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DETERMINING DRIVING RESISTANCE WITH REBOUND OF PILE-TOP DURING PILE DRIVING

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Abstract: This paper presents a method to determine driving resistance with rebound of pile-top during pile driving. The soil around the pile shaft is assumed to be rigid-plastic, while that under the pile-tip is assumed to be ideally elastoplastic. The driving force acting on the pile-top is simplified to a triangular impact force. The kinematic equation of the pile-tip is established. From the one-dimensional wave equation, the movements of the pile-tip and pile-top are determined. The rebound at the pile-top can be written in a very concise form. It is shown that the shaft resistance makes the rebound at the pile-top decrease. In particular, when the pile is very long or the soil around the pile is very stiff, the decrease is very obvious.

Key words: pile, pile driving, soil resistance, Smith model, rebound
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INTRODUCTION

It had been recognized that during pile driving the rebound of a pile-top gives more information about resistance than the penetration (Chen et al., 1996; Chen et al., 1997; Uto et al., 1992). The development of the theoretical model of the point resistance obtained from PD- and PDA- measurements (Chen et al., 1996, Weele et al., 1994) makes it possible to estimate the static point resistance during pile driving. In the method, the shaft resistance was neglected as the transverse vibration of a pile greatly reduces the shaft resistance. The point resistance was assumed to be the main driving resistance. The soil under the pile-tip was supposed to be an ideal elastoplastic material, with ultimate static resistance of R_s. In order to make the analysis simpler, the impact force caused by the hammer is simplified to a triangular force. The driving model is shown in Fig. 1(a). The kinematic equation of the pile-tip was easily established and solved. Then the point resistance can be written as (Chen et al., 1996)

$$R_s = C_s \frac{R_e Z}{t_0} \qquad (1)$$

where C_s is a dimensionless constant, approximately equals to 1.3; t_0 is the duration time of the contact between the hammer and the pile-top; Z is the impedance of the pile. By integrating twice the recorded acceleration of the pile-top with respect to time, the pile penetration can be obtained. As it may take approximately 200 ms before the pile-top is at rest after each blow and permanent penetration is reached, double integration of acceleration will not yield a perfect solution. Fortunately such problem had been solved by IFCO (Weele et al., 1994; Chen et al., 1996; Chen et al., 1997).

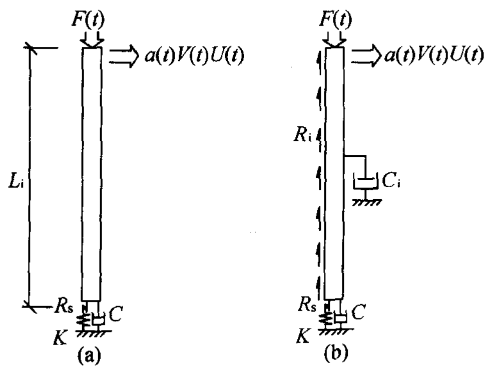

Fig.1 Pile driving models
(a) pile driving model proposed by Chen et al. (1996, 1997) without shaft resistance;
(b) the present model with shaft resistance

In Eq. (1), the impedance of the pile is known before pile driving, and t_0 for each blow can be obtained from the pile-top velocity. The rebound can be obtained from the pile-top movement. The total penetration and the permanent penetration for each blow are as shown in Fig.2. Fig.3 shows the typical point resistance versus depth, which clearly indicates the thickness of each soil-layer and agrees well with the cone resistance of CPT (Cone Penetration Test). The method requires only a single, small and robust sensor and the data collection and interpretation are done automatically by the field computer (Weele et al., 1994). PDA-analyses in combination with the CASE- or CAPWAP-method are difficult, as they require long time for analysis and also more complicated instrumentation. It also shows that the dropheight, cushion stiffness and soil damping have little effect on the point resistance compared with other methods, such as CASE- and CAPWAP- method. The method seemed very effective when put into use (Weele et al., 1994). A shortcoming was that the shaft resistance was not taken into account. In practice, the soil resistance generally includes the shaft and point resistances during pile driving. If the pile penetrates into stiff soil or is very long, the shaft resistance may be significant (Chen and Chen, 2000). In this paper, the shaft resistance is taken into account to improve the method proposed by Weele et al. (1994) and Chen et al. (1996, 1997).

