)(5)}{=} Cq_B \tag{6}$$

where

$$\begin{cases} C = [(1 - c_1)E + (\alpha l)^{-1}c_3\tilde{u},\ c_1E - (\alpha l)^{-1}c_3\tilde{u},\ c_2E] \\ q_B = [\boldsymbol{r}_i^T, \boldsymbol{r}_j^T, \boldsymbol{u}^T]^{\mathrm{T}} \end{cases}$$

and E is 3×3 identity matrix.

Then the vector $\boldsymbol{r}_0$ can be expressed as follows

$$\boldsymbol{r}_0 \underset{(5)(6)}{=} Dq_B \tag{7}$$

where D are functions of $\boldsymbol{r}_{ij}, \boldsymbol{u}$, and can be expressed as

$$D = [(1 - a_1)E + (\alpha l)^{-1}a_3\tilde{u},\ a_1E - (\alpha l)^{-1}a_3\tilde{u},\ a_2E] \tag{8}$$

Where $\boldsymbol{a} = [a_1, a_2, a_3]^{\mathrm{T}} = m^{-1}\int \boldsymbol{c}\,\mathrm{d}m = -\Gamma\hat{\boldsymbol{\rho}}_i$,

$\Gamma = \dfrac{1}{\alpha l}\begin{bmatrix} \alpha & -\beta & 0 \\ 0 & l & 0 \\ 0 & 0 & \alpha l \end{bmatrix}$, $\hat{\boldsymbol{\rho}}_i$ is the projection of $\boldsymbol{Oi}$ to the local base. So the mass center velocity of the typical body can be written as

$$\boldsymbol{v}_k = \frac{\mathrm{d}\boldsymbol{r}_0}{\mathrm{d}t} = \phi\dot{q}_B \tag{9}$$

and $\phi = [(1 - a_1)E + (\alpha l)^{-1}a_3\tilde{u},\ a_1E - (\alpha l)^{-1}a_3\tilde{u},\ a_2E + (\alpha l)^{-1}a_3\,\tilde{r}_{ij}]$

According to (Huston 1994), velocity can be expressed in terms of partial velocities and generalized speeds as given below

$$\boldsymbol{v}_k = v_{klm}y_l\boldsymbol{n}_{om} \tag{10}$$

But from Eq. (9), $\boldsymbol{r}_k$ is only relevant with $\boldsymbol{r}_i, \boldsymbol{r}_j$ and $\boldsymbol{u}$, so v_{klm} are nonzero just when $l = 3i - 2, 3i - 1, 3j - 2, 3j - 1, 3j, 3u - 2, 3u - 1, 3u$.

† Numbers beneath signs of equality refer to equations numbered correspondingly.

Here it is presumed that i, j and u mean separately global serial number. Body B's mass center partial velocities can be arranged as follows

$$v_{k,3i-2} \underset{(1,9,10)}{=} \begin{Bmatrix} 1-a_1 \\ \frac{a_3}{\alpha l}x_{u,3} \\ -\frac{a_3}{\alpha l}x_{u,2} \end{Bmatrix} \quad v_{k,3i-1} = \begin{Bmatrix} -\frac{a_3}{\alpha l}x_{u,3} \\ 1-a_1 \\ \frac{a_3}{\alpha l}x_{u,1} \end{Bmatrix}$$

$$v_{k,3i} = \begin{Bmatrix} \frac{a_3}{\alpha l}x_{u,2} \\ -\frac{a_3}{\alpha l}x_{u,1} \\ 1-a_1 \end{Bmatrix} \quad v_{k,3j-2} = \begin{Bmatrix} a_1 \\ -\frac{a_3}{\alpha l}x_{u,3} \\ \frac{a_3}{\alpha l}x_{u,2} \end{Bmatrix}$$

$$v_{k,3j-1} = \begin{Bmatrix} \frac{a_3}{\alpha l}x_{u,3} \\ a_1 \\ -\frac{a_3}{\alpha l}x_{u,1} \end{Bmatrix} \quad v_{k,3j} = \begin{Bmatrix} -\frac{a_3}{\alpha l}x_{u,2} \\ \frac{a_3}{\alpha l}x_{u,1} \\ a_1 \end{Bmatrix}$$

$$v_{k,3u-2} = \begin{Bmatrix} a_2 \\ -\frac{a_3}{\alpha l}(x_{i,3}-x_{j,3}) \\ \frac{a_3}{\alpha l}(x_{i,2}-x_{j,2}) \end{Bmatrix}$$

$$v_{k,3u-1} = \begin{Bmatrix} \frac{a_3}{\alpha l}(x_{i,3}-x_{j,3}) \\ a_2 \\ -\frac{a_3}{\alpha l}(x_{i,1}-x_{j,1}) \end{Bmatrix}$$

$$v_{k,3u} = \begin{Bmatrix} -\frac{a_3}{\alpha l}(x_{i,2}-x_{j,2}) \\ \frac{a_3}{\alpha l}(x_{i,1}-x_{j,1}) \\ a_2 \end{Bmatrix} \tag{11}$$

