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Three dimensional Couette flow and heat transfer through a porous medium with variable permeability

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Abstract: This paper reports research on the effects of variations in injection velocity and permeability on the heat transfer and flow through a highly porous medium between two horizontal parallel plates situated at constant distance with constant suction by the upper plate. Due to this type of variation in injection velocity and in permeability the flow becomes three dimensional. The governing equations are solved by adopting complex variable notations to obtain the expressions for the velocity and temperature field. The skin-friction along the main flow direction and rate of heat transfer are discussed with the help of graphs.

Key words: Couette flow, Porous medium, Heat transfer, Permeability injection/suction.
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INTRODUCTION

The flow of a viscous liquid over and through a porous medium has been the subject of intensive studies in recent years because of its natural occurrence in the movement of water and oil inside the earth and the flow of rivers through porous banks. It has applications in many engineering and biomedical problems. In view of the geophysical applications the flows through porous medium, a series of investigations conducted by Raptis et al. (1981a; 1981b; 1982) into the steady flow past a vertical wall. Raptis (1983) and Raptis et al. (1985) further studied the unsteady free convective flow through highly porous medium bounded by an infinite porous plate. Recently Singh et.al (1983) studied a free convective flow through a porous medium with periodic permeability variation. Assuming periodic permeability variation Singh et al.(1995a) studied the oscillatory flow through a porous medium. Singh et. al (1995b) also studied the effect of transverse periodic variation of the permeability on the steady flow and heat transfer. Singh (1999) studied three dimensional couette flow with transpiration cooling. Also Singh et al. (2000) studied the three dimensional fluctuating flow and heat transfer through a porous medium with variable permeability. However, to the best of the author's knowledge, the applications of the variations of injection velocity and permeability on three dimensional Couette flow has not yet appeared in the literature. Therefore the object of this paper is to investigate the simultaneous effects of injection and permeability variations on three dimensional Couette flow and heat transfer through porous variable permeability medium.

GOVERNING EQUATIONS

An unsteady Couette flow of a viscous incompressible fluid between two parallel flat highly porous plates is considered. The upper plate in uniform motion U is subjected to a constant suction V_0 and the lower to a variation of injection velocity of the form:

$$V^*(z^*, t^*) = V_0[1 + \varepsilon\cos(\pi z^*/d - \omega^* t^*)] \tag{1}$$

Also the permeability variations of the porous medium is of the form

$$k^*(z^*, t^*) = k_0^*/[1 + \varepsilon\cos(\pi z^*/d - \omega^* t^*)] \tag{2}$$

Where k_0^* is the mean permeability of the medium, ω^* is the frequency of fluctuations, t^* is the time and ε is a positive constant quantity ($<<1$). Without any loss of generality the distance d between the plates equals to the wave length of injection velocity and permeability distribution. The lower and upper plates are assumed to be at constant temperature T_0 and T_1 respectively, with $T_1 > T_0$. All physical quantities are independent of x^* for this problem of fully developed laminar flow but the flow remains three dimensional due to the variations of injection velocity and permeability of the porous medium.

Thus, denoting velocity components u, v, w in the x, y, z directions, respectively and the temperature by θ, the flow through a highly porous medium is governed by the following non-dimensional equations:

$$v_y + w_z = 0, \tag{3}$$

$$\omega u_t/\lambda + v u_y + w u_z = (u_{yy} + u_{zz})/\lambda - \lambda(u - 1)/k(z,t), \tag{4}$$

$$\omega v_t/\lambda + v v_y + w v_z = -p_y + (v_{yy} + v_{zz})/\lambda - \lambda v/k(z,t), \tag{5}$$

$$\omega w_t/\lambda + v w_y + w w_z = -p_z + (w_{yy} + w_{zz})/\lambda - \lambda w/k(z,t), \tag{6}$$

$$\omega\theta_t/\lambda + v\theta_y + w\theta_z = (\theta_{yy} + \ldots\theta_{zz})/\lambda Pr \tag{7}$$

