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Application of chaotic theory to parameter estimation*

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Abstract: High precision parameter estimation is very important for control system design and compensation. This paper utilizes the properties of chaotic system for parameter estimation. Theoretical analysis and experimental results indicated that this method has extremely high sensitivity and resolving power. The most important contribution of this paper is apart from the traditional engineering viewpoint and actualizing parameter estimation just based on unstable chaotic systems.

Key words: control system, chaos, parameter estimation, nonlinearity, existing engineering method
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INTRODUCTION

There are many reports (Ott et al., 1990; Chen, 1993) on chaotic phenomena, but none on the application of chaotic theory to the practice of control. The most important characteristics of a chaotic system are the initial value sensitivity and the parameter sensitivity that determine the instability and inscrutability of the chaotic system. One tiny disturbance may trigger a tremendous change in the state trajectory of the chaotic system. Pacora, the inventor of chaotic synchronization, put forward this key problem in 1993 (Pecora, 1993). But it has not been solved so far, there is a long way to go in applying chaotic theory to practice. It is not an unconquerable obstacle, measuring with chaos (Tong et al., 1999; Tong et al., 2000; Chen et al., 2000) is a good illustration.

The existing methods of parameter estimation, including nonlinear reconstruction, are all based on the principle of least square; and must measure abundant data first, then can do estimation, but the error inevitable in the measuring process will influence the resolving power of estimation.

In this study, the existing engineering methods for estimating parameters are discarded in favor of study and analysis of the parameters in an unstable system, which is a new method for parameter estimation based on chaotic theory. The control system becomes chaotic by addition of a nonlinear unit, and then the parameter space is partitioned based on the distance definition of symbol space. The search area is shrunk step by step to get the estimating parameter finally. This method does not need any measuring instruments, simplifies the estimating process, and has extremely high sensitivity and resolving power. So it has a good future as an estimating method.

REALIZATION OF CONTROL SYSTEM CHAOS

Any control system, without regard to the order of the system, can reach the chaotic state by addition of a nonlinear unit (dashed part of Fig.1).

The simplest first order system is used as an example (Fig.1), where $W(s) = \frac{K}{Ts + 1}$.

We add a nonlinear unit to the standard first order system. (dashed part of Fig. 1). The switch P will connect with the negative input R_2 when a new pulse of pulse series δ arrives. The procedure is as follows:

1. After the switch P connects with the initial value R_3, $C(t) = C_{(0)} > R_0$ is at equilibrium.

2. When the pulse series δ with the period

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h (see Fig. 1) arrives, the switch P connects with R_2 (<0)(which is equivalent to $D(t)$ producing a negative step change), and makes $C(t)$ begin to descend (Fig. 2, the segment $C_{(0)} - A$). Note: If the value of $C(t)$ does not descend to R_0(Fig.2, the point B), the switch P will R_2 connect with even if a new pulse comes.

3. When $C(t) = R_0$ (Fig. 2, point A), $E(t)$ changes from a value less than zero to one greater than zero, and makes $N(t)$ change, from -1 to $+1$. Then switch P connects with R_1 (>0), $D(t)$ produces a positive step change $C(t)$, making begin to ascend(Fig.2, segment $A - C_{(1)}$).

4. When the next pulse series δ arrives (Fig. 2, point $C_{(1)}$), the second period of $C(t)$ will begin. Fig.2 shows the behavior of the system output $C(t)$.

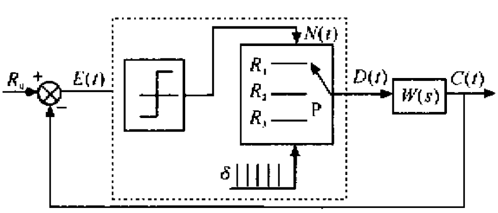

Fig.1 The components of a chaotic system

Now we quantitatively analyze the whole process.

Assuming $R_0 = 0$ for the sake of convenience.

When $D(t)$ is a positive step change, assuming $R_1 = \eta R$ ($\eta > 0$, R is a positive constant). The initial value of $C(t_0) = R_0 = 0$, then we get

$$C(t) = \eta KR(1 - e^{\frac{-(t-t_0)}{T}}). \tag{1}$$

When $D(t)$ is a negative step change, and the initial value is assumed to be

$$C(t_0) = C_{(n)},\quad R_2 = R,$$

We have

$$C(t) = -KR + (C_{(n)} + KR)e^{\frac{-(t-t_0)}{T}} \tag{2}$$

Assuming that when $t = t_n$, $C(t) = R_0 = 0$, the solution of the Eq.(2) is found to be

$$t_n = T\ln(\widetilde{C}_{(n)} + 1) + t_0 \tag{3}$$

Where

$$\widetilde{C}(n) = \frac{C(n)}{KR} \tag{4}$$

Assuming the ascend-descend period of $C(t)$ can be finished in 1 – 2 pulse period h, according to Eqs.(1) and (3), we can get the following relation between $\widetilde{C}_{(n)}$ and $\widetilde{C}_{(n+1)}$.