Further, one can get mass center acceleration from Eq. (10)

$$\boldsymbol{a}_k \underset{(10)}{=} (v_{klm}\ddot{y}_l + \dot{v}_{klm}\dot{y}_l)\boldsymbol{n}_{om} \tag{12}$$

Here $\dot{v}_{klm}$ means the time-derivative of v_{klm}, and in fact it is easy to deduce $\dot{v}_{klm}$ from Eq. (11). According to (Kane, 1985), the angular velocity of B in A, denoted by ${}^A\boldsymbol{\omega}^B$, is defined as

$${}^A\boldsymbol{\omega}^B \triangleq \boldsymbol{e}_1\frac{{}^A\mathrm{d}\boldsymbol{e}_2}{\mathrm{d}t}\cdot\boldsymbol{e}_3 + \boldsymbol{e}_2\frac{{}^A\mathrm{d}\boldsymbol{e}_3}{\mathrm{d}t}\cdot\boldsymbol{e}_1 + \boldsymbol{e}_3\frac{{}^A\mathrm{d}\boldsymbol{e}_1}{\mathrm{d}t}\cdot\boldsymbol{e}_2 \tag{13a}$$

Using dots to denote time-differentiation in A, one can rewrite Eq. (13) as

$${}^A\boldsymbol{\omega}^B = [\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3]\begin{bmatrix} \boldsymbol{e}_3^{\mathrm{T}}\dot{\boldsymbol{e}}_2 \\ \boldsymbol{e}_1^{\mathrm{T}}\dot{\boldsymbol{e}}_3 \\ \boldsymbol{e}_2^{\mathrm{T}}\dot{\boldsymbol{e}}_1 \end{bmatrix} \tag{13b}$$

Substitution of Eq. (3) into Eq. (13b) yields

$$\boldsymbol{e}_3^{\mathrm{T}}\dot{\boldsymbol{e}}_2 = \frac{1}{\alpha^2 l^2}[-\beta(\tilde{\boldsymbol{u}}\boldsymbol{r}_{ij})^{\mathrm{T}}, \beta(\tilde{\boldsymbol{u}}\boldsymbol{r}_{ij})^{\mathrm{T}}, -l(\tilde{\boldsymbol{u}}\boldsymbol{r}_{ij})^{\mathrm{T}}]\,\dot{q}_B$$

$$\boldsymbol{e}_1^{\mathrm{T}}\dot{\boldsymbol{e}}_3 = \frac{1}{\alpha^2 l^2}[-\alpha(\tilde{\boldsymbol{u}}\boldsymbol{r}_{ij})^{\mathrm{T}}, \alpha(\tilde{\boldsymbol{u}}\boldsymbol{r}_{ij})^{\mathrm{T}}, 0]\,\dot{q}_B$$

$$\boldsymbol{e}_2^{\mathrm{T}}\dot{\boldsymbol{e}}_1 = \frac{1}{\alpha^2 l^2}[-\alpha(l\boldsymbol{u}-\beta\boldsymbol{r}_{ij})^{\mathrm{T}}, \alpha(l\boldsymbol{u}-\beta\boldsymbol{r}_{ij})^{\mathrm{T}}, 0]\,\dot{q}_B \tag{14}$$

With the aid of Eq. (4) and Eq. (14) one can rewrite Eq. (13b) as

$${}^A\boldsymbol{\omega}^B = \varphi\dot{q}_B \tag{15}$$

where

$$\varphi = \frac{1}{\alpha^3 l^3}\begin{bmatrix} \varphi_{i,11} & \varphi_{i,12} & \varphi_{i,13} & \varphi_{j,11} & \varphi_{j,12} & \varphi_{j,13} & \varphi_{u,11} & \varphi_{u,12} & \varphi_{u,13} \\ \varphi_{i,21} & \varphi_{i,22} & \varphi_{i,23} & \varphi_{j,21} & \varphi_{j,22} & \varphi_{j,23} & \varphi_{u,21} & \varphi_{u,22} & \varphi_{u,23} \\ \varphi_{i,31} & \varphi_{i,32} & \varphi_{i,33} & \varphi_{j,31} & \varphi_{j,32} & \varphi_{j,33} & \varphi_{u,31} & \varphi_{u,32} & \varphi_{u,33} \end{bmatrix}$$

