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Study of interaction between shock wave and unsteady boundary layer*

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Abstract: This paper reports theoretical and experimental study of a new type of interaction of a moving shock wave with an unsteady boundary layer. This type of shock wave-boundary layer interaction describes a moving shock wave interaction with an unsteady boundary layer induced by another shock wave and a rarefaction wave. So it is different from the interaction of a stationary shock wave with steady boundary layer, also different from the interaction of a reflected moving shock wave at the end of a shock tube with unsteady boundary layer induced by an incident shock. Geometrical shock dynamics is used for the theoretical analysis of the shock wave-unsteady boundary layer interaction, and a double-driver shock tube with a rarefaction wave bursting diaphragm is used for the experimental investigation in this work.

Key words: Moving shock wave, Unsteady boundary layer, Interaction
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INTRODUCTION

Shock wave reflection from a wall surface, diffraction along a curved surface, propagation through a channel or a tube with varying cross-sectional area etc, under the condition of moving gases ahead of the shock waves comprise a type of widespread shock dynamics phenomena. In these phenomena mentioned above, a new type of shock wave-unsteady boundary layer interaction is concerned, which is different from conventional shock wave-boundary layer interaction including steady and unsteady flows with inverse pressure gradient. The new type shock wave-boundary layer interaction is very important for the reflection of shock wave from surfaces and diffraction of shock waves around a curved wall with flows ahead of them, because in this case, unsteady boundary layer or mixed boundary layer (from steady and unsteady boundary layer interaction) may influence the behaviour and wave pattern of incident and reflected shock waves. The new type of shock wave-boundary layer interaction can be divided into two cases.

The first case is the interaction of a moving shock wave with an unsteady boundary layer induced by another shock wave and a rarefaction wave. This interaction can be usually studied in a double driver shock tube. Up to now, the double driver shock can be tested in electrically controlled bursting diaphragm mode (Han et al., 1982; 1985; Yang et al., 1994) or rarefaction wave bursting diaphragm mode (Ruetenik et al., 1966; Dong et al., 2000; 2002) for synchronization of two shock waves; In the two shock waves, the first shock and rarefaction wave is to induce an unsteady boundary layer, and the second shock is to make an interaction with the unsteady boundary layer. The second case is more complicated and usually involves the phenomena of shock reflection, shock diffraction, and shock propagation under the condition of moving gases ahead of the shock waves. In this case, an interaction of a shock wave with a mixed boundary layer occurs. The mixed boundary layer is formed by the interaction of an

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unsteady boundary layer induced by the first shock with a steady boundary layer formed from the tip of a body (plate, wedge, cone, and so on).

The first case is basic, and this paper deals only with a moving shock wave-unsteady boundary layer interaction. Geometrical shock dynamics is used for theoretically analyzing the shock wave-unsteady boundary layer interaction. The interaction can be regarded as a diffraction of a moving shock wave around a curved wall, which results from the unsteady boundary layer. A double driver shock tube with rarefaction wave bursting diaphragm was used for experimentally producing the shock wave-unsteady boundary layer interaction.

INTERACTION OF SHOCK WAVE WITH UNSTEADY BOUNDARY LAYER

The interaction of a shock wave with an unsteady boundary layer can be simulated by using a double driver shock tube with rarefaction wave bursting diaphragm as shown in Fig. 1. In the double driver shock tube diaphragm D_1 is first ruptured, and the first shock wave, rarefaction wave as well as the induced unsteady boundary layer are formed. When the rarefaction wave arrives at the location of the second diaphragm, diaphragm D_2 is ruptured, and the second shock wave is formed. At the same time, the interaction of the second shock wave with the unsteady boundary layer occurs.

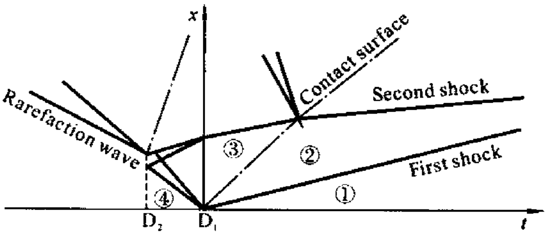

Fig.1 The physical figures of double driver shock tube with a rarefaction bursting diaphragm
(①, ②, ③, and ④ are four regions; D_1 and D_2 are two diaphragms)

It is evident that in the interaction of the shock wave with the unsteady boundary layer above, the direction of pressure gradient of shock wave is the same as the flow direction in the boundary layer, which is different from conventional shock wave-boundary layer interaction with an inverse pressure gradient.

