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Car-following models of vehicular traffic

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Abstract: The Car-following models is a kind of microscopic simulation model for vehicular traffic, which describe the one-by-one following behaviors of vehicles in the same traffic lane. As a common traffic phenomenon, following behavior is of great importance in the micro-study of intelligent traffic control. Compared with other traffic-flow models, car-following model embodies the human factors and reflects the real traffic situation in a better way. This paper gives a systematic review of the development and actuality of car-following models by introducing and analyzing in detail the advantages and disadvantages of GHR model, OV model, CA model and fuzzy-logic model. In addition, local stability and asymptotic stability of car-following models are discussed in this paper.

Key words: Car-following model, Stability analysis, Traffic control
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INTRODUCTION

As the conflict between resources and environment and the demand of transportation become more apparent, more and more attention is paid to the study of traffic flow theory. In order to investigate various traffic phenomena more deeply and to optimize traffic control more effectively, great importance should be attached to the computer simulation of traffic systems, which simulates the various behaviors of vehicles and pedestrians, and reflects the characteristics of traffic flow. Car-following models, which describe the one-by-one following process of vehicles in the traffic flow, had been studied in the last nearly a half century (Pipes, 1953). Car-following itself is one of the main processes of various kinds of microscopic simulation models, and thus has been widely investigated by researchers in the fields of traffic systems modeling and control. Consequently, car-following models have a significant position in intelligent transportation systems (ITS) and also are of great potential value in collision-warning and collision-avoidance (CW&CA) systems. Obviously, the further understanding and systematic review of car-following models, done in this paper, become more and more important as advanced technologies develop.

CAR-FOLLOWING MODELS

1. Gazis-Herman-Rothery (GHR) model

In the car-following theories, each individual vehicle always decelerates or accelerates as a response to surrounding stimulus. So the equation of motion of the n-th vehicle can be summarized as follows:

$$[Response]_n \propto [Stimulus]_n \quad (1)$$

The types of the actual model vary with the definitions of the outside stimulus. Generally speaking, the stimulus may include the speed of the vehicle, the relative speed and spacing between the nth and the $(n-1)$th vehicle (the vehicle immediately in front of it). GHR model is the most representative stimulus-response model first brought forward by Chandler et al., (1958). The model specifies the stimulus as the relative velocity of a pair of vehicles; that is, each vehicle tends to move at the same speed as that of the vehicle immediately in front of it. In this way the model can be expressed as

$$a_n(t) = c\Delta v(t-T), \quad (2)$$

Where a_n denotes the acceleration of vehicle n at time t, Δv is the relative speed of the vehicle n to its front vehicle at earlier time of $t - T$, T represents the driver reaction time, and c is the sensitivity coefficient which measures the response intensity of the driver to unit stimulus. In 1959, Gazis, Herman, and Potts derived the macroscopic relationship of traffic flow and velocity from Eq. (2). Nevertheless, the situation described by Eq. (2) is much different from the real world and it cannot depict the phenomenon of high-density traffic since in Eq. (2) the driver has nothing to do with the relative spacing between the pair of the vehicles. In order to make the model more applicable, a term of $1/\Delta x$ is introduced into the sensitivity coefficient ($c \rightarrow c/\Delta x$). That is, the closer each other the vehicles are, the higher the sensitivity coefficient is. Therefore, Eq. (2) becomes

$$a_n(t) = c\,\frac{\Delta v(t - T)}{\Delta x(t - T)}, \tag{3}$$

Where Δx is the spacing between two vehicles. In 1960, Edie modified the model again. He thought that the velocity of the vehicle itself influences the behavior of its driver. Thus the GHR model can be more generally written as:

$$a_n(t) = c v_n^m\,\frac{\Delta v(t - T)}{\Delta x^{l}(t - T)}, \tag{4}$$

Where v_n is the speed of the nth vehicle, and m, l are constants to be determined.

The most vital part of the GHR model is the specification of the parameters m and l. In their experiments for the investigation of the sensitivity of the traffic macroscopic relationships obtained from Eq. (4), Gazis et al. (1961) divided the traffic flow into two parts: congestion and non-congestion. Through analysis of 18 datasets, they found the most favorable combination of m and l was $m = 0$ to 2, and $l = 1$ to 2. In the following 15 years, a large amount of work on the definition of the 'best' combination of m and l was done, but without an uniform result. There are possibly two reasons accounting for this: the complexity of traffic itself, and the measurement methods taken in the experiments. Much more data have been obtained under special conditions, such as lower velocities and stop-start that cannot reflect car-following behavior in general situations. This hinders people from the extensive applications of the GHR model. A summary of valuable combinations of the parameters m and l is given in the following table (Brackstone et al., 1999.)

