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Robust predictive control of uncertain intergrating linear systems with input constraints*

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Abstract: This paper presents a two-stage robust model predictive control (RMPC) algorithm named as IRMPC for uncertain linear integrating plants described by a state-space model with input constraints. The global convergence of the resulted closed loop system is guaranteed under mild assumption. The simulation example shows its validity and better performance than conventional Min-Max RMPC strategies.

Key words: Model predictive control, Robust control, Input constraints, Convex programming
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INTRODUCTION

Model predictive control (MPC), which is also known as moving horizon control or receding horizon control, has captured much attention of both industrial and academic control communities in the last couple of decades because of its widely successful applications in the processing industries (Qin et al., 1997; Morari et al., 1999; Mayne et al., 2000, Li et al., 2001). Currently, MPC is generally considered as an effective means to deal with multivariable constrained control problems. While MPC is successful practically and its theoretical properties such as feasibility of the on-line optimization, stability and performance are also being explored deeply with regard to linear systems, very little is known about the rigorous analysis of its closed-loop performance and stability properties in the presence of plant-model mismatch.

The introduction of uncertainty in the system description raises the question of robustness, i.e. the maintenance of certain properties such as stability and performance in the presence of uncertainty, which are dealt with in robust control theories. Most studies on robustness consider unconstrained systems. The problem of stabilizing robustly a linear system subject to input and output constraints, especially in the context of MPC, has received much attention in the recent control literature. Various modifications proposed in these literature can be generally grouped into two main categories: the first one is to minimize the worst-case controller cost and named as Min-Max algorithms (Lee et al., 1997). The main disadvantage of the Min-Max approaches is that the computational control performance may be too conservative for some cases. The second category is to minimize a nominal cost under robust constraints. A significant disadvantage of most robust control algorithms is that all possible plant dynamics are considered to be equally likely. Typical statistical assumptions for a process identification experiment lead to an estimate of the joint probability distribution function for the plant parameters, providing a clear indication of which parameter set is most likely to be encountered. So the most robust control algorithms, such as Min-Max controllers, generally lead to too conservative performance. Optimizing performance objective based on a nominal model which is generally the most likely to occur, while guaranteeing the closed-loop robust stability, is preferable from the practical point of view.

Keeping to the above argument, Badgwell and his coworkers developed an alternative

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method for achieving robust stability using cost function constraints. This approach involves optimizing process behavior using the most likely parameter vector (nominal model), while adding constraints to guarantee robust stability. Badgwell (1997) showed that the use of cost function constraints leads to robust stability for a finite set of stable, multivariable systems, subject to hard input and soft state constraints. Ralhan and Badgwell (2000a) extended the algorithm to the case of an infinite dimensional uncertainty description. Borrowing from the ideas first presented by Lee and Cooley (2000), Ralhan and Badgwell (2000b) also showed that the cost function constraints based approach proposed originally for the stable uncertain systems can be generalized to the linear integrating systems with hyperellipsoid uncertain parameters set. But the convergence property of their methodology is weak.

This paper develops further Badgwell and coworkers' algorithm for linear integrating systems with a bounded set of input matrices. In order to obtain better dynamic performance, we use a two-stage optimization algorithm, which can guarantee the global asymptotical stability of the closed-loop system by introducing a contraction factor to the first stage optimization. Eventually, one simulation example based on a simple scalar integrating plant is used to validate its effectivity and compare its dynamic performance to the conventional Min-Max robust MPC method.

PROBLEM DEFINITION

Consider discrete-time linear systems with simple integrators. For convenience of exposition, we assume that the model is given in the following form:

$$\underbrace{\begin{bmatrix} z_{k+1}^S \\ z_{k+1}^I \end{bmatrix}}_{z_{k+1}} \underbrace{\begin{bmatrix} A_S & 0 \\ 0 & I \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} z_k^S \\ z_k^I \end{bmatrix}}_{z_k} + \underbrace{\begin{bmatrix} B_S(\vartheta_k) \\ B_I(\vartheta_k) \end{bmatrix}}_{B(\vartheta_k)} u_k \tag{1}$$