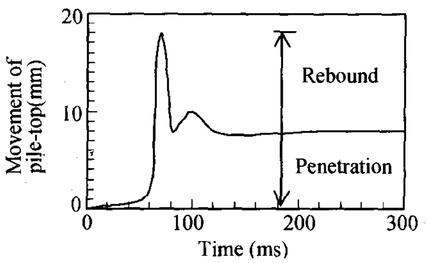

Fig.2 Movement of pile-top

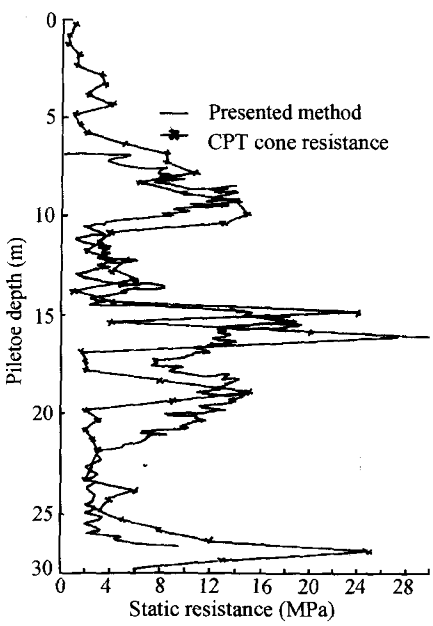

Fig.3 Point resistance vs. depth (Chen et al., 1996)

THEORETIC ANALYSIS

1. Theoretical model of pile driving

As shown in Figs. 1(b) and 4, the soil around the pile shaft is postulated to be rigid-plastic, and that at the pile-tip is postulated to be elastoplastic using the Smith soil model (Smith, 1960). The soil around the pile shaft is homogenous, and the shaft resistance distributes evenly around the pile. The driving model is shown in Figs. 1 and 4, where R_i is the static shaft resistance, R_s the static point resistance, and C_i the damping coefficient at the pile shaft. The driving force is simplified to a triangular impact force, as shown in Fig.5, where F_0 is the amplitude of the driving force, and t_1 the loading time. Denote the length of the pile as L and the stress wave velocity as c. We have $c = \sqrt{E/\rho}$, where E is the elastic modulus and ρ the density of the pile material. The reflection wave at the pile-top is not considered. Up-going velocity is supposed to reach pile-top without attenuation.

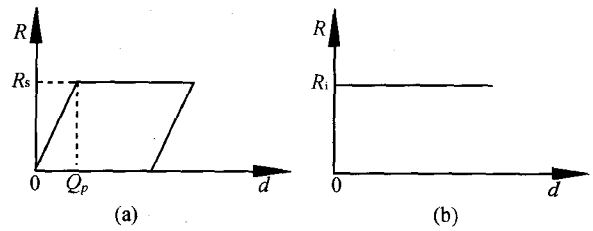

Fig.4 Models of static shaft and point resistances

(a) model of static shaft resistance, where d is the displacement between pile and soil;

(b) model of static point resistance, where d is the movement of the pile-toe

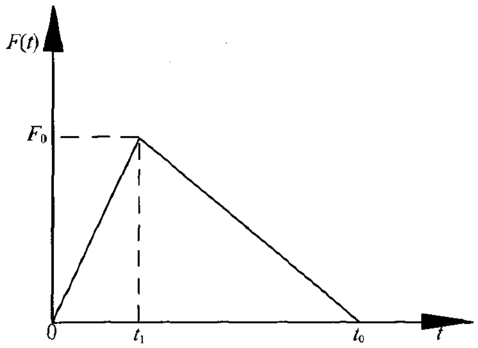

Fig.5 Driving force, simplified to a triangular force

2. Kinematic equation of pile-tip

The kinematic equation of the pile-tip can be described as

$$CV(t) + K[U(t) - U_p(t)] = F_d(t) + F_u(t) \quad (2)$$

where $U(t)$ and $V(t)$ are the movement and the velocity of the pile-tip respectively, $U_p(t)$ the permanent movement of the pile-tip, $F_d(t)$ down-going force, Z the impedance of the pile, C the damping coefficient of soil at the pile-tip, and K the stiffness of soil at the pile-tip. In particular, $F_u(t)$ is the up-going force, which can be described as