$$\widetilde{C}_{(n+1)} = \begin{cases} \eta(1 - e^{-\frac{h-t_n}{T}}) = \\ \eta[1 - e^{\frac{-h}{T}} - e^{\frac{-h}{T}}\widetilde{C}_{(n)}] & \widetilde{C}_{(n)} < \widetilde{C}_c \\ \eta(1 - e^{-\frac{2h-t_n}{T}}) = \\ \eta[1 - e^{\frac{-2h}{T}} - e^{\frac{-2h}{T}}\widetilde{C}_{(n)}] & \widetilde{C}_{(n)} < \widetilde{C}_c \end{cases} \tag{5}$$

Where C_c is as shown in Fig.2.

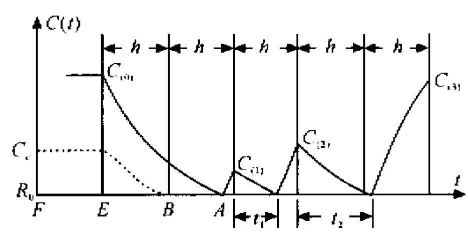

Fig.2 Chaotic orbit diagram

$$\widetilde{C}_c = (e^{\frac{h}{T}} - 1) \tag{6}$$

$$C_c = KR\widetilde{C}_c \tag{7}$$

The iteration relation of Eq.(5) is shown in Fig.3, where $C_a = \eta KR(1 - e^{\frac{-h}{T}})$, $C_b = C_a[1 + (1 - \eta e^{\frac{-h}{T}})e^{\frac{-h}{T}}]$. It is an inverse saw-tooth mapping, which is a typical chaotic system (Hao, 1989).

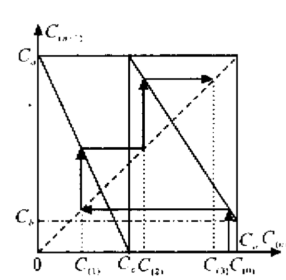

Fig.3 Chaotic mapping

If the control system is not a first-order system but is the second-order or higher order system, or a nonlinear system, we can choose the proper period h of the pulse δ to make the ascend curve and the descend curve change monotonically. The analyzing process is similar, except that there is a minor change for the Eq. (5).

PARAMETER SPACE, TRAJECTORY SPACE AND SYMBOLIC SPACE

Assuming an initial value $C_{(0)}$, according to Eq. (5) we can get one trajectory:

$$C_{(0)}, C_{(1)}, C_{(2)}, C_{(3)}, \cdots = \{C_{(i)}\}$$
$$i = 0, 1, 2, \cdots$$

According to symbolic dynamics (Hao, 1989), we can get a symbolic sequence:

$$S_0, S_1, S_2, \cdots = \{S_i\} \quad S_i = 0 \text{ or } 1$$

Where

$$S_i = \begin{cases} 1, & C_{(i)} > C_c \\ 0, & C_{(i)} > C_c \end{cases} \tag{8}$$

All parameters changed under permitted conditions form a parameter space Γ, the initial value of which is kept constant. Every $T \in \Gamma$, corresponds to one trajectory b_i. Different parameter corresponds to different trajectory. The set of all trajectory b_i forms the trajectory space β.

Each trajectory $\{C_{(i)}$ can be represented by a symbol $\{S_i$ according to Eq. (8); then from trajectory space β we have symbolic space ζ. In space ζ there are two symbolic sequences:

$$\{a_i\} = a_0, a_1, \cdots \text{ and } \{b_i\} = b_0, b_1, \cdots$$

The definition of distance between them is defined as follows:

$$d(a_0, b_0) = \left| \sum_{i=0}^{\infty} \frac{a_i - b_i}{2^i} (-1)^{-i+1} \right| \tag{9}$$

Obviously, this definition satisfied those three conditions (Tong et al., 1999). The proof is omitted.

SINGLE PARAMETER SYSTEM ESTIMATING

We adopt the system shown in Fig. 1 as an example. Here, the parameter T needs to be estimated. According to Eq. (9), we know that the distance $d = 0$ means the superposition of the two trajectories. If we get the trajectory $\{a_i\}$, get $\{b_i\}$ from the assumed parameter T by computer simulation, adjust T step by step until $d(a_0, b_0)$ approach zero infinitely, this value of T is the estimation we require.