where $z_k \in R^n$ represents the state of the system and $u_k \in \mathrm{U} \subset R^p$ is the limited input to the plant at time k. The assumption here is that the current state z_k can be measured perfectly. A_s has all eigenvalues strictly inside the unit disk. The above form of model can be derived easily by performing an appropriate state coordinate transformation. All the elements of $B(\vartheta)$ are assumed to be affine functions of ϑ, which is an uncertain parameter vector that belongs to a convex, compact set Ω. The goal of the control system is to bring the state of the system from an initial nonzero value to the origin with input constraints when the input matrix $B(\vartheta)$ is not known exactly. Let us assume that we have selected one plant from the set Ω as the most likely plant that will be encountered by the controller, and that we will use this as the model for the MPC algorithm. We will refer to this as the nominal model $\tilde{\vartheta}$. Simultaneously, $\bar{\vartheta}$ denotes the actual plant parameter, which is not known exactly. The input constraints set U is defined as the following special convex polyhedral:

$$U = \{u: u^{\min} \leqslant u \leqslant u^{\max}\} \tag{2}$$

CONTROLLER COST FUNCTIONS

MPC controllers typically regulate the plant state by minimizing a cost function that penalizes deviations of the state and input away from their targets. Rawlings and Muske (1993) showed that infinite horizon cost function based MPC strategies have good stability property. Similarly to their formulation, we define the following optimization formulation for the proposed system (1):

$$\min_{\pi \in U^m} \Phi(z_0, \pi, \vartheta) \doteq \sum_{j=0}^{\infty} [(z_j^S)^{\mathrm{T}} (z_j^I)^{\mathrm{T}}] \underbrace{\begin{bmatrix} Q_S & Q_{SI} \\ Q_{SI} & Q_I \end{bmatrix}}_{Q} \begin{bmatrix} z_j^S \\ z_j^I \end{bmatrix} + u_j^{\mathrm{T}} R u_j \tag{3}$$

with

$$z_j = A^j z_0 + \sum_{i=1}^{j} A^{j-1} B(\vartheta) u_{i-1} \tag{4}$$

$$u_j \in U, \ j = 0, \cdots, m-1 \tag{5}$$

$$u_{m+l} = 0, 0 \leqslant l \tag{6}$$

$$\pi = [u_0^{\mathrm{T}}, u_1^{\mathrm{T}} \cdots u_{m-1}^{\mathrm{T}}]^{\mathrm{T}}, \quad U^m \doteq \underbrace{U \times U \times \cdots \times U}_{m} \tag{7}$$

Here, z_0 represents the current measured state value of the plant, z_j are the predicted future states, and u_j are the future inputs considered by the controller; the constraint $u_{m+l}=0$, $0\leqslant l$, is artificially imposed in order to control the size of optimization. The weight matrices Q and R are chosen positive definite and $Q_{IS}{}^{\mathrm{T}}=Q_{SI}$. Under this constraint, for the stable plant, its state asymptotically converges to the origin and we can easily compute the infinite horizon controller cost. For integrating plants, however, setting the input to the origin does not imply that the plant state will converge to the origin. Hence, for the integrating plant case we cannot, in general, bound the infinite horizon controller cost. One way to solve this problem is to impose an end-point equality constraint on the optimization formulation to force a finite cost value. However, this end-point equality constraint is usually infeasible for robust controller designing. In order to overcome this difficulty, a robust MPC controller considered here uses two cost functions: one for the primary optimization to consider the steady performance, and the other one for the secondary optimization to guarantee the dynamic performance over the whole control horizon. This two-stage optimization strategy was proposed first by Lee and Cooley (2000), then developed further by Ralhan and Badgwell (2000a, 2000b) for the open-loop stable plants and pure integrating plants to design more practical robust MPC controllers. In this paper, we also adopt this two-stage optimization strategy:

Definition 1: IRMPC At time step k, the two-stage integrating robust model predictive controller (IRMPC) is defined as:

Stage I: Steady-State error minimization

$$\min_{w_k\in R^p}\Psi(z_k,w_k.\tilde{\vartheta}_k)\doteq(\tilde{z}^I_{k+m})^{\mathrm{T}}Q_I\tilde{z}^I_{k+m} \quad (8)$$

subject to

$$\Psi(z_k,w_k,\vartheta_k)\leqslant\alpha\Psi(z_k,\hat{w}_k,\vartheta_k),\ \forall\vartheta_k\in\Omega \quad (9)$$

$$w_k\in W;\ w_k=u_k+u_{k+1}+\cdots+u_{k+m-1} \quad (10)$$

$$\hat{w}_k=I_1S\pi^*_{k-1};\ \hat{w}_0=[0\cdots0]^{\mathrm{T}} \quad (11)$$

where:

$$\tilde{z}^I_{k+m}=z^I_k+B_I(\tilde{\vartheta})w_k;\ 0<\alpha<1;$$

$$S=\begin{bmatrix}0 & I & \cdots & 0\ 0\\ \vdots & \ddots & \ddots & \vdots\\ \vdots & & \ddots\quad\ddots & \vdots\\ 0 & \cdots\cdots & & 0\ I\\ 0 & \cdots\cdots & & 0\ 0\end{bmatrix};\ I_1=[1\cdots1]^{\mathrm{T}} \quad (12)$$

Stage II: Dynamic error minimization:

$$\min_{\pi\in U^m}\Phi(z_k,\pi_k,\tilde{\vartheta}_k)\doteq\sum_{i=1}^{m-1}\tilde{z}^{\mathrm{T}}_{k+i}Q\tilde{z}_{k+i}+(\tilde{z}^S_{k+m})^{\mathrm{T}}\bar{Q}_S\tilde{z}^S_{k+m}+2(\tilde{z}^S_{k+m})^{\mathrm{T}}\bar{Q}_{SI}\tilde{z}^I_{k+m}+\sum_{j=0}^{m-1}u^{\mathrm{T}}_{k+j}Ru_{k+j} \quad (13)$$

subject to:

$$\Phi(z_k,\pi_k,\vartheta_k)\leqslant\Phi(z_k,\hat{\pi}_k,\vartheta_k),\ \forall\vartheta_k\in\Omega \quad (14)$$

$$I_1\pi_k=w^*_k \quad (15)$$

$$U^m\doteq\underbrace{U\times U\times\cdots\times U}_{m} \quad (16)$$

$$u_{m+l}=0,\ 0\leqslant l, \quad (17)$$

$$\hat{\pi}_k=S\pi^*_{k-1},\hat{\pi}_0=[0\cdots0]^{\mathrm{T}} \quad (18)$$

Here z_k is the current measured value of the plant state; $\hat{w}_k$ and $\hat{\pi}_k$ represent respectively the feasible solutions for the two optimizations at current time step k; w^*_k and π^*_k represent respectively their optimal solutions. The current actual input v_k is set as: $v_k=u^*_k=[I,0\cdots0]\cdot\pi^*_k$. The weight matrices $\bar{Q}_S$ and $\bar{Q}_{SI}$ are respectively the solutions to the following matrix equations:

$$A^{\mathrm{T}}_S\bar{Q}_SA_S+Q_S=\bar{Q}_S;\ \bar{Q}_{SI}=Q_{SI}+A^{\mathrm{T}}_S\bar{Q}_{SI} \quad (19)$$

Comment In the above, w_k represents the sum of u_k through u_{k+m-1} and W represents the feasible set for $u_k+u_{k+1}+\cdots+u_{k+m-1}$ as defined by U^m. Obviously, W is a convex compact set provided that the input constraint set is selected as the special convex polyhedral Eq. (2).

As defined above, the two-stage robust MPC algorithm consists of two semi-infinite programs. In stage-I the overall control action w_k is determined to minimize steady-state offset. The optimal input vector π^*_k is then computed in stage-II over the entire prediction horizon with the constraint that the sum of the predicted input vector satisfies optimal solution in stage-I.

ROBUST STABILITY

In this section, We show that the state vector of the closed-loop system comprised of the above IRMPC algorithm acting on the plant is convergent to the origin. To show this, let us first consider the feasibility of the optimization propositions in IRMPC formulation under certain conditions.