$\varphi_{i,11} = \alpha\beta x_{ij,1}(x_{u,3}x_{ij,2} - x_{u,2}x_{ij,3})$

$\varphi_{i,12} = \alpha\beta x_{ij,2}(x_{u,3}x_{ij,2} - x_{u,2}x_{ij,3}) + \alpha l[x_{ij,3}(x_{u,1}^2 + x_{u,2}^2) - x_{u,3}(x_{ij,1}x_{u,1} + x_{ij,2}x_{u,2})]$

$\varphi_{i,13} = \alpha\beta x_{ij,3}(x_{u,3}x_{ij,2} - x_{u,2}x_{ij,3}) + \alpha l[-x_{ij,2}(x_{u,1}^2 + x_{u,3}^2) + x_{u,2}(x_{ij,1}x_{u,1} + x_{ij,3}x_{u,3})]$

$\varphi_{j,11} = \alpha\beta x_{ij,1}(x_{u,2}x_{ij,3} - x_{u,3}x_{ij,2})$

$\varphi_{j,12} = \alpha\beta x_{ij,2}(-x_{u,3}x_{ij,2} + x_{u,2}x_{ij,3}) + \alpha l[-x_{ij,3}\cdot(x_{u,1}^2 + x_{u,2}^2) + x_{u,3}(x_{ij,1}x_{u,1} + x_{ij,2}x_{u,2})]$

$\varphi_{j,13} = \alpha\beta x_{ij,3}(-x_{u,3}x_{ij,2} + x_{u,2}x_{ij,3}) + \alpha l[x_{ij,2}(x_{u,1}^2 + x_{u,3}^2) - x_{u,2}(x_{ij,1}x_{u,1} + x_{ij,3}x_{u,3})]$

$\varphi_{u,11} = \alpha l x_{ij,1}(-x_{ij,3}x_{u,2} + x_{ij,2}x_{u,3})$

$\varphi_{u,12} = \alpha l x_{ij,1}(x_{ij,3}x_{u,1} - x_{ij,1}x_{u,3})$

$\varphi_{u,13} = \alpha l x_{ij,1}(-x_{ij,2}x_{u,1} + x_{ij,1}x_{u,2})$

$\varphi_{i,21} = \alpha\beta x_{ij,1}(x_{u,1}x_{ij,3} - x_{u,3}x_{ij,1}) + \alpha l[-x_{ij,3}\cdot(x_{u,1}^2 + x_{u,2}^2) + x_{u,3}(x_{ij,1}x_{u,1} + x_{ij,2}x_{u,2})]$

$\varphi_{i,22} = \alpha\beta x_{ij,2}(x_{u,1}x_{ij,3} - x_{u,3}x_{ij,1})$

$\varphi_{i,23} = \alpha\beta x_{ij,3}(x_{u,1}x_{ij,3} - x_{u,3}x_{ij,1}) +$

$al[x_{ij,1}(x_{u,2}^2 + x_{u,3}^2) - x_{u,1}(x_{ij,2}x_{u,2} + x_{ij,3}x_{u,3})]$

$\varphi_{j,21} = \alpha\beta x_{ij,1}(-x_{u,1}x_{ij,3} + x_{u,3}x_{ij,1}) + al[x_{ij,3} \cdot (x_{u,1}^2 + x_{u,2}^2) - x_{u,3}(x_{ij,1}x_{u,1} + x_{ij,2}x_{u,2})]$

$\varphi_{j,22} = \alpha\beta x_{ij,2}(-x_{u,1}x_{ij,3} + x_{u,3}x_{ij,1})$

$\varphi_{j,23} = \alpha\beta x_{ij,3}(-x_{u,1}x_{ij,3} + x_{u,3}x_{ij,1}) + al[-x_{ij,1} \cdot (x_{u,2}^2 + x_{u,3}^2) + x_{u,1}(x_{ij,2}x_{u,2} + x_{ij,3}x_{u,3})]$