$$F_u(t) = F_d(t) - ZV(t) \quad (3)$$

Then Eq. (2) is rewritten as

$$[C + Z]V(t) + K[U(t) - U_p(t)] = 2F_d(t) \quad (4)$$

or

$$[C + Z]\frac{dU(t)}{dt} + K[U(t) - U_p(t)] = 2F_d(t) \quad (5)$$

The movement of the pile-tip (or the soil at the pile-tip) $U(t)$ can be divided into three stages: elastic movement, plastic movement and rebound. When the movement of the pile-tip $U(t)$ is less than the maximum elastic movement of the soil at the pile-tip (denoted as Q_p), there is only elastic movement of the soil. The static soil resistance increases linearly with the elastic movement of the pile-tip until it reaches the maximum static resistance R_s. In this case, Eq. (5) is simplified to

$$[C + Z]\frac{dU(t)}{dt} + KU(t) = 2F_d(t) \quad (6)$$

When the movement of the pile-tip $U(t)$ exceeds Q_p, plastic movement takes place. Then the maximum static soil resistance remains unaltered, and Eq. (5) becomes

$$[C + Z]\frac{dU(t)}{dt} + R_s = 2F_d(t) \quad (7)$$

When the movement of the pile-tip $U(t)$ reaches its limit, then rebound takes place. The static soil resistance decreases when the movement of the pile-tip $U(t)$ decreases. In this case, the movement of the pile-tip is described by Eq. (5). Eq. (5) can be solved step by step, to obtain the condition of the deformation compatibility at the end of each stage.

3. Velocity and movement at pile-tip

Seven dimensionless parameters are introduced as follows:

$$\eta = t_1/t_0, n = R_s/F_0, n_i = R_i/F_0,$$
$$m_1 = Z/(Kt_0), m_2 = C/(Kt_0),$$
$$m_{2i} = C_i/(Kt_0), \ T = 2L/(ct_0).$$

With the assumption of the Smith damping law, we have

$$m_1 = Q_p/(nV^t_{max}t_0) \quad (8)$$

and

$$m_2 = J_p Q_p/t_0 \quad (9)$$

where J_p is the Smith damping coefficient. V^t_{max} is the maximum velocity of the pile-top:

$$V^t_{max} = F_0/Z.$$

When the pile hammer impacts on the pile-top, the impact force propagates downward along the pile. When the stress wave reaches a point x, as shown in Fig.6, the shaft resistance is excited . Then the tensile and compressive stress

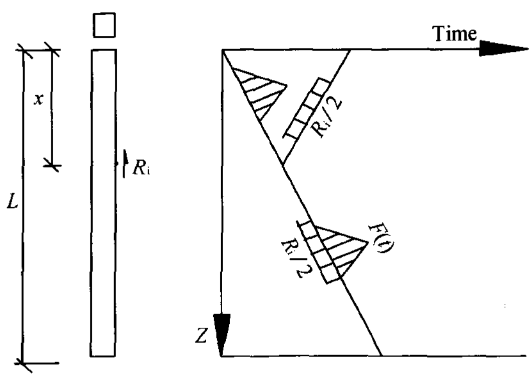

Fig.6 Stress wave propagation in pile

wave propagating in opposite directions are generated. The tensile wave propagates downwards and the compressive wave propagates upwards. The amplitudes of the two kinds of stress waves equal to half of the shaft resistance amplitude. When the driving force reaches the pile-tip, the amplitude of the shaft resistance equals to half of the total shaft resistance, i.e. $R_i/2$. The dynamic shaft resistance is the product of velocity due to the driving force and the damping coefficient at the pile shaft. Therefore the down-going wave caused by soil resistance at the pile shaft is

$$V_d^R(t) = \begin{cases} -\dfrac{F_0 C_i t}{2Z^2 t_1} - \dfrac{R_i}{2Z} & 0 \leqslant t < t_1 \\ -\dfrac{F_0 C_i (t - t_0)}{2Z^2 (t_1 + t_0)} - \dfrac{R_i}{2Z} & t_1 \leqslant t < t_0 \\ -\dfrac{R_i}{2Z} & t_0 \leqslant t \end{cases} \quad (10)$$