Lemma 1 Assume that for all $\vartheta \in \Omega$, $B_1(\vartheta)$ has full row rank and the individual matrix elements have the same sign, $w_k = [0 \cdots 0]^T$ is the interior-point of the overall input constraint set W; then there exists some $0 < \alpha < 1$ for which the constraints (9) − (12) in the stage-I optimization remain feasible throughout for any initial state value and for all possible parameter values $\vartheta \in \Omega$.

Proof At the initial time step, we adopt the feasible solution $\hat{w}_0 = [0 \cdots 0]^T$ as in (11) such that the initial cost function value for any model is:

$$\Psi(z_0, \hat{w}_0, \vartheta) = (z_0^I)^T Q_I z_0^I \tag{20}$$

If the initial state z_0^I is zero, the constraints Eqs. (9) − (12) are obviously feasible; Otherwise, we can estimate the gradient of the cost function under zero input as follows:

$$\nabla_{w_0} \Psi(z_0, w_0, \vartheta)|_{w_0 = \hat{w}_0} = 2B_I^T(\vartheta) Q_I z_0 \tag{21}$$

Since $B_I^T(\vartheta)$ has full row rank and its elements have the same sign for all $\vartheta \in \Omega$, and z_0 is nonzero and $Q_I > 0$. On the other hand, the overall input constraint set W is a convex, compact set that includes the origin as an interior point. Hence, there exists $w_0 \in W$ such that

$$\nabla_{w_0} \Psi(z_0, w_0, \vartheta)|^T_{w_0 = \hat{w}_0} w_0 = (2B_I^T(\vartheta) \cdot Q_I z_0)^T w_0 < 0, \ \forall \vartheta \in \Omega \tag{22}$$

So there exists $w_0 \neq \hat{w}_0$, $w_0 \in W$; $0 < \alpha < 1$ such that:

$$\Psi(z_0, w_0, \vartheta) \leqslant \alpha \Psi(z_0, \hat{w}_0, \vartheta), \ \forall \vartheta \in \Omega \tag{23}$$

It means that the constraints Eqs. (9) − (12) of the stage-I optimization are always feasible for any initial state value z_0 with $\hat{w}_0 = [0 \cdots 0]^T$ at the initial time step.

Assume that we have found the optimal solution at time step $k-1$ ($k >= 1$), given by w_{k-1}^* for the first stage and π_{k-1}^* for the second stage, then the feasible solution to the stage-I optimization at time k is:

$$\hat{w}_k = I_1 S \pi_{k-1}^* - v_{k-1} \tag{24}$$

where v_{k-1} is the plant actual input at $k-1$, so the feasible cost for any model at time step k can be written as

$$\begin{aligned}\hat{\Psi}_k \doteq \Psi(z_k, \hat{w}_k, \vartheta) &= (z_{k+N}^I(\hat{w}_k))^T Q_I \cdot (z_{k+N}^I(\hat{w}_k)) = \\ &(z_k^I + B_I(\vartheta)\hat{w}_k)^T Q_I (z_k^I + B_I(\vartheta)\hat{w}_k) = \\ &\| z_k^I + B_I(\vartheta)(w*_{k-1} - v_{k-1}) \|_{Q_I}^2 = \\ &\| z_{k-1}^I + B_I(\vartheta) w_{k-1}^* \|_{Q_I}^2 = \\ &(z_{k-1+m}^I(w*_{k-1}))^T Q_I z_{k-1+m}^I(w_{k-1}^*)\end{aligned} \tag{25}$$

If the predicted state $z_{k-1+m}^I(w_{k-1}^*)$ is zero, the constraints Eqs. (9) − (12) are obviously feasible; Otherwise, we can estimate the gradient of the cost function under the feasible input $\hat{w}_k$ at time step k:

$$\nabla_{w_k} \Psi(z_k, w_k, \tau)|_{w_k = \hat{w}_k} = 2B_I^T(\vartheta) Q_I (z_k^I + B_I \hat{w}_k) = 2B_I^T(\vartheta) Q_q z_{k-1+m}^I(w_{k-1}^*) \tag{26}$$

Similar to the argument at the initial time step, we can see that there exists $w_k \neq \hat{w}_k$, $w_k \in w$; $0 < \alpha < 1$ such that:

$$\Psi(z_k, w_k, \vartheta) \leqslant \alpha \Psi(z_k, \hat{w}_k, \vartheta), \ \forall \vartheta \in \Omega \tag{27}$$

In light of the above argument, the constraints Eqs. (9) − (12) in the stage-I optimization remain feasible throughout for any initial state value and for all possible parameter values $\vartheta \in \Omega$.