THEORETICAL ANALYSIS OF THE INTERACTION

The physical figures of an unsteady boundary layer induced by a shock wave and a rarefaction wave after bursting a diaphragm in a shock tube is shown in Fig. 1. The shock velocity is W_{S1}, the flow velocities outside the boundary layer in regions ② and ③ are equal ($u_2 = u_3$), and the velocity of the head of rarefaction wave is a_4. If a coordinate system is attached to the first shock wave, in this coordinate system the first shock is at rest, the flow velocity ahead of the shock is equal to W_{S1}, the wall surface moves leftward with the velocity W_{S1}, and the velocities in regions ② and ③ are ($W_{S1} - u_2$) and ($W_{S1} - u_3$), respectively. Obviously, ($W_{S1} - u_2$) = ($W_{S1} - u_3$).

The distribution of flow velocity in the unsteady boundary layer is different from the case in steady flow, so the displacement thickness of the unsteady boundary layer is negative, and the wall surface in the shock tube becomes concave. Now when the second shock in the double driver shock tube propagates through this region, the curved wall will disturb it. The channel first diverges, and then converges, so the second shock is disturbed first by shock-expansion, and then by shock-compression.

If the time at which the second shock impinges the head of rarefaction wave is regarded as the initial time t_0, the positions of the second shock and the unsteady boundary layer at t_0, t_1, t_2, t_3 and t_4 can be illustrated (Dong et al., 2002). After superposing the curves of displacement thickness of the unsteady boundary layer at different time, it can be seen that in fact, the second shock wave diffracts around a curved wall as shown in Fig. 2. It follows from Fig. 2 that the shock-expansion, which disturbs the second shock wave, is strong; the shock-compression disturbs the second shock wave gradually. So the second shock wave curves backward strongly, and then curves forward weakly.

In order to calculate the shape of second shock wave disturbed by the curved wall, the

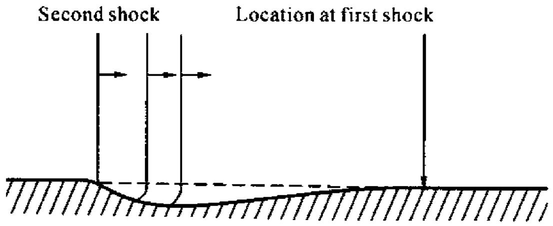

Fig. 2 **The diffraction of the second shock wave around the curved wall formed by the superposition of the unsteady boundary layer at different time**

expressions that describe relations among first shock wave, rarefaction waves, second shock wave, and unsteady boundary layer, under the condition of the frame being attached to the first shock, are given as follows:

The flow velocity in regions ② and ③ is

$$V_2 = V_3 = W_{S1} - u_2 = W_{S1} - u_3 \qquad (1)$$

The second shock velocity relative to the first shock velocity is

$$V_{S2} = W_{S2} + u_3 - W_{S1} \qquad (2)$$

The velocity of the head of the rarefaction wave is

$$V_r = W_{S1} + a_4 \qquad (3)$$

where a_4 is the speed of sound in the region ④.

For convenience in analysis, we assume that the first shock wave is formed immediately when diaphragm D_1 is ruptured, also assume that the second shock is formed when the rarefaction wave impinges to diaphragm D_2.

The time interval that the rarefaction wave arrives at the location of D_2 from the location of D_1 after D_1 is ruptured is

$$t_r = L_2 / a_4 \qquad (4)$$

where L_2 is the length of the intermediate section.

The maximum distance between first and second shock waves when the second shock begins to be disturbed is

$$L_{SS} = (W_{S1} + a_4) t_r \qquad (5)$$

The time needed for the second shock to overtake the first shock is

$$t_{SS} = \frac{L_{SS}}{V_{S2}} = \frac{(W_{S1} + a_4) L_2}{(W_{S2} + u_3 - W_{S1}) a_4} \qquad (6)$$

The length of the unsteady boundary layer when diaphragm D_2 is ruptured by the rarefaction wave is

$$L_{b0} = V_r t_r = (W_{S1} + a_4) L_2 / a_4 \qquad (7)$$

The length of the unsteady boundary layer when the second shock overtakes the first shock is

$$L_{bf} = V_r t_{SS} = \frac{(W_{S1} + a_4)^2 L_2}{(W_{S2} + u_2 - W_{St}) a_4} \qquad (8)$$

If both driven section and intermediate section are filled with the same gas, Eqs. (7) and (8) can be rewritten as

$$L_{b0} = (W_{S1} + 1) L_2 \qquad (9)$$

$$L_{bf} = \frac{(M_{S1} + 1)}{(W_{S2} + u_3)/a_4 - M_{S1}} \qquad (10)$$

According to the geometrical shock dynamic theory, the problem mentioned above belongs to the diffraction of a moving shock wave around a curved wall with a uniform gas flow ahead of the shock wave. The shock dynamic equations in this condition (Han et al. 1989b, 1993) can be expressed as follows

$$\frac{\partial}{\partial x}\left[\frac{M\cos\theta + m}{(M + m\cos\theta)A'}\right] + \frac{\partial}{\partial y}\left[\frac{M\sin\theta}{(M + m\cos\theta)A'}\right] = 0 \qquad (11a)$$

$$\frac{\partial}{\partial x}\left[\frac{\sin\theta}{M + m\cos\theta}\right] - \frac{\partial}{\partial y}\left[\frac{\cos\theta}{M + m\cos\theta}\right] = 0 \qquad (11b)$$

$$\frac{dA'}{dM} = \frac{2M}{(M^2 - 1)K(M)} \qquad (11c)$$

where M and m are shock Mach number (in the present case, for second shock) and flow Mach number in the region ahead of the second shock, respectively, that is, $M = M_{S2}$, $m = (W_{S1} - u_2)/a_2$ or $m = (W_{S1} - u_3)/a_3$. θ is the angle between the normal to the second shock and x-axis. $K(M)$ is a slowly varying function. A' is the cross-sectional area of the ray tube in the frame which is attached to the flow ahead of the second shock.

The characteristic relations of the Eqs. (11a, 11b, 11c) can be expressed as

$$\theta + \int_1^M \left[\frac{2}{(M^2 - 1)K(M)}\right]^{1/2} \mathrm{d}M = K_1 \quad (12a)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \tan(\theta + \nu_1) \quad (12b)$$

$$\theta - \int_1^M \left[\frac{2}{(M^2 - 1)K(M)}\right]^{1/2} \mathrm{d}M = K_2 \quad (12c)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \tan(\theta + \nu_2) \quad (12d)$$

where

$$\tan\nu_1 = \frac{\left[\frac{1}{2}(M^2 - 1)K(M)\right]^{1/2} - m\sin\theta}{M + m\cos\theta}$$

$$\tan\nu_2 = \frac{\left[\frac{1}{2}(M^2 - 1)K(M)\right]^{1/2} + m\sin\theta}{M + m\cos\theta}$$

The Eqs. (12a, 12b) represent upward disturbance waves on the surface of shock wave; the Eqs. (12c, 12d) represent downward disturbance waves. The shock wave which is disturbed by the curved wall can be divided into three regions: in the first region, the disturbance wave has not arrived, the shock wave is plane; in the second region, the shock wave is disturbed by only upward or downward wave, but not both; this is a simple wave region; in the third region, the shock wave is disturbed by both of upward and downward waves simultaneously, this is a double wave region.

Under the condition of uniform flow ahead of the shock, the simple wave relations for the upward wave can be expressed as

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \tan(\theta + v_1) \quad (13a)$$

$$\tan\nu_1 = \frac{\left[\frac{1}{2}(M^2 - 1)K(M)\right] - m\sin\theta}{M + m\cos\theta} \quad (13b)$$

$$\theta = \int_{M_0}^M \left[\frac{2}{(M^2 - 1)K(M)}\right]^{1/2} \mathrm{d}M \quad (13c)$$

In the double wave region, according to the given boundary and initial conditions, and the shock dynamic equations, we can obtain the strength, orientation, and shape of the second shock.

EXPERIMENTAL INVESTIGATION IN A DOUBLE DRIVER SHOCK TUBE

The wave pattern in a double driver shock tube with a rarefaction wave bursting diaphragm is shown in Fig. 1. The double driver shock tube with square cross-section is divided into six parts, the driver section, intermediate section, driven section, test section, extension section, and vacuum tank. The driver section is 2.00 m long, the intermediate section is 0.35 m long, the driven section is 3.00 m long, the test section is 1.00 m long, and the extension section is 2.00 m long. The extension section is used to prevent the reflection wave of the first shock at the entrance of the vacuum tank from entering the test section. The cross-sectional area of the shock tube is 94 × 94 mm^2. The driver section, intermediate section, and driven section are filled with air. There is a single mylar diaphragm (D_2) between driver section and intermediate section. In order to control the pressure difference of bursting diaphragm between driven section and intermediate section, a mechanism with double diaphragms was used. This mechanism (for convenience, we call it D_1) can give more accurate pressure difference. Three different thickness mylar diaphragms, 0.025 mm, 0.05 mm, 0.1 mm, were used for different testing conditions (Dong et al., 2000). A Schlieren optical system was used for flow visualization, and piezoelectric crystal transducer was used for shock speed measurement and transient pressure measurement.

The Schlieren photograph and pressure waveforms of the first and secood shocks in the above conditions are given in Fig. 3. It follows from Fig. 3 that the disturbed second shock bends backward in the vicinity of wall surface.

CONCLUSIONS

1. The shock dynamic theory was used for analyzing the new type of shock wave-unsteady boundary layer interaction. The interaction problem can be regarded as the diffraction of a shock wave along a curved wall surface formed by the negative displacement thickness of an unsteady boundary layer.

2. The double driver shock tube with rarefaction wave bursting diaphragm can be used for simulating the interaction of shock wave with unsteady boundary layer. The experimental results showed that this facility is useful for study of

such kind of shock dynamic phenomena.

3. Both theoretical analysis and experimental results showed the same tendency of change in shape of shock wave disturbed by boundary layer.

4. The results of theoretical analysis and experimental investigation of the interaction of a shock wave with an unsteady boundary layer reported in this paper are only preliminary. There is still a lot of work for us to do regarding the new type of shock wave-unsteady boundary layer interaction.

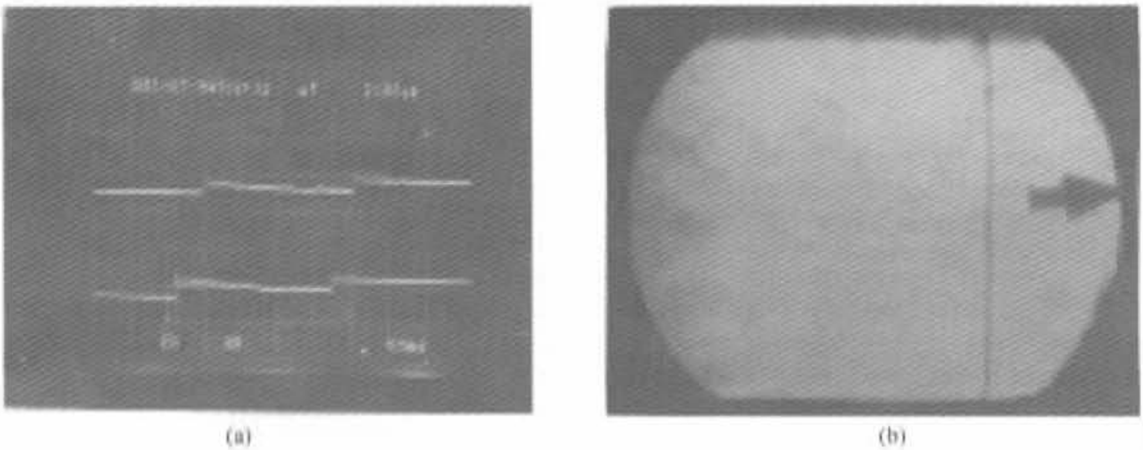
(a) (b)

Fig.3 The interaction of second shock with unsteady boundary layer
(a) the first shock; (b) the second shock

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