The down-going force $F_d(t)$ can be described as

$$F_d(t) = F(t) + ZV_d^R(t) \quad (11)$$

Furthermore, the up-going wave reaching the pile-top is expressed as

$$V_u^R(t) = \begin{cases} -\dfrac{F_0 C_i t}{2Z^2 t_1} - \dfrac{R_i t}{2ZT} & 0 \leqslant t < t_1 \\ -\dfrac{F_0 C_i (t - t_0)}{2Z^2 (t_1 + t_0)} - \dfrac{R_i t}{2ZT} & t_1 \leqslant t < t_0 \\ -\dfrac{R_i}{2Z} t_0 \leqslant t \end{cases} \quad (12)$$

Substituting Eq. (10) into Eqs. (11), (6), (7) and (5) step by step, the dimensionless form of the movement and the velocity at the pile-tip are derived as

$$\frac{K}{F_0} U(\tau) =$$

$$\begin{cases} \dfrac{2}{\eta}\tau - n_i - \dfrac{2m}{\eta} - \dfrac{m_{2i}}{m_1 \eta}(\tau - m) + \left(n_i + \dfrac{2m}{\eta} - \dfrac{m m_{2i}}{m_1 \eta}\right) e^{-\frac{\tau}{m}} & 0 \leqslant \tau < \eta \\ \dfrac{2}{(\eta - 1)}(\tau - 1) - n_i - \dfrac{2m}{(\eta - 1)} - \dfrac{m_{2i}}{m_1(\eta - 1)}(\tau - m - 1) + \left[\left(n_i + \dfrac{2m}{\eta} - \dfrac{m m_{2i}}{m_1 \eta}\right) e^{-\frac{\eta}{m}} + \dfrac{m}{\eta(\eta - 1)}\left(2 - \dfrac{m_{2i}}{m_1}\right)\right] e^{-\frac{\tau - \eta}{m}} & \eta \leqslant \tau < \tau_n \\ \dfrac{(\tau - 1)^2}{m(\eta - 1)} - \dfrac{(n + n_i)}{m}\tau - \dfrac{m_{2i}(\tau - 1)^2}{2mm_1(\eta - 1)} + w_1 & \tau_n \leqslant \tau < \tau_e \\ \dfrac{2(\tau - 1)}{(\eta - 1)} - n_i + \dfrac{Kg}{F_0} - \dfrac{2m}{(\eta - 1)} - \dfrac{m_{2i}}{m_1(\eta - 1)}(\tau - m - 1) + w_2 e^{-\frac{\tau}{m}} & \tau_e \leqslant \tau \leqslant 1 \end{cases} \quad (13)$$

and

$$\frac{Kt_0}{F_0} V(\tau) =$$

$$\begin{cases} \dfrac{2}{\eta} - \dfrac{m_{2i}}{m_1 \eta} - \left(\dfrac{n_i}{m} + \dfrac{2}{\eta} - \dfrac{m_{2i}}{m_1 \eta}\right) \cdot e^{-\frac{\tau}{m}} & 0 \leqslant \tau < \eta \\ \dfrac{2}{(\eta - 1)} - \dfrac{m_{2i}}{m_1(\eta - 1)} - \left[\left(\dfrac{n_i}{m} + \dfrac{2}{\eta} - \dfrac{m_{2i}}{m_1 \eta}\right) \cdot e^{-\frac{\eta}{m}} + \dfrac{(2m_1 - m_{2i})}{m_1 \eta(\eta - 1)}\right] e^{-\frac{\tau - \eta}{m}} & \eta \leqslant \tau < \tau_n \\ \dfrac{2(\tau - 1)}{m(\eta - 1)} - \dfrac{(n + n_i)}{m} - \dfrac{m_{2i}(\tau - 1)}{mm_1(\eta - 1)} & \tau_n \leqslant \tau < \tau_e \\ \dfrac{2}{(\eta - 1)} - \dfrac{m_{2i}}{m_1(\eta - 1)} - \dfrac{w_2}{m} e^{-\frac{\tau}{m}} & \tau_e \leqslant \tau \leqslant 1 \end{cases} \quad (14)$$

where $m = m_1 + m_2$, $\tau = t/t_0$, τ_n and τ_e are the dimensionless time when the plastic movement and the rebound take place, w_1 and w_2 the integration constants, g residual movement when rebound takes place. From Eqs. (13) and (14), τ_n and τ_e can be written as