Steps are as follows:

1. We have the peak value sequence $C_{(0)}$, $C_{(1)}, \cdots, C_{(i)}, \cdots$ by experiment on the system shown in Fig. 1; then get the symbolic sequence according to Eq. (2). In fact, we can obtain $\{a_i\}$ directly from the position of switch P in nonlinear tache and pulse δ, without calculating $C_{(i)}$.

2. During the process of computer simulation, we choose parameter T arbitrarily, obtain the symbolic sequence $\{b_i\}$ according to Eqs. (5) and (8).

3. $\{a_i\}$ is compared with $\{b_i\}$ bit by bit. a_0 and b_0 are first compared. There are two states for different values of T: $a_0 = b_0$ or $a_0 \neq b_0$, according to which parameter space Γ is divided into two parts: Γ_1 and Γ_2 ($\Gamma_1 \cup \Gamma_2$) $= \Gamma$. We use H_1 to symbolize the subspace in which $a_0 = b_0$, that is to say we assume $H_1 = \Gamma_1$. Then continue the same process, when a_1 and b_1 are compared, there are also two states: $a_1 = b_1$ or $a_1 \neq b_1$. Parameter space H_1 is divided into two: H_{11} and H_{11}($H_{11} \cup H_{12} = H_1$). We assume $H_2 = H_{11}$, in which and $a_0 = b_0$, and $a_1 = b_1$ repeat this process until $\{b_i\} = \{a_i\}$, when $H_1 \supset H_2 \supset H_3 \supset H_4 \cdots \supset H_i$. In theory, under the condition that the digit of $\{b_i\}$ and $\{a_i\}$ is long enough, H_i can converge into one arbitrarily small region. The parameter in H_i is the estimation we need.

ESTIMATION OF MULTI-PARAMETER SYSTEM WHICH INCLUDES A DELAY TACHE

The system $W(s)$ in Fig. 1 is a multi-parameter system with a delay tache. $W(s)$ is described as follows:

$$\dot{C}(t) = \alpha C^2(t) + \beta C(t) + \gamma = KD(t - \tau) \tag{10}$$

Where

τ is time delay. K, α, β, γ are system parameters, unchanged with respect to time.

The estimation of system with delay tache is different in some degree to that of ordinary system. The lag of switch P caused by the delay tache leads to the difference. Let us see Fig.4. We divide trajectory space and symbolic space into four cases correspondingly, where the case (A) is denoted as 0, case (B) is denoted as symbol 2, the case (C) is denoted as symbol 1, the case (D) is denoted as symbol 3. That is to say, the estimation of this system with a delay

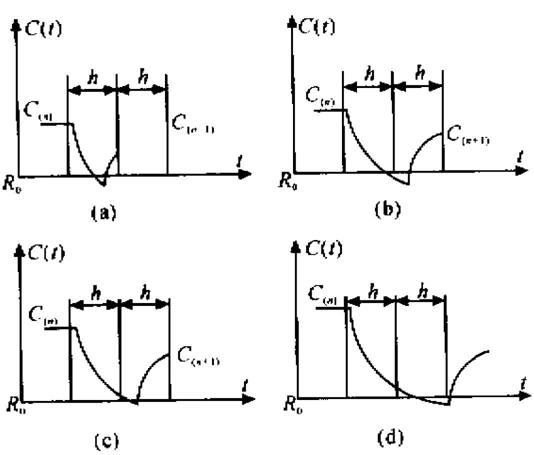

Fig.4 The four cases corresponding to the first order system with a delay tache
(a) symbol 0; (b) symbol 2;
(d) symbol 1; (d) symbol 3

tache corresponding to four symbolic sequences. The definition of the distance in Eq.(9) remains the same, except that the values range of a_i and b_i change accordingly.

RESULTS OF COMPUTER SIMULATION

We use the first-order system with a delay tache as an example to simulate. For convenience in discussion, we fix the value of γ and τ, choose α, β as the parameters to be estimated.

We assume $h = 1.5s$, Time delay $\tau = 0.3s$, $\gamma = 0.5$. We first choose one group of parameters $\alpha = 0.1100$, $\beta = 0.5040$, then apply them to the system. By calculating, we obtain the symbolic sequence $\{a_i\}$

$$\{a_i\} = 000001110011111111111011111111 \quad (11)$$

Now we begin to estimate α and β. The process of finding α_s, β_s is the process for finding the value of the required parameters. We adjust α_s, β_s, to make the first symbol obtained by simulation equal to the first symbol of $\{a_i\}$. This means that the parameters we choose lie in the range of H_1. Then we continue to adjust α_s, β_s, to make the second one equal to the second symbol of $\{a_i\}$, which means that the parameters lie in the range of H_2. By repeating this process, we can get the value of α_s and β_s. We draw the suitable region H_i in Fig.5 so as to see conveniently. As the symbolic sequence is too long to draw out, we only give several simple diagrams to explain this question.