where $k(z,t) = k_0/[1 + \varepsilon\cos(\pi z - t)]$,
k_0(permeability parameter) $= k_0{}^* V_0{}^2/\nu^2$
$y = y^*/d$, $z = z^*/d$, $t = \omega^* t^*$,
$\omega = \omega^* d^2/\nu$, $p = p^*/\rho V_0^2$
$u = u^*/U$, $v = v^*/V_o$, $w = w^*/V_0$,
P_r(Prandtl number) $= \nu/\alpha$,
λ(Injection parameter) $= V_0 d/\nu$, $\theta = (T^* - T_0)/(T_1 - T_0)$
are the dimensionless quantity and ρ, ν, α and p are respectively , density, kinematic viscosity, thermal diffusivity and pressure. The (*) stands for the dimensional quantities. The boundary conditions of the problem, in dimensionless form, are:

$$\left.\begin{array}{l} y = 0:\ u = 0,\ v(z) = 1 + \varepsilon\cos(\pi z - t), \\ \quad w = 0,\ \theta = 0 \\ y = 1:\ u = 1,\ v = 1,\ w = 0,\ \theta = 1 \end{array}\right\} \tag{8}$$

METHOD OF SOLUTION

Since the amplitude injection velocity variation and permeability variation $\varepsilon(<<1)$ is very small, we now assume the solution of the following form:

$$f(y,z,t) = f_0(y) + \varepsilon f_1(y,z,t) + \varepsilon^2 f_2(y,z,t) + \cdots \tag{9}$$

where f stands for u, v, w, p and θ. When $\varepsilon = 0$ the problem is reduced to the well known two dimensional flow with constant injection/suction and constant permeability. In this case Eqs.(3) to (7) reduce to

$$v_{0y} = 0 \tag{10}$$

$$u_{y0} - \lambda v_0 u_{0y} - \lambda^2 u_0/k_0 = -\lambda^2/k_0 \tag{11}$$

$$\theta_{0yy} - v_0\lambda Pr\theta_{0y} = 0 \tag{12}$$

The corresponding boundary conditions become

$$\left.\begin{array}{l} y = 0:\ u_0 = 0,\ v_0 = 1,\ \theta_0 = 0 \\ y = 1:\ u_0 = 1,\ v_0 = 1,\ \theta_0 = 1 \end{array}\right\} \tag{13}$$

The solutions of Eqs.(10) to (12) under the boundary condition Eq.(13) are

$$\left.\begin{array}{l} u_0(y) = 1 + [e^{(\beta+\alpha y)} - e^{(\alpha+\beta y)}]/(e^\alpha - e^\beta), \\ \quad v_0 = 1, w_0 = 0 \\ \theta_0(y) = (e^{\lambda Pry} - 1)/(e^{\lambda Pry} - 1),\ p_0 = \text{constant}, \end{array}\right\} \tag{14}$$

where $\alpha = \lambda[1 + (1 + 4/k_0)^{1/2}]/2$,
$\beta = \lambda[1 - (1 + 4/k_0)^{1/2}]/2$.

When $\varepsilon \neq 0$, substituting Eq.(9) in Eqs.(3) to (7) and comparing the coefficients of identical power of ε, neglecting those of ε^2, ε^3, etc., the following first order equations are obtained with the help of solution Eq.(14)

$$v_{1y} + w_{1z} = 0 \tag{15}$$

$$\omega u_{1t}/\lambda + v_1 u_{0y} + u_{1y} = (u_{1yy} + u_{1zz})/\lambda - \lambda\{u_1 + (u_0 - 1)\cos(\pi z - t)\}/k_0 \tag{16}$$

$$\omega v_{1t}/\lambda + v_{1y} = -p_{1y} + (v_{1yy} + v_{1zz})/\lambda - \lambda\{v_1 + \cos(\pi z - t)\}/k_0 \tag{17}$$

$$\omega w_{1t}/\lambda + w_{1y} = -p_{1z} + (w_{1yy} + w_{1zz})/\lambda - \lambda w_1/k_0 \tag{18}$$

$$\omega\theta_{1t}/\lambda + v_1\theta_{0y} + \theta_{1y} = (\theta_{1yy} + \theta_{1zz})/Pr\lambda \tag{19}$$

The corresponding boundary conditions become

$$\left.\begin{array}{l} y = 0:\ u_1 = 0,\ v_1 = \cos(\pi z - t), w_1 = 0, \theta_1 = 0 \\ y = 1:\ u_1 = 0, v_1 = 0, w_1 = 0, \theta_1 = 0 \end{array}\right\} \tag{20}$$