$$\tau_n = \frac{nm_1 m^2}{m_1 n_i \eta + 2mm_1 - mm_1 n_i - mm_{2i}} \quad (15)$$

and

$$\tau_e = \frac{(n + n_i)(\eta - 1)}{2 - m_{2i}} \quad (16)$$

From the down-going wave velocity and the velocity at the pile-top, the up-going wave velocity can be derived as

$$V_u(t) = V(t) - V_d(t) \quad (17)$$

With the assumption, the up-going wave velocity will reach the pile-top without attenuation. Fig. 7 shows the typical solution of Eq. (2).

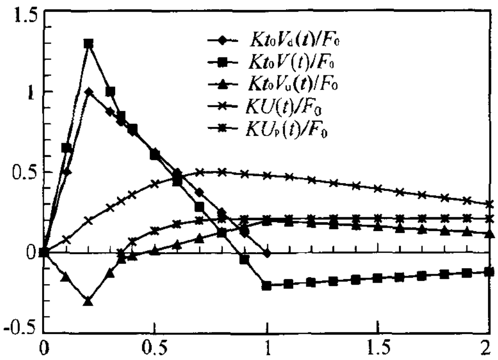

Fig.7 Typical solution of Eq. (2)

4. Rebound of pile-top

If there is no residual compression in the pile, the final residual movement of the pile-tip is equal to that of the pile-top. The rebound of the pile-top can be expressed as

$$R_e = U^t_{max} - U^b_{max} + Q_p \qquad (18)$$

where R_e is the rebound of the pile-top, U^t_{max} and U^t_{max} the maximum movements of the pile-top and pile-tip respectively. U^t_{max} and U^b_{max} can be derived by integrating the velocities at the pile-tip and pile-top as shown in Fig. 8. Because the

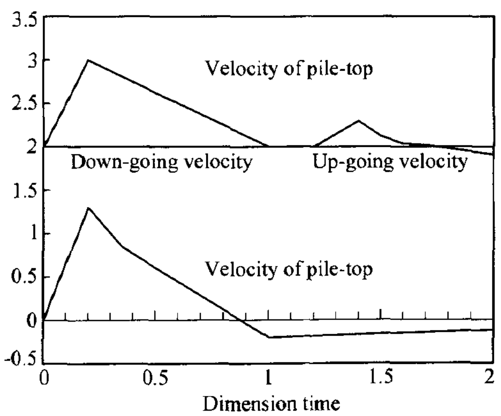

Fig.8 Velocity and movement of pile-top and pile-tip

velocity caused by the up-going soil resistance, $V^R_u(t)$, is small compared with $V^t(t)$ and $V_u(t)$, it is assumed that the movement of the pile-top reaches its maximum value when the up-going velocity $V_u(t)$ reaches zero.

Denote β as the dimensionless time when the up-going velocity reaches zero. As shown in Fig. 7 there exists

$$\frac{K}{F_0}[U^t_{max} - U^b_{max}] = \int_0^1 V^t(\tau)d\tau + \int_0^\beta V_u(\tau)d\tau - \int_0^{\tau+\beta} V^R_u(\tau)d\tau - \int_0^\beta V_u(\tau)d\tau = \frac{(1-\beta)(n+n_i)}{2(2-m_{2i})} - \left[\frac{(T^2-\beta^2)}{4T} + \frac{\beta}{2} + \frac{m_{2i}(1-\beta^2)}{2m_1 n_i(1-\eta)}\right]\frac{n_i}{m_1} = \alpha_1\frac{n}{m_1} - \alpha_2\frac{n_i}{m_1} \qquad (19)$$

where

$$\alpha_1 = \frac{(1-\beta)}{2(2-m_{2i})} \qquad (20)$$

and

$$\alpha_2 = -\alpha_1 + \frac{(T^2-\beta^2)}{4T} + \frac{\beta}{2} + \frac{m_{2i}(1-\beta)^2}{2m_1 n_i(1-\eta)} \qquad (21)$$

Eq. (19) can be written as

$$U^t_{max} - U^b_{max} = \alpha_1\frac{R_s t_0}{Z} - \alpha_1\frac{R_i t_0}{Z} \qquad (22)$$