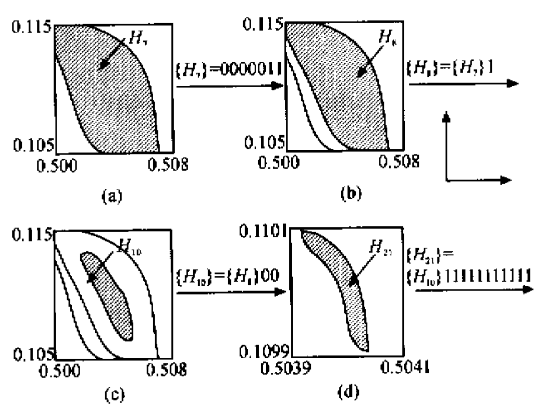

Fig.5 The diagram of the principle of parameter estimating
(a) the region of H_7; (b) the region of H_8;
(c) the region of H_{10}; (d) the region of H_{21}
(Where $\{H_i\}$ presents the first i-bit symbolic sequence corresponding to the range of H_i)

$\{b_i\}$, obtained by simulation, is compared with $\{a_i\}$. The first seven bits are the same, as shown in Fig.5, where H_{21} is the region after 21 times comparison between symbols. We can conclude from the figure that $H_7 \supset H_8 \supset H_{10} \supset H_{21}$. When the sequence is long enough ($\{b_i\} \to \{a_i\}$), H_i will be a small enough range. The value in H_i is the estimation.

When we choose the length of symbolic sequence as bits, the range of H_{30} is $(\alpha, \beta) = (0.1099982 - 0.1100015, 0.5039994 - 0.5040009)$. This is the estimation of the system. The estimating precision of α is

± 0.00000015, that of β is ± 0.0000009.

The example above is the estimation of two parameters. They can be shown in one plane. When the number of the parameter a needed to be estimated is more than 2, we can divide the n-dimension space to complete the multi-parameter estimation. The problem can be solved by the same method. In the example above, we explain the estimating process by shrinking the searching area, but in practice we need not do so much computing, we can choose optimizing method to find the result.

ESTIMATING PRECISION

Precision is related to the length of the sequence in theory without respect to the error from calculation. When the length of the sequence tends to infinity, the estimating error is zero. Under the conditions of our experiment, if we choose the length of the sequence as 30, the precision can reach to 10^{-6}.

The calculation ability for a computer commonly should be of higher precision to satisfy the requirements of engineering. The largest error in simulation is the experiment error. As a result of various kinds of interference during the experiment, trajectory excursion occurs, then the symbolic sequence is influenced, and the precision of parameter estimating consequently influenced.

Therefore, we define a value for trajectory excursion.

If we repeat the experiment, we can get many symbolic sequences under interference. We calculate the distance between every two sequences, which is $d_1, d_2, d_3, \cdots, d_i, \cdots$

The definition of trajectory excursion of a system is:

$$F = \max\{d_1, d_2, d_3, \cdots, d_i, \cdots\} \quad (12)$$

The system error of experiment is less than or equal to F. The smaller the F, the smaller the excursion between the experiment trajectory and the trajectory without interference.

If we get the average of the points of these trajectories in symbolic space, the average represents the trajectory, which is the closest one to the true trajectory. Estimation using by this sequence can yield higher precision. We will discuss this aspect in another paper.

CONCLUSIONS

We can find the characteristics of this method, from the simulation results above.

1. The most important contribution of this paper is apart from the traditional engineering viewpoint and actualizing parameter estimation just based on unstable chaotic systems. The traditional engineering viewpoint is a linear viewpoint. It emphasizes stability, equilibrium, ordering and consistency. We can not imagine how to apply the unstable system to engineering. However, chaotic system is nonlinear system. The important characteristics of chaotic system are instability, imbalance and inconsistency.

2. If only the changes of the peak of chaotic trajectory do not span the critical point C_c (Fig. 2), the symbol will keep unchanged, which means that the trajectories in our method are permitted to change in some certain region with the symbolic sequence and the distance unaffected. Then parameter estimation is not influenced.

3. The sequences obtained by experiment directly apply to estimation in our method. Here is no need to do any measuring instrument. Thus the measuring error is avoided.

In the end, we must say this work is only a preliminary study of application of chaotic theory to parameter estimating.

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