Remark 1 Though the feasibility of stage-I optimization formulation can be guaranteed throughout for all the initial state values under certain mild hypothesis, there is a possibility that the constraints of stage-II optimization formulation become infeasible because of the constraint Eq. (15) on π_k (which may be different from $\hat{w}_k$). Hence, it should be implemented as a soft constraint. But according to the following lemma, the optimal plant cost function of the stage-I optimization formulation converges exponentially to zero and $w_k^* \rightarrow \hat{w}_k$ such that the constraint can be guaranteed to be feasible eventually without softening.

Lemma 2 Assume that for all $\vartheta \in \Omega$, $B_1(\vartheta)$ has full row rank and that the individual matrix elements have the same sign. $w_k = [0 \cdots 0]^{\mathrm{T}}$ is the interior-point of the overall input constraint set W, then the optimal plant cost of stage-I optimization formulation converges exponentially to zero and $w_k^* \to \hat{w}_k$ for all initial conditions z_0 and for all possible parameter values $\vartheta \in \Omega$.

Proof Let us assume that we have found the optimal solution w_k^* of stage-I optimization formulation at time step k, then the optimal plant cost at time step k is given by:

$$\bar{\Psi}_k^* \doteq \Psi(z_k, w_k^*, \bar{\vartheta}) = (\bar{z}_{k+m}^I)^{\mathrm{T}} Q_I \bar{Z}_{k+m}^I = (z_k + B_I(\bar{\vartheta}) w_k^*)^{\mathrm{T}} Q_I (z_k + B_I(\bar{\vartheta}) w_k^*) \tag{28}$$

At time step $k+1$, we can obtain the feasible solution as: $\hat{w}_{k+1} = I_1 S \pi_k^* = w_k^* - u_k^*$ and its relevant feasible plant cost can be written as:

$$\hat{\bar{\Psi}}_{k+1} \doteq \Psi_{k+1}(z_{k+1}, \hat{w}_{k+1}, \bar{\vartheta}) = (z_{k+1}^I + B_I(\bar{\vartheta}) \cdot \hat{w}_{k+1})^{\mathrm{T}} Q_I (z_{k+1}^I + B_I(\bar{\vartheta}) \hat{w}_{k+1}) \tag{29}$$

The predicted state of the plant at time step $k+1$ is given as:

$$z_{k+1}^I = z_k^I + B_I(\bar{\vartheta}) u_k^* \tag{30}$$

Substituting for z_{k+1}^I in Eq. (29), we get:

$$\hat{\bar{\Psi}}_{k+1} = \| z_k^I + B_I(\bar{\vartheta}) u_k^* + B_I(\bar{\vartheta}) \hat{w}_{k+1} \|_{Q_I}^2 = \| z_k^I + B_I(\bar{\vartheta}) w_k^* \|_{Q_I}^2 = \bar{\Psi}_k^* \tag{31}$$

On the other hand, the robustness constraint Eq. (9) holds for all the model in the uncertainty domain,

$$\bar{\Psi}_{k+1}^* \leqslant \alpha \hat{\bar{\Psi}}_{k+1} = \alpha \bar{\Psi}_k^* \tag{32}$$

Hence,

$$\bar{\Psi}_k^* \leqslant \alpha^k \bar{\Psi}_0^*$$

Since the plant cost is bounded below by zero and $0 < \alpha < 1$, we can conclude that the optimal plant cost $\bar{\Psi}_k^*$ converges exponentially to zero and $w_k^* \to \hat{w}_k$.

Then the robust convergence of the state vector of the closed loop system comprised of the IRMPC algorithm acting on the plant is summarized as the following theorem.