Table 1 Valuable combinations of the parameters m and l in the GHR model

Source	m	l	Note
Chandler et al. (1958)	0	0	
Herman & Potts (1959)	0	1	
Hoefs (1972)	1.5/0.2/0.6	0.9/0.9/3.2	Deceleration without the use of brakes/ Deceleration with the use of brakes/Acceleration
Treiterer&Myers (1974)	0.7/0.2	2.5/1.6	Deceleration/Acceleration
Ceder&May(1976)	0/0	3/0~1	Uncongested/Congested
Ozaki (1993)	0.9/−0.2	1/0.2	Deceleration/Acceleration

2. Optimal velocity (OV) models

The OV model is another kind of stimulus-response model. As we know, when the driver receives a surrounding stimulus, he computes the desired velocity first instead of acceleration. The mathematical formula for that behavior is given by:

$$a_n - c[V_n^{\text{desired}}(t) - v_n(t)] \tag{5}$$

Where $V_n^{\text{desired}}(t)$ represents the desired velocity of the nth vehicle at time t. In the GHR model, $V_n^{\text{desired}}(t)$ equals the velocity of the front vehicle, i.e., $v_{n+1}(t)$. In the OV model, however, the desired velocity is considered to be relevant to the relative spacing, that is, $V_n^{\text{desired}}(t) = V^{\text{opt}}(\Delta x_n(t))$. Thus

$$a_n(t) = c[V^{\text{opt}}(\Delta x_n(t)) - v_n(t)] \tag{6}$$

It can be inferred from the OV model that the driving strategy of the driver is to maintain a safe velocity according to relative position, while the driver in the GHR model keeps a safe distance according to relative velocity. There are many specific forms of $V^{opt}(\Delta x_n(t))$, and the following is a popular choice:

$$V^{opt}(\Delta x)=\begin{cases}0 & \Delta x<\Delta x_A \\ f(\Delta x) & \Delta x_A<\Delta x<\Delta x_B \\ v_{max} & \Delta x_B<\Delta x\end{cases} \quad (7)$$

The simulations (Sugiyama et al., 1998) showed that the OV model might reproduce many aspects of traffic flow, though the Eq. (7) is a little bit over-simplified. For instance, by $\Delta x<\Delta x_A$ it means the traffic is under congestion and the vehicles should stop driving; and when $\Delta x_B<\Delta x$ the traffic flow density is lower and thus the vehicle can run at its maximum speed.

3. Collision avoidance (CA) models

The CA model is also called the safe distance model. Different from the GHR model, this model is aimed at obtaining a safe following distance instead of describing a stimulus-response type function. According to the model, collision would be unavoidable if the distance is shorter than the safe distance and the behavior of the front vehicle is unpredictable. On the basis of the Newtonian movement law, the original equation (Kometani et al., 1959) could be expressed as follows:

$$\Delta x(t-T)=\alpha v_{n-1}^2(t-T)+\beta_1 v_n^2(t)+\beta v_n(t)+b_0 \quad (8)$$

Subsequently, Gipps (1981) modified the CA model by introducing some mitigating factors such as safe reaction time $T/2$ (collision could be avoided within this reaction time), the braking rates of $-1/2b_n$ (b_n is the expected maximum braking rate of the nth vehicle), the braking rate of the front vehicle of $-1/2b^*$ (b^* is the possibly maximum braking rate of the front vehicle predicted by the nth vehicle). By using psychological method in studying the behavior of the driver, Winsum (1999) suggested that the safe driving distance is correlated with the time headway t_p:

$$D_p=t_p v_i \quad (9)$$

Where D_p refers to the safe distance, and t_p is independent of the vehicle speed. The study also showed that the value of t_p changes along with the driving skill and relates to the weather and the state of the driver (e.g. fatigue). If the distance to the front vehicle is smaller than D_p, the driver will choose to decelerate until D_p is reached. The deceleration can be described as:

$$a_i=cTTC_{est}+d+\varepsilon \quad (10)$$

Where TTC_{est} represents the TTC (Time-to-Collision) estimated by the driver; c and d are constants, and ε is a random error term.