Substituting Eq. (22) into Eq. (18), the static resistance at the pile-tip can be obtained

$$R_s = C_s\frac{R_e Z}{t_0} + C_d R_i \qquad (23)$$

where

$$C_s = (\alpha_1 + m_1)^{-1} \qquad (24)$$

and

$$C_d = \frac{\alpha_2}{(\alpha_1 + m_1)} \qquad (25)$$

Denoting $\lambda = R_i/R_s$, Eq. (23) can be written as

$$R_s = \frac{C_s}{(1-\lambda C_d)}\frac{R_e Z}{t_0} \qquad (26)$$

or

$$R_e = \frac{R_s t_0}{Z}\frac{1-\lambda C_d}{C_s} \qquad (27)$$

EFFECT OF SHAFT RESISTANCE ON REBOUND

The effect of λ on the point resistance at the pile-tip and the rebound of the pile-top is shown in Fig.9. When $\lambda = 0$, it means that the shaft resistance is neglected. When $\lambda = 1$, the shaft resistance is the same as the point resistance. It can be seen from Fig.9 that the shaft resistance has a very obvious effect on the rebound. Particularly if the error due to the neglect of the shaft resistance is restricted within ten percent, λ must be less than 0.16, which means the shaft resistance is less than thirteen percent of the point resistance. The soil resistance at the pile-tip obtained with consideration of the soil resistance at the pile shaft is bigger than that obtained without such a consideration.

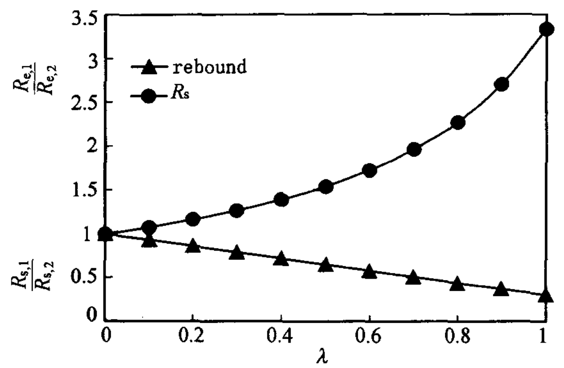

Fig.9 Effect of shaft resistance on rebound and the point resistance, where subscripts 1 and 2 of R_s, R_e mean results estimated by the present method and Chen et al. (1996) respectively

Eq. (23) shows the influence of the shaft resistance on the rebound. The shaft resistance distinctly reduces the pile-top rebound. The greater the shaft resistance is, the more obvious the decrease is. When $\lambda = 0.7$, the rebound is only half of that when the shaft resistance is neglected. In-situ measurement showed that when the driving of the pile is stopped for certain reasons, the rebound will decrease greatly during redriving. It is due to the increase of the shaft resistance during the break.

STATIC POINT AND SHAFT RESISTANCE

If $\lambda = 0$, Eq. (23) can be rewritten as

$$R_s = C_s \frac{R_e Z}{t_0} \tag{28}$$

which is the same as that obtained by Chen et al. (1996, 1997). Commonly, during pile driving we have $t_0 = (10 - 15)$ ms, $Q_p = (5 - 10)$ mm, $J_p = (0.5 - 1.5)$ s/m, and $V^t_{max} = (2.5 - 4.0)$ m/s. Therefore the ranges of the dimensionless parameters can be determined: $n = (0.1 - 0.5)$, $n_i = (0.03 - 0.02)$, $\eta = (0.1 - 0.5)$, $m_1 = (0.08 - 0.25)/n$, $m_2 = (0.15 - 1.5)$, $m_{2i} = (0.05 - 0.5)$, $T = (1.0 - 2.0)$. Substitution of the dimensionless parameters into Eq. (25) can reveal the range of C_d: $C_d = (0.5 \sim 0.9) \cong 0.7$. C_s can be estimated from Eq. (24). In-situ measurement also showed that C_s varied slightly, and was approximately 1.3 (Chen et al., 1996, 1997). In Eq. (26), the rebound of the pile-top and the duration time of the driving force t_0 can be obtained from the velocity of the pile-top, (Chen et al., 1996). Eq. (26) is rewritten as

$$R_s = \frac{1.3}{(1 - 0.7\lambda)} \frac{R_e Z}{t_0} \tag{29}$$

The curve of the point resistance versus the depth is very similar to that of the cone resistance of CPT, see Fig.3. The curve can be used to conveniently determine the depth of the bearing stratum as well as the pile capacity.

