ISSN 1009 – 3095 *Journal of Zhejiang University SCIENCE* V.4, No.2, P.195 – 201, Mar. – Apr., 2003
http://www.periodicals.com.cn; http://www.zju.edu.cn/English
http://www.zjupress.com; http://lib.zju.edu.cn/eindex.htm; jzus@zju.edu.cn

Computation of one-dimensional consolidation of double layered ground using differential quadrature method*

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Received Feb.20, 2002; revision accepted Apr.26, 2002

Abstract: The authors give the solution to the problem of one-dimensional consolidation of double-layered ground with the use of the differential quadrature method. Case studies showed that the computational results for pore-water pressure in soil layer agreed with those of analytical solution; and that in the computational results for the interface of soil layer also agreed with those of the analytical solution except for the small discrepancies during shortly after the start of computation. The advantages of the solution presented in this paper are that compared with the analytical solution, it avoids the cumbersome work in solving the transcendental equation for eigenvalues, and in the case of the Laplace transform solution, it can resolve the precision problem in the numerical solution of long time inverse Laplace transform. Because of the matrix form of the solution in this paper, it is convenient for formulating computational program for engineering practice. The formulas for calculating double-layered ground consolidation may be easily extended to the case of multi-layered soils.

Key words: Double-layered ground, One-dimensional consolidation, Differential quadrature method
Document code: A **CLC number:** TU433

INTRODUCTION

It is important to forecast and calculate the role of consolidation in strengthening soft soil ground. For convenient use in engineering, the model of consolidation was reduced to a one-dimensional problem in spatial domain. Since Terzaghi established one-dimensional linear consolidation theory and gave the corresponding analytical solution to predict the consolidation in one layered soil under constant load, many researchers and engineers made contributions to develop his theory. Wilson et al. (1974) and Baligh et al. (1978) respectively developed an analytical solution for consolidation of a soil layer subjected to a cyclic square load. Alonso et al. (1974) considered a random loading solution for consolidation. Wu et al. (1988) gave the solution under arbitrary cyclic loading using Laplace transform method. Gray (1945) considered that when two adjoining compressible strata are made to consolidate under an applied load, the behavior of each stratum is influenced by the presence and action of the other, established a one-dimensional linear consolidation model of double-layered soil and gave its analytical solution under a constant load. Xie (1994) developed an analytical solution for consolidation of double-layered ground with arbitrary distribution of initial pore-water pressure and casual variation of surface load with time; Xie et al. (1999) presented a fully explicit analytical solution for consolidation of partially drained boundaries two-layered soil subjected to a constant load.

The computation methods for one-dimensional linear consolidation of multi-layered soil based on Terzaghi's theory mainly include analytical method and Laplace transform method. For the analytical solution, because the eigen-equation deduced by variables separation becomes more complicated with increase of layer number, the solution of its eigen values is cumbersome. To get a Laplace-transformed solution, the numerical inverse Laplace transform method has to be adopted due to the difficulty in getting the direct solution.

* Project (No. RC9609) supported by the Zhejiang Provincial Natural Science Foundation of China

In this paper, the differential quadrature method (DQM from here on) based on Xie's work (1994) will be introduced for calculating the one-dimensional linear consolidation of double-layered ground. In the following section, the basic mathematical concepts underlying DQM are first presented; then the formulations for calculating the consolidation of double-layered ground by using DQM are given. Finally, several comparisons between the analytical solution and the DQM solution are presented.

DIFFERENTIAL QUADRATURE METHOD

Consider a function $\Psi(x)$ in which $0 \leqslant x \leqslant a$. Let the function values fall