The CA model works well in describing the propagation of disturbances in traffic flow. It may be calibrated according to the instinctive judgement on the behavior of the driver, and need only the parameters b_n and b^* in most cases. As a result, The CA model is widely applied to traffic simulation models including UK DoT's SISTM model, SPEACS model, INTRAS model and so on. However, there are many problems at the same time, such as the concept of "safe headway". In fact, it is not a totally valid standing point. The driver might take the behavior of several front vehicles into account and predict to what extent the front car might decelerate on this basis.

4. Fuzzy-logic models

The fuzzy-logic model is a milestone in the history of car-following theories. In substance, the vehicle behavior is actually the behavior of its driver. Thus an accurate description of the driver characteristics is of great significance to the effectiveness of the model. As is well known, the logical thought of a human being is cursory and qualitative and consequently the delivery and receipt of information is not precise. Hence, the concept of fuzzy logic fits the human observation, thinking, understanding, and decision-making processes much better.

In the fuzzy model, the driver is abstracted as a fuzzy controller with the status of the front car as its input and the decision made through a series of thinking as its output. The first fuzzy model was put forward by Kikuchi et al., (1992) who tried to 'fuzzify' the GHR model. The inputs are Δx, Δv and a_{n+1}, divided into 6, 6 and 12 'fuzzy sets', respectively, with different membership functions. Take the fuzzy assertion "the distance is too close" as an example. When $\Delta x<0.5$, the corresponding membership degree is 1; while $\Delta x=2$, the distance is not so short and thus the membership degree is set to 0.

A value of $\Delta x \in (0.5, 2)$ also has its corresponding membership degree in $(0, 1)$. When the membership degree of the model input is determined, we can derive an equivalent fuzzy output via a reference machine. For instance, the rule:

If $\Delta x = {}'ADEQUATE{}'$,
Then $a_{n,i} = (\Delta v_i + a_{n-1} x T)/\gamma$

where $T(=1)$ is the reaction time, $\gamma(=2.5)$ is the expected time of the driver to catch up with the front vehicle. Assume $\Delta x \neq ADEQUATE$, a_i will change by translating the membership function up or down according to the extent of deviation from $ADEQUATE$. Compared with the GHR model, this model may reflect reality better but some problems still exist. For example, the vehicle acceleration is often hard to be measured, and its linear dependence upon Δx is extremely small. In recent years, lots of research efforts have been made on the fuzzy model of traffic flow, but little has been done on the membership function itself, which, in a sense, is the most important part of the model. Furthermore, real traffic situations are so complex, leading a huge number of fuzzy rules, which in fact block up the comprehensive applications of this model.

STABILITY ANALYSIS

For all types of models, stability analysis is definitely an important issue. It determines if the model has an applicable value. As we know, when a system is disturbed, there will usually be three possible responses: 1. the disturbances tend to be damped; 2. the disturbances are restrained; 3. the disturbances are enlarged. For the first two cases, the system (and, equivalently, the model of the system) is regarded as stable. For the last case we say the model is unstable. In car-following models, the stability analysis distinguishes two types: the local stability and the asymptotic stability (Gartner et al., 1998). The local stability is focused on the response of a vehicle to the disturbances of its front vehicle, while the asymptotic stability is concerned with the propagation of the disturbances among a platoon of vehicles. Before discussing them in detail, we would like to point out that (1) for convenience, the discussion will be confined to a single-lane linear car-following model; (2) in real situation, car-following behavior is extremely common and generally stable.

Local stability

We modify the Eq. (1) by using a differential form as follows:

$$\ddot{x}_f(t) = c[\dot{x}_l(t-T) - \dot{x}_f(t-T)] \qquad (11)$$

Where x_f and x_l denote the positions of the following vehicle and the leading vehicle, respectively. Let $t = \tau T$ and $D = cT$, Eq. (11) can then be rewritten as follows:

$$\ddot{x}_f(t) = D[\dot{x}_l(\tau-1) - \dot{x}_f(\tau-1)] \qquad (12)$$

By Laplace transform and inverse Laplace transform for Eq. (12) (Chandler et al., 1958; and Kikuchi et al., 1992.) we can obtain the analytical solutions for the position, velocity, and acceleration of the following vehicle. The solutions are so complicated that the description of its physical characteristics is relatively difficult to be extracted from them. Under some specific initial conditions, however, the solutions could be simplified. Assume the following vehicle and the leading vehicle both initiate at a speed u, and then the acceleration of the following vehicle can be expressed as below:

$$a_f(s) = L^{-1}[D(D + se^s)^{-1}s] \qquad (13)$$

As is well known in modern control theories, the stability of Eq. (13) can be depicted by the roots of $D + se^s = 0$. Obviously, the roots will be different with the different values of D. Only when the real part of the root is negative, will the following vehicle's acceleration be stable. The following conclusions were drawn by setting the root as $s = a + ib$ (Gartner et al., 1998):

1) If $D \leqslant e^{-1} (=0.3680)$, then $a \leqslant 0$, $b = 0$. In this case the motion of the following vehicle is non-oscillatory and the acceleration of it is exponentially damped.