$\varphi_{u,21} = alx_{ij,2}(-x_{ij,3}x_{u,2} + x_{ij,2}x_{u,3})$

$\varphi_{u,22} = alx_{ij,2}(x_{ij,3}x_{u,1} - x_{ij,1}x_{u,3})$

$\varphi_{u,23} = alx_{ij,2}(-x_{ij,2}x_{u,1} + x_{ij,1}x_{u,2})$

$\varphi_{i,31} = \alpha\beta x_{ij,1}(x_{u,2}x_{ij,1} - x_{u,1}x_{ij,2}) + al[x_{ij,2}(x_{u,1}^2 + x_{u,3}^2) - x_{u,2}(x_{ij,1}x_{u,1} + x_{ij,3}x_{u,3})]$

$\varphi_{i,32} = \alpha\beta x_{ij,2}(x_{u,2}x_{ij,1} - x_{u,1}x_{ij,2}) + al[-x_{ij,1} \cdot (x_{u,2}^2 + x_{u,3}^2) + x_{u,1}(x_{ij,2}x_{u,2} + x_{ij,3}x_{u,3})]$

$\varphi_{i,33} = \alpha\beta x_{ij,3}(-x_{u,1}x_{ij,2} + x_{u,2}x_{ij,1})$

$\varphi_{j,31} = \alpha\beta x_{ij,1}(x_{u,1}x_{ij,2} - x_{u,2}x_{ij,1}) + al[-x_{ij,2} \cdot (x_{u,1}^2 + x_{u,3}^2) + x_{u,2}(x_{ij,1}x_{u,1} + x_{ij,3}x_{u,3})]$

$\varphi_{j,32} = \alpha\beta x_{ij,2}(x_{u,1}x_{ij,2} - x_{u,2}x_{ij,1}) + al[x_{ij,1} \cdot (x_{u,2}^2 + x_{u,3}^2) - x_{u,1}(x_{ij,2}x_{u,2} + x_{ij,3}x_{u,3})]$

$\varphi_{j,33} = \alpha\beta x_{ij,3}(x_{u,1}x_{ij,2} - x_{u,2}x_{ij,1})$

$\varphi_{u,31} = alx_{ij,3}(-x_{ij,3}x_{u,2} + x_{ij,2}x_{u,3})$

$\varphi_{u,32} = alx_{ij,3}(x_{ij,3}x_{u,1} - x_{ij,1}x_{u,3})$

$\varphi_{u,33} = alx_{ij,3}(-x_{ij,2}x_{u,1} + x_{ij,1}x_{u,2})$

$x_{ij,r} = x_{j,r} - x_{i,r} \quad r = 1,2,3$

As indicated in Eq. (10), one can formulate the angular velocity in terms of partial angular velocities and generalized speeds as follows

$$\boldsymbol{\omega}_k = \omega_{klm} y_l \boldsymbol{n}_{om} \tag{16}$$

where one can obtain ω_{klm} from comparing Eq. (16) with Eq. (15) and Eq. (1) readily. One can also get angular acceleration of typical body B as

$$\boldsymbol{\alpha}_k \underset{(16)}{=} (\dot{\omega}_{klm} y_l + \omega_{klm} \dot{y}_l) \boldsymbol{n}_{om} \tag{17}$$

CONSTRAINT EQUATIONS AND JACOBI MATRICES

As described in (Bayo, 1991), the FCC method simplifies the formulation of element or rigid-body constraint equations and pair constraint equations. Moreover the constraint equations are linear or quadratic, very easy to evaluate, and they never contain transcendental functions as in other formulations. So constraint equations can be expressed as

$$\Phi_i(x_l, t) = 0, \; i = 1, N_4, l = 1, N_3 \tag{18}$$

where N_3 means total number of generalized coordinate and N_4 total constraint equation number. First and second time-differentiation of Eq. (18) produce

$$B_{il} y_l + g_i = 0 \tag{19}$$

$$B_{il} \dot{y}_l = - \dot{g}_i - \dot{B}_{il} y_l \tag{20}$$

where $B_{il} = \dfrac{\partial \Phi_i}{\partial x_l}$, $g_i = \dfrac{\partial \Phi_i}{\partial t}$, and B_{il} is named as Jacobi matrix of constraint equations.