Theorem 1 Assume that for all $\vartheta \in \Omega$, $B_I(\vartheta)$ has full row rank and that the individual matrix elements have the same sign. When the feedback law $v_k = u^*(z_k)$, where $u^*(z_k)$ represents the optimal solution to the two stage IRMPC algorithm, is applied to the system Eq. (1) with $\vartheta_k = \vartheta \, \forall k$ then $z_k \to 0$ and $v_k \to 0$ as $k \to \infty$ for all initial conditions z_0 and for all possible parameter values $\vartheta \in \Omega$.

Proof According the robustness constraint (14) of the stage-II optimization formulation and the weighting matrix Eq. (19), we note that:

$$\Phi_{k+1}(z_{k+1}, \pi_{k+1}^*, \bar{\vartheta}) \leqslant \Phi_{k+1}(z_{k+1}, \hat{\pi}_{k+1}, \bar{\vartheta}) = \Phi_k(z_k, \pi_k^*, \bar{\vartheta}) - z_{k+1}^{\mathrm{T}} Q z_{k+1} - v_k^{\mathrm{T}} R v_k + \bar{\Psi}_k^* \tag{33}$$

Since $\bar{\Psi}_k^*$ is an exponentially converging sequence as indicted by the above lemma 2, there exist $\alpha < 1$ and $b < \infty$ such that:

$$\bar{\Psi}_k^* \leqslant b\alpha^k \, \forall k \tag{34}$$

Hence,

$$\bar{\Phi}_{k+1}^* \leqslant \bar{\Phi}_k^* - z_{k+1}^{T} Q z_{k+1} - v_k^{\mathrm{T}} R v_k + b\alpha^k \leqslant \bar{\Phi}_0^* - \sum_{i=0}^{k} (z_{i+1}^{\mathrm{T}} Q z_{i+1} + v_i^{\mathrm{T}} R v_i) + \sum_{i=0}^{k} b\alpha^i \tag{35}$$

Since the stage-II cost function is nonnegative, i.e. $\Phi_k > 0$, then:

$$\sum_{i=0}^{k} (z_{i+1}^{\mathrm{T}} Q z_{i+1} + v_i^{\mathrm{T}} R v_i) \leqslant \bar{\Phi}_0^* + \sum_{i=0}^{k} b\alpha^i, \quad \forall k > 0 \tag{36}$$

For any $0 < \alpha < 1$, the above right-hand side sum is a positive finite term when $k \to \infty$, which means the left-hand side sum $\sum_{i=0}^{k} (z_{i+1}^{\mathrm{T}} Q z_{i+1} + v_i^{\mathrm{T}} R v_i)$ also converges to a finite term in the limit, which implies

$$z_k \to 0 \text{ and } v_k \to 0 \text{ as } k \to \infty.$$

Remark 2 Note that Ralhan and Badgwell (1999, 2000) gave some two-stage RMPC algorithms respectively for stable or pure-integrating systems without input constraint. The robust stability guarantee of their algorithms, however, were all weaker, in the sense that if the system reaches steady-state, it reaches the correct steady-state (no steady-state offset). Our improved algorithm presented here guarantees the global convergence of the integrating system with input constraint.

Remark 3 In Lee and Cooley (2000) a two-stage Min-Max RMPC algorithm is given for integrating system with input constraints. Their algorithm can only guarantee the robust stability for the systems with time-varying uncertain parameter. Thus it leads to conservative result for time-invariant uncertain parameter. Though it can also improve the stability for time-invariant parameter by adding contraction constraints, the resulted optimization proposition is computationally intractable.

COMPUTATIONAL ISSUES

In general, the analytic solution of the IRMPC algorithm defined above cannot be obtained explicitly so that it is necessary to develop a reliable numerical solution method. However, the two semi-infinite programs included in the IRMPC algorithm add difficulty in solving effectively this algorithm numerically. Kassmann (1999) concluded five solution methods for this class of problems. Ralhan and Badgwell (2000) adopted methods based on discretization and local reduction for the general case of uncertain variable B. Discretization methods rely on replacing the infinite-dimensional uncertain set Ω with a finite-dimensional approximation. However, this is not always the case and in general, there is no discretization set to guarantee that the solutions of the two optimization propositions are identical. Local reduction methods rely on replacing the semi-infinite constraint with a single constraint based on the most limiting parameter, which is obtained by an inner optimizer in the set. In the most general case, though, the inner optimization must be solved numerically, it is possible in some cases to obtain the analytic solution by a conservative approximation.

SIMULATION EXAMPLE

In this section, we will validate the performance of the IRMPC algorithm compared to the algorithm proposed by Lee and Cooley (2000). For the sake of clarity, Let us define the corresponding algorithm given by Lee and Cooley (2000) as below:

Definition 2 Min-Max RMPC. At time step k, the Min-Max RMPC is defined as,

Stage I Steady-state error minimization:

$$\min_{w_k\in W}\max_{\vartheta\in\Omega}\Psi(z_k,\vartheta,w_k)\doteq(z^I_{k+m})^T Q_I z^I_{k+m} \quad (37)$$

Stage II Dynamic error minimization:

$$\min_{\pi\in U^m}\max_{\vartheta\in\Omega}\Phi(z_k,\pi_k,\vartheta)\doteq\sum_{i=1}^{m-1} z^T_{k+i}Qz_{k+i}+(z^S_{k+m})^T\overline{Q}_S z^S_{k+m}+2(z^S_{k+m})^T\overline{Q}_{SI}z^I_{k+m}+\sum_{j=0}^{m-1}u^T_{k+j}Ru_{k+j} \quad (38)$$

Subject to:

$$I_1\pi_k=w^*_k \quad (39)$$

$$u_j\in U,\ j=0,\cdots,m-1, \quad (40)$$

$$u_{m+l}=0,0\leqslant l, \quad (41)$$

We consider the case of a scalar integrating plant:

$$z_{k+1}=z_k+bu_k \quad (42)$$

The actual plant input parameter $\bar{b}$ is only known to lie in the range.

$$1<b_{\min}\leqslant\bar{b}\leqslant b_{\max} \quad (43)$$

The plant input is limited as follows:

$$-0.15\leqslant u(k)\leqslant 0.15 \quad (44)$$

To compare performance for a specific case, let us assume $b_{\min}=4$ and $b_{\max}=12$. When the nominal model $\tilde{b}$ and actual plant input parameter $\bar{b}$ are both chosen as $b_{\min}$, i.e. there no model mismatch exists, the plant responses with IRMPC and Min-Max RMPC controllers under the following controller parameters settings are as shown in Fig. 1.

$$Q=1;\ R=0.1;\ m=3;\ \alpha=0.95 \quad (45)$$

Both the IRMPC and Min-Max RMPC algorithms bring the state to the origin from an initial state of 1. The IRMPC algorithm responds faster than Min-Max RMPC as the latter still considers the worst objective in the whole uncertain range, though there is no model mismatch exists practically.

Fig. 2 shows the same test for the case when the nominal model $\tilde{b}=1/2(b_{\min}+b_{\max})=8$ and the actual plant parameter $\bar{b}=b_{\max}=12$, i.e. the worst possible plant on the uncertain inner-zone. Here the tuning controller parameters are selected as:

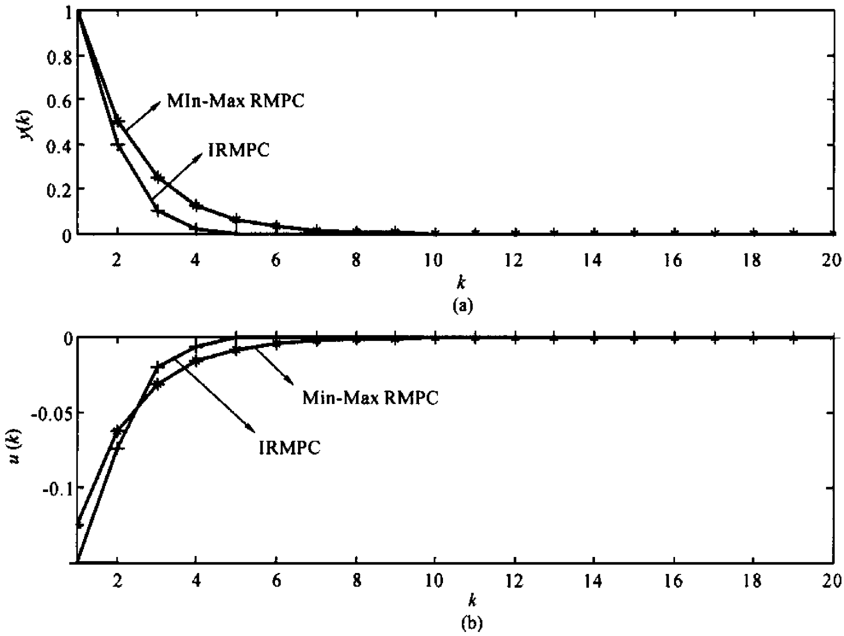

Fig. 1 Scale integrating example responses with no model mismatch

(a) actual plant state response; (b) actual plan input

"+—": IRMPC-based response curves; "*—": Min-Max RMPC-based response curves

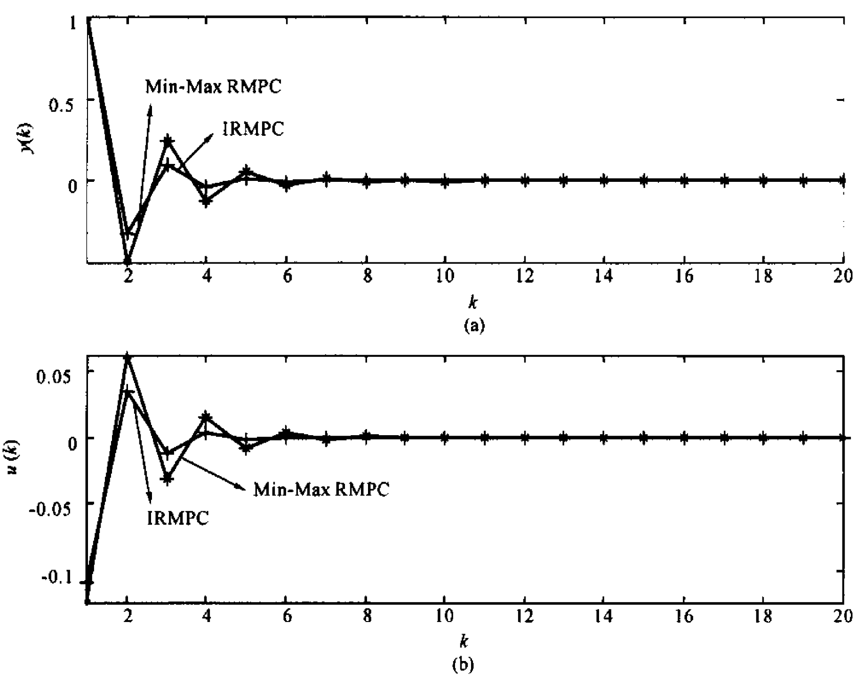

Fig. 2 Scale integrating example responses with worst model mismatch

(a) actual plant state response; (b) actual plan input

"+—": IRMPC-based response curves; "*—": Min-Max RMPC-based response curves

$$Q=1;\ R=10;\ m=3;\ \alpha=0.8 \tag{46}$$

Again, both the controllers also bring the state to the origin and IRMPC algorithm still gives better performance than Min-Max RMPC.

In addition, from the numerical computation point of view, IRMPC replaces the muti_objective nonlinear optimization proposition, which is involved in the conventional Min-Max RMPC algorithm, with a conventional convex nonlinear programming problem so that it is more computationally efficient.

CONCLUSIONS

A two-stage robust MPC algorithm (IRMPC) has been presented for uncertain integrating systems subject to input constraints. Through introducing a contraction factor to the robust constraint of the first-stage optimization, the global asymptotical convergence of the closed-loop system can be guaranteed under mild assumption. Simulation results for a scale uncertain integrating system showed that the algorithm performs well when compared with conventional Min-Max RMPC algorithms. Further research will be carried out on the disturbance rejection property of the proposed algorithm.

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