2) If $e^{-1} < D < \pi/2$, then $a < 0$, $b > 0$, the motion is oscillatory with exponential damping to an ultimately stable driving speed.

3) If $D = \pi/2$, then $a = 0$, $b > 0$, the motion is oscillatory with a constant amplitude.

4) If $D > \pi/2$, then $a > 0$, $b > 0$, the motion is oscillatory with an increasing amplitude.

By invoking the meaning of c and T defined in

Eq.(2) and $D = cT$ we can draw a conclusion from the above analysis that both the over-sensitiveness and the over-sluggishness of the driver response to the speed change of his front vehicle are dangerous for driving. From the solution for the relative spacing between two vehicles, we can also find that its stability is also determined by the roots of $D + se^s = 0$. For example, if $D < e^{-1}$, the change of the relative spacing is non-oscillatory. Using U, V, to represent the start and the stop speeds of the pair of the vehicles, respectively, we have the following equation:

$$\Delta S = \frac{1}{c}(V - U) \tag{14}$$

Where ΔS denotes the relative spacing change due to the motion disturbances of the front vehicle. It can be inferred from Eq.(14) that if the leading vehicle stops, that is $V = 0$, then $\Delta S = -U/c$. So the following vehicle may avoid a collision only when the relative spacing is not less than U/c. On the other hand, however, for road occupation efficiency the relative spacing should be as small as possible. So the value of c should be large enough within the stable limit. Ideally, the best choice of c is $(eT)^{-1}$.

Consider the following equation used to describe more general car-following behaviors:

$$\ddot{x}_f(\tau) = D\frac{d^m}{dt^m}[x_l(\tau - 1) - x_f(\tau - 1)],\quad m = 0, 1, 2, \cdots \tag{15}$$

Eq.(12) is a special case of Eq.(15) with $m = 1$. Similarly, the stability of Eq.(15) can be derived by the roots of Eq.(12):

$$D + s^m e^s = 0. \tag{16}$$

The analysis based on different values of m showed that all of these roots are larger than zero when m is even, which implies that local stability may be reached only when m is odd. The result also indicate that an acceleration response directly proportional to relative spacing stimulus will result in unstable driving behaviors.

Asymptotic stability

In fact, each of a platoon of vehicles behaves similarly as a single vehicle. As a result, we have

$$\ddot{x}_{n+1}(t) = c[\dot{x}_n(t - T) - \dot{x}_{n+1}(t - T)],\quad n = 0, 1, 2, \cdots, N \tag{17}$$

Where N is the number of the vehicles in the platoon. Any specific solution to these equations depends on the velocity of the leading vehicle in the platoon, denoted by u_0, and the two parameters c and T. For any relative spacing, if a disturbance grows in amplitude, then a "collision" would eventually occur somewhere back in the platoon of vehicles.

By using the Fourier analysis, the velocity of a vehicle can be expressed as follows (Ceder et al., 1976):

$$u_n(t) = a_0 + f_n e^{i\omega t},\quad n = 0, 1, 2, \cdots, N \tag{18}$$

Substituting Eq.(18) into Eq.(17) yields

$$u_n(t) = a_0 + \left[1 + \left(\frac{\omega}{c}\right)^2 + 2\left(\frac{\omega}{c}\right)\sin(\omega T)\right]^{-n/2} e^{i\Omega(\omega, c, T, n)} \tag{19}$$

In the case of asymptotic stability for a platoon of vehicles, the disturbance should be attenuated in propagation. So the amplitude of u_n should decrease with n increased. Therefore we can derive the following inequality:

$$\frac{\omega}{c} > 2\sin(\omega T) \tag{20}$$

Assume $\omega \to 0$, then c must satisfy the following inequality:

$$cT < \frac{1}{2}[\lim_{\omega \to 0}(\omega T)/\sin(\omega T)] \tag{21}$$

Accordingly, the asymptotic stability is guaranteed for all frequencies where this inequality is satisfied. Notice that the local stability without oscillation results in the asymptotic stability.