The shaft resistance is determined by the following expression

$$R_i = \frac{\lambda C_s}{(1 - \lambda C_d)} \frac{R_e Z}{t_0} = \frac{1.3\lambda}{(1 - 0.7\lambda)} \frac{R_e Z}{t_0} \tag{30}$$

In Eqs. (28) and (29), the parameter λ is essential for determining the point resistance and shaft resistance. As λ is the ratio of the static shaft resistance to the static point resistance, it is best to estimate λ with the cone and shaft resistance of CPT with different depth of the soil, which will introduce a little error.

Fig.10 is the result of estimation presented by Chen et al. (1997), for which no shaft resistance was considered. With the consideration of shaft resistance, the estimated point resistance is much larger than that of Chen et al. (1997). The shaft resistance can also be estimated, as shown in Fig.11. When the pile does not penetrate very deeply, the shaft resistance is not very large, and the point resistances in Fig.10 and

Fig.11 are almost the same. But when the pile penetrates very deeply into the soil, the shaft resistance cannot be neglected. The point resistance is estimated with the method presented by Chen et al (1997), as shown in Figs.10 and 11 indicating high estimated shaft resistance.

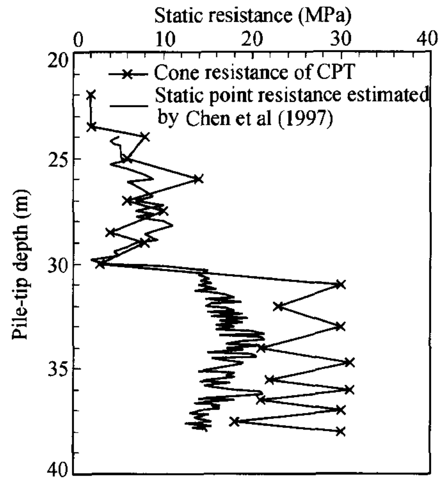

Fig.10 Static point resistance estimated by Chen et al.(1997)

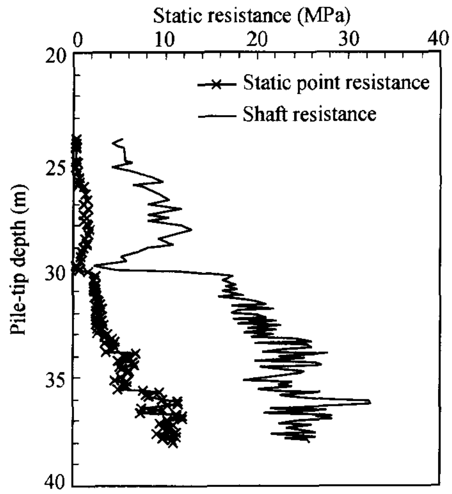

Fig.11 Static shaft resistance and point resistance estimated by the present method

CONCLUSIONS

This paper presents a method to determine driving resistance with the rebound of pile-top during pile driving. The effect of the shaft resistance on the pile-top rebound during pile driving is discussed. The soil around the pile shaft is assumed to be viscous-rigid and that under the pile-tip is viscous-elastic. The driving force acting on the pile - top is simplified to a triangular impact force. The kinematic equation of the pile-tip is established. The movements of the pile-tip and pile-top are obtained from the one-dimensional wave equation. The rebound at the pile-top can be written in a very concise form. It is shown that the shaft resistance reduces the rebound at the pile-top. When the pile is very long or the soil around the pile is very stiff, the rebound decreases significantly. The neglect of the shaft resistance may lead to a large error, and the point resistance is underestimated with the method presented by Chen et al. (1996, 1997). The present method can be used to estimate the point resistance and the shaft resistance.

The ratio of shaft resistance to point resistance is employed to estimate the composition of the soil resistance. The precision of estimation depends mainly on the precision of the ratio. In this paper the ratio is estimated from the results of CPT, i.e. the ratio of the shaft resistance to the point resistance of CPT. At the same site of construction, if PDA can be carried out to distinguish the shaft resistance and the point resistance at different depths, the precision of estimation will be greatly improved.

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