DYNAMICS

Kane's equations may be used to obtain the governing dynamical equations based on the theory that the sum of the respective generalized forces is zero. That is

$$f_l + f_l^* = 0 \quad l = 1, \cdots, N_3 \tag{21}$$

where f_l is the generalized applied force associated with generalized speed y_l [see Eq. (1)], and f_l^* is the generalized inertia force for y_l. Put in matrix form Eq. (21) can be rewritten as

$$\begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1N_3} \\ \vdots & \vdots & \ddots \\ \alpha_{N_31} & \cdots & \alpha_{N_3N_3} \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \vdots \\ \dot{y}_{N_3} \end{bmatrix} = \begin{bmatrix} f_1 \\ \vdots \\ f_{N_3} \end{bmatrix} - \begin{bmatrix} h_1 \\ \vdots \\ h_{N_3} \end{bmatrix} \tag{22}$$

where:

$$\begin{cases} \alpha_{lp} = \sum_{k=1}^{N} (m_k v_{klm} v_{kpm} + I_{kmn} \omega_{klm} \omega_{kpn}) \\ f_l = \sum_{k=1}^{N} (v_{klm} F_{km} + \omega_{klm} T_{km}) \\ h_l = \sum_{k=1}^{N} (m_k v_{klm} \dot{v}_{kpm} y_p + I_{kmn} \omega_{klm} \dot{\omega}_{kpn} y_p + \\ \qquad e_{rsm} I_{ksn} \omega_{klm} \omega_{kpr} \omega_{kqn} y_p y_q), \end{cases}$$

m_k and I_k are the mass and central inertia dyadic of typical body B, and $e_{rsm} = (r - s)(s - m)(m - r)/2$. Then using the method described in Section 2, one can solve Eq. (1), (20), (22) for $x_{i,1}$, $x_{i,2}$, $x_{i,3}$ and y_l.

EXAMPLE

Fig. 2 shows a deployable structure widely used in spaceflight. When point k in body 4 moves on body 2, the mechanism is deployed or folded. Body 1's mass, m_1, is 139.32 g. Then $m_2 = 69.66$ g, $m_3 = 139.32$ g, $m_4 = 155.76$ g. Center inertia is I.

$$I_{11} = I_{21} = I_{31} = I_{41} = 653.06 \text{ gmm}^2.$$
$$I_{12} = I_{13} = I_{32} = I_{33} = 1161959.2 \text{ gmm}^2.$$
$$I_{22} = I_{23} = 1377895.8 \text{ gmm}^2.$$
$$I_{42} = I_{43} = 16224013.5 \text{ gmm}^2.$$

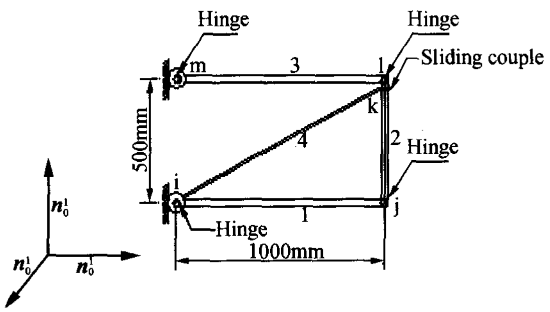

Fig.2 Deployable structure

The forces and moments on the mass center of every body are given as

$$F_1 = (0.0, 1000.0, 0.0)^T,$$
$$M_1 = (0.0, 500.0, 0.0)^T$$
$$F_2 = (0.0, -1000.0, 0.0)^T,$$
$$M_2 = (0.0, -500.0, 0.0)^T$$

At the beginning the structure is still. Fig. 3 shows the x-direction motion of the mass center of body 4.

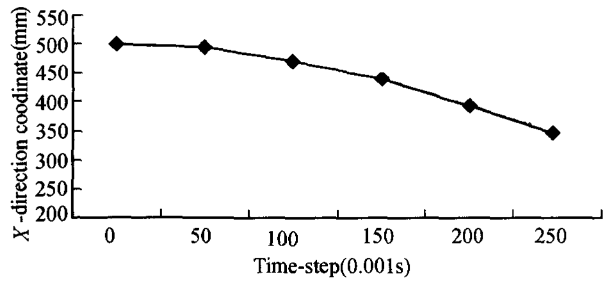

Fig.3 Mass center motion of body 4

CONCLUSIONS

1. It is very difficult to describe a multibody system with a good few closed loops by using Huston's form of Kane's equation.

2. The dynamic characters of a multibody system can be expressed by the relative points and unitary vectors based on the analytical expressions of the typical body mass center velocity and body angular velocity.

3. Kane's equations can be used to formulate the dynamics equations expediently.

4. The FCC method can be used to avoid equating the closed loop to an open one, and the structure's points and unitary vectors can be shared in order to reduce the number of the unknown variables.

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