CONCLUSIONS

The car-following models have been developed for more than 40 years, from the coupled differential equation models in the 50s' to the fuzzy models in the 90s'. The steering ideology has been changed and the psychological characteristics of human drivers have been considered in the models. Although the models fit real traffic situations better when the human factors are introduced into them, there are still some prevalent problems. The following two are the key points. First, the models are usually mainly based on the rationality of the driver, who can perceive the distance, velocity, and acceleration correctly. Nevertheless, psychological researchers found that the roughly estimation of the status of the front vehicle is often influenced by heuristic human behaviors, with adaptability and flexibility. Second, many models take into account optimization, while in fact drivers may not following this principle. And, though some models introduce factors like noise to minimize the discrepancy between traffic practice and those models, they are still based on some flawed assumptions. Presently, the acceptable viewpoint is that drivers have their own expected satisfactory degree of their driving behavior. When the satisfactory degree is reached, the state of the vehicle will be kept by its driver and the vehicle will not go forward to the optimal level. Bore (1999) brought forward a significant theory, i.e., the Task Scheduling Theory to describe that phenomenon. According to the Task Scheduling Theory, drivers do not observe the front cars all the time of driving to control their own acceleration response, they instead just take a glance at the front vehicles time to time to judge their appropriate behaviors.

The car-following theory is an attractive research field in transportation projects. If the single-lane car-following model succeeds, it may be able to be extended to multiple-lane cases and the study on overtaking and turning will have a solid foundation. At present, most research work focus on the stability analysis of car-following models and the implication of each of the relationships to macroscopic flow properties. The models would be much more applicable if such factors as "personal motivation" and "attitude" could involved to explain the individuality of drivers. This should be a quite promising research direction.

References

Aycin, M. F., Benekohal, R. F., 2001. Stability and performance of car following models in congested traffic. *Journal of Transportation Engineering*, 127(1): 2–12.

Boer, E. R., 1999. Car following from the driver's perspective. *Transportation Research Part F*, **2**: 201–206.

Brackstone, M., McDonald, M., Wu, J., 1997. Development of a fuzzy logic based microscopic motorway simulation model. *In*: Proceedings of the IEEE Conference on Intelligent Transportation Systems (ITSC97), Boston, USA.

Brackstone, M., McDonald, M., 1999. Car-following: a historical review. *Transportation Research* (*Part F*), **2**: 181–196.

Ceder, A., May, Jr. A. D., 1976. Further evaluation of single and two-regime traffic flow models. *Transportation Research Record*, **567**: 1–30.

Chandler, R. E., Herman, R., Montroll, E. W., 1958. Traffic dynamics: studies in car following. *Operations Research*, **6**: 165–184.

Chowdhury, D., Santen, L., Schadschneider, A., 2000. Statistical physics of vehicular traffic and some related Systems. *Physics Reports* **329**: 199–329.

Edie, L. C., 1960. Car following and steady state theory for non-congested traffic. *Operations Research*, **9**: 66–76.

Erwin, R. B., 1999. Car following from the driver's perspective. *Transportation Research* (*Part F*), **2**: 201–206.

Gartner, N., Messer, C. J., Rathi, A., 1998. Car following, Traffic Flow Theory. *TRB Special Report* 1654: 1–39.

Gazis, D. C., Herman, R., Potts, R. B., 1959. Car following theory of steady state traffic flow. *Operations Research*, **7**: 499–505.

Gipps, P. G., 1981. A behavioral car following model for computer simulation. *Transportation Research* (*Part B*), **15**: 105–111.

Hancock, P. A., 1999. Is car following the real question—are equations the answer? *Transportation Research* (*Part F*), **2**: 97–199.

Herman, R., Montroll, E. W., Potts, R. B., Rothery, R. W. 1959. Traffic dynamics: analysis of stability in car Following. *Operations Research*, **7**: 86–106.

Kikuchi, C., Chakroborty, P., 1992. Car following model based on a fuzzy inference system. *Transportation Research Record*, **1365**: 82–91.

Kometani, E., Sasaki, T., 1959. Dynamic behavior of traffic with a nonlinear spacing-speed relationship. *In*: Proceedings of the Symposium on Theory of Traffic Flow, Research Laboratories, General Motors, Elsevier, p. 105–119.

Sugiyama, Y., Yamada, H., 1998. Aspects of Optimal Velocity Model for Traffic Flow. *In*: Proceedings of International Workshop of Traffic and Granular Flow' 97, Springer, Singapore, p. 301–318.

Winsum, W. V., 1999. The human element in car following models. *Transportation Research* (*Part F*), **2**: 207–211.