The partial differentional Eqs. (15) to (19) describe the three dimensional flow. The solution of these equations will be obtained in complex variable notations, the real part of which will have physical significance. Thus we write u_1, v_1, w_1, θ_1 and p_1 as

$$\left.\begin{aligned} u_1(y,z,t) &= u_{11}(y)e^{i(\pi z - t)}, \\ v_1(y,z,t) &= v_{11}(y)e^{i(\pi z - t)}, \\ w_1(y,z,t) &= -v_{11}(y)e^{i(\pi z - t)}/i\pi \\ p_1(y,z,t) &= p_{11}(y)e^{i(\pi z - t)}, \\ \theta_1(y,z,t) &= \theta_{11}(y)e^{i(\pi z - t)} \end{aligned}\right\} \quad (21)$$

The forms of the cross flow velocities $v_1(y,z,t)$ and $w_1(y,z,t)$ in Eq.(21) are so chosen that the Eq. continuity Eq.(15) is satisfied, substituting the Eq.(21) into Eqs.(16) to (19) reduces them to ordinary differentional equations:

$$u_{11yy} - \lambda u_{11y} - (\pi^2 + \lambda^2/k_0 - i\omega)\, u_{11} = \lambda v_{11} u_{0y} + \lambda^2(u_0 - 1)/k_0 \quad (22)$$

$$v_{11yy} - \lambda v_{11y} - (\pi^2 + \lambda^2/k_0 - i\omega) v_{11} = \lambda p_{11y} + \lambda^2/k_0 \quad (23)$$

$$v_{11yyy} - \lambda v_{11yy} - (\pi^2 + \lambda^2/k_0 - i\omega)\, v_{11y} = \lambda \pi^2 p_{11} \quad (24)$$

$$\theta_{11yy} - \lambda Pr\theta_{11y} - (\pi^2 - i\omega Pr)\theta_{11} = \lambda Pr v_{11}\theta_{0y} \quad (25)$$

Solving these equations under the corresponding transformed boundary conditions, we get the following expressions for u_1, v_1, w_1, p_1 and θ_1 as:

$$u_1 = [Re^{\gamma y} + Se^{\delta y} + \lambda\{A_1(\alpha e^{\beta}e^{(\alpha+\gamma)y}/c_1 - \beta e^{\alpha}e^{(\beta+\gamma)y}/c_2) + A_2(\alpha e^{\beta}e^{(\alpha+\delta)y}/c_3 - \beta e^{\alpha}e^{(\beta+\alpha)y}/c_4) - A_3(\alpha e^{\beta}e^{(\alpha+\pi)y}/c_5 - \beta e^{\alpha}e^{(\beta+\pi)y}/c_6) - A_4(\alpha e^{\beta}e^{(\alpha-\pi)y}/c_7 - \beta e^{\alpha}e^{(\beta-\pi)y}/c_8) - AB(\alpha e^{\beta}e^{\alpha y} - \beta e^{\alpha}e^{\beta y})/c_9\}/\{A(e^{\alpha} - e^{\beta})\} - \lambda^2(e^{(\beta+\alpha y)} - e^{(\alpha+\beta y)}/\{k_0 c_9(e^{\alpha} - e^{\beta})\}]e^{i(\pi z - t)} \quad (26)$$

$$v_1 = [A_1 e^{\gamma y} + A_2 e^{\delta y} - A_3 e^{\pi y} - A_4 e^{-\pi y} + AB]\cdot e^{i(\pi z - t)}/A \quad (27)$$

$$w_1 = -[A_1\gamma e^{\gamma y} + A_2\delta e^{\delta y} - A_3\pi e^{\pi y} + A_4\pi e^{-\pi y}]e^{i(\pi z - t)}/Ai\pi \quad (28)$$

$$p_1 = [A_3\{\lambda\pi + (\lambda^2/k_0 - i\omega)\} + A_4\{\lambda\pi - (\lambda^2/k_0 - i\omega)\}]e^{i(\pi z - t)}/A\pi\lambda \quad (29)$$

$$\theta_1 = [Te^{\xi y} + Me^{\eta y} + \lambda^2 Pr^2\{A_1 e^{(\gamma + Pr\lambda)y}/c_{10} + A_2 e^{(\delta + Pr\lambda)y}/c_{11} - A_3 e^{(\pi + Pr\lambda)y}/c_{12}Pr + A_4 e^{(Pr\lambda - \pi)y}/c_{13}Pr + ABe^{Pr\lambda y}/c_{14}\}/\{A(e^{\lambda Pr} - 1)\}]e^{i(\pi z - t)} \quad (30)$$

where

$$\gamma = [\lambda + (\lambda^2 + 4\pi^2 + 4\lambda^2/k_0 - 4i\omega)^{1/2}]/2$$

$$\delta = [\lambda - (\lambda^2 + 4\pi^2 + 4\lambda^2/k_0 - 4i\omega)^{1/2}]/2$$

$$A = 2(\delta - \gamma)(1 + e^{(\gamma+\delta)}) + (\pi + \gamma - \delta - \gamma\delta/\pi)\cdot(e^{(\gamma-\pi)} + e^{(\delta+\pi)}) - (\pi - \gamma + \delta - \gamma\delta/\pi)(e^{(\gamma+\pi)} + e^{(\delta-\pi)})$$

$$A_1 = (\pi - \delta)(1 - B)e^{(\delta+\pi)} - (\pi + \delta)(1 - B)\cdot e^{(\delta-\pi)} + 2\delta(1 - B) - (\pi - \delta)Be^{-\pi} + (\pi + \delta)\cdot Be^{\pi} - 2B\delta e^{\delta}$$

$$A_2 = (\pi + \gamma)(1 - B)e^{(\gamma-\pi)} - (\pi - \gamma)(1 - B)\cdot e^{(\gamma+\pi)} - 2\gamma(1 - B) + (\pi - \gamma)Be^{-\pi} - (\pi + \gamma)\cdot Be^{\pi} + 2B\gamma e^{\gamma}$$

$$A_3 = (\gamma - \delta)(1 - B)e^{(\gamma+\delta)} + (\delta + \gamma\delta/\pi)(1 - B)\cdot e^{(\gamma-\pi)} - (\gamma + \gamma\delta/\pi)(1 - B)e^{(\delta-\pi)} - B(e^{\delta} - e^{-\pi})(\delta + \gamma\delta/\pi) + B(e^{\gamma} - e^{-\pi})(\gamma + \gamma\delta/\pi)$$

$$A_4 = (\gamma - \delta)(1 - B)e^{(\gamma+\delta)} + (\delta - \gamma\delta/\pi)(1 - B)e^{(\gamma+\pi)} - (\gamma - \gamma\delta/\pi)(1 - B)e^{(\delta+\pi)} + B(e^{\delta} - e^{\pi})(\gamma\delta/\pi - \delta) + B(e^{\gamma} - e^{\pi})(\gamma - \gamma\delta/\pi)$$

$$B = -\lambda^2(\pi^2 + \lambda^2/k_0)/[k_0\{(\pi^2 + \lambda^2/k_0)^2 + \omega^2\}]$$

$$\xi = [\lambda Pr + (\lambda^2 Pr^2 + 4\pi^2 - 4i\omega Pr)^{1/2}]/2$$

$$\eta = [\lambda Pr - (\lambda^2 Pr^2 + 4\pi^2 - 4i\omega Pr)^{1/2}]/2$$

$$c_1 = (2\alpha\gamma + \lambda^2/k_0),\quad c_2 = (2\beta\gamma + \gamma^2/k_0)$$

$$c_3 = (2\alpha\delta + \lambda^2/k_0),\quad c_4 = (2\beta\delta + \gamma^2/k_0)$$

$$c_5 = (2\pi\alpha - \lambda\pi + i\omega),\ c_6 = (2\pi\beta - \lambda\pi + i\omega)$$

$$c_7 = (\lambda\pi - 2\pi\alpha + i\omega),\ c_8 = (\lambda\pi - 2\pi\beta + i\omega)$$

$$c_9 = (\pi^2 - i\omega),\quad c_{10} = \lambda\gamma(Pr + 1) + i\omega(Pr - 1) + \lambda^2/k_0$$

$$c_{11} = \lambda\delta(Pr + 1) + i\omega(Pr - 1) + \lambda^2/k_0$$

$$c_{12} = (\pi\lambda + i\omega),\ c_{13} = (\pi\lambda - i\omega),\ c_{14} = (i\omega Pr - \pi^2)$$

The coefficients R, S, T and M in Eqs.(26), (30) respectively are obtained by using transformed boundary conditions but are not presented here for the sake of brevity.

RESULTS AND DISCUSSIONS

The important characteristics of the problem are the shear stress and the rate of heat transfer at the plates. Knowing the velocity field, we can obtain the component of shear in the non-dimensional form in the main flow direction as

$$\tau_x = \tau_x^* / UV_{0\rho} = (1/\lambda) \text{ real part of } (\partial u/\partial y)_{y=0} = \tau_1 + \varepsilon |L| \cos(\pi z - t + \varphi_1)) \quad (31)$$

where τ_1 is the sinusoidal skin-friction and $|L|$ is the amplitude of skin-friction in the main flow direction as given by

$$\tau_1 = (\alpha e^{\beta} - \beta e^{\alpha})/\lambda(e^{\alpha} - e^{\beta}), |L| = (L_r^2 + L_i^2)^{1/2},$$
$$\tan \varphi_1 = L_i / L_r$$
$$L = L_r + iL_i = (R_\gamma + S\delta)/\lambda + [A_1 \{\alpha e^{\beta}(\alpha + \gamma)/c_1 - \beta e^{\alpha}(\beta + \gamma)/c_2\} + A_2 \{\alpha e^{\beta}(\alpha + \delta)/c_3 - \beta e^{\alpha}(\beta + \delta)/c_4\} - A_3 \{\alpha e^{\beta}(\alpha + \pi)/c_5 - \beta e^{\alpha}(\beta + \pi)/c_6\} - A_4 \{\alpha e^{\beta}(\alpha - \pi)/c_7 - \beta e^{\alpha}(\beta - \pi)/c_8\} - AB(\alpha^2 e^{\beta} - \beta^2 e^{\alpha})/c_9] / \{A(e^{\alpha} - e^{\beta})\} - \lambda(\alpha e^{\beta} - \beta e^{\alpha}) / \{k_0 c_9 (e^{\alpha} - e^{\beta})\} \quad (32)$$

The amplitude $|L|$, of the skin-friction component in the main flow direction shown in Fig.1, It is observed that it decreases with the

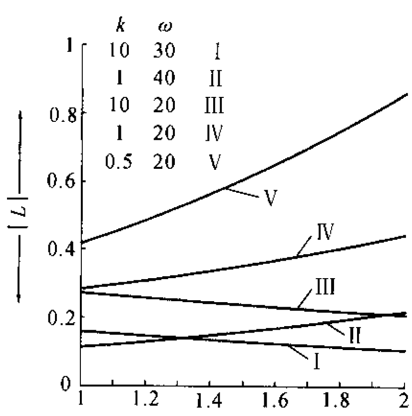

Fig.1 The amplitude, $|L|$, of the skin friction in the main flow direction

increase of the permeability of the porous medium k or the increase of the frequency ω of permeability and injection fluctuations . The phase $\tan \phi_1$ of the skin-friction component in the main flow direction is presented in Fig.2 showing that for small values of injection parameter ($\lambda = 0.5$), there is a phase lag and for $\lambda = 1$, there is a phase lead. The phase lag decreases with the increase of the permeability of the medium while phase lead decreases. The phase lead of skin-friction increases with ω. Fig.3 shows the sinusoidal skin-friction. It is noticed that it decreases with increasing permeability parameter and injection parameter; and that the sinusoidal skin-friction increases with decreasing ω.

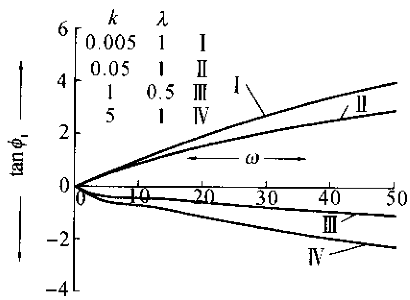

Fig.2 The tangent of phase angle $\tan \phi_1$ of main flow skin friction

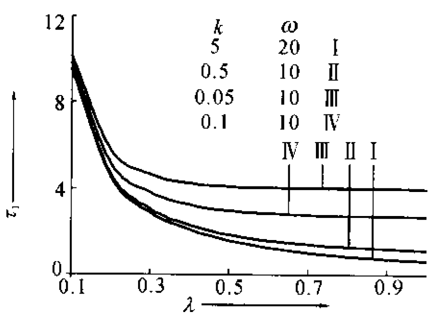

Fig.3 Sinusoidal skin-friction τ_1, in main flow direction

From the temperature field, the rate of heat transfer in term of the Nusselt Number can be obtained as:

$$Nu = q_w^* / \rho V_0 C_p (T_1 - T_0) = -\{k / d_\rho V_0 C_p\}(\partial\theta/\partial y)_{y=0} = 1/(e^{\lambda Pr} - 1) + \varepsilon |H| \cos(\pi z - t + \varphi_2) \quad (33)$$

$$|H| = (H_r^2 + H_i^2)^{1/2}, \tan \varphi_2 = H_i / H_r$$
$$H = H_r + iH_i = (T\xi + M\eta)/\lambda Pr + \lambda Pr[A_1(\gamma + Pr\lambda)/c_{10} + A_2(\delta + Pr\lambda)/c_{11} - A_3(\pi + Pr\lambda)/c_{12} Pr + A_4(Pr\lambda - \pi)/c_{13} Pr + ABPr\lambda/c_{14}] / [A(e^{\lambda Pr} - 1)] \quad (34)$$

The amplitude $|H|$ of rate of heat transfer is shown in Fig.4 and Fig.5 for $Pr = 7$ and $Pr = 0.71$ respectively. Fig.4 reveals that amplitude decreases with increasing injection parameter. The values of amplitude $|H|$ are lower when we increase the frequency of fluctuation. It is interesting to note that the amplitude $|H|$, of rate of heat transfer vanishes for higher injection rate. Fig. 5 shows that amplitude increases with increase of permeability parameter k and decreases with increase in ω. In the case of air ($Pr = 0.71$) for higher injection rate, the magnitude decreases. It is interesting to note that higher injection rate has no influence for the case of higher fluctuating frequency $|\omega|$. The values of the amplitude $|H|$ are lower in the case of water ($Pr = 7$) than in the case of air as evident from Fig.4 and Fig.5.

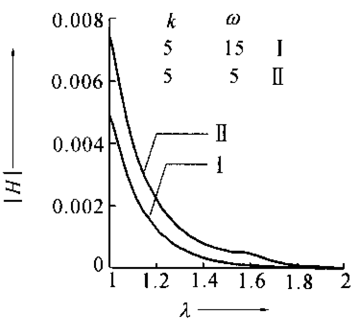

Fig.4 The amplitude $|H|$ of rate of heat transfer (For $Pr = 7$)

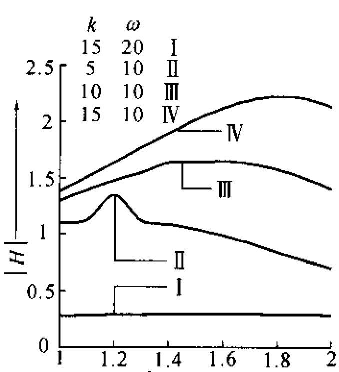

Fig.5 The amplitude $|H|$ of the rate of heat transfer (For $Pr = 0.71$)

In Fig. 6, we have plotted the tangent of phase shift tan φ_2 in Nusselt Number against ω. The figure clearly shows that there is a phase lag for $Pr = 0.71$ (air) and phase lead for $Pr = 7$ (water). It is observed that the phase lead increased with increase in λ and for small/large value of k. The phase lag is more pronounced for large λ. In the case of air the phase lag almost vanishes for large fluctuations of permeability and injection while there always remains phase lead in the case of water which goes on decreasing.

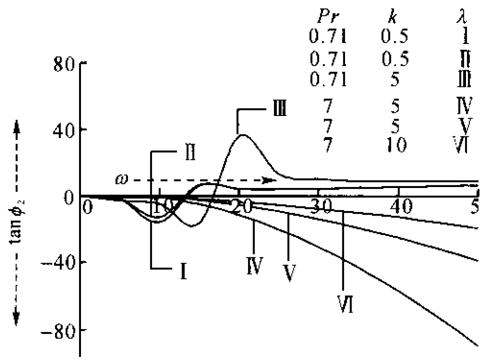

Fig. 6 The trangent of phase angle tan ϕ_2, of the heat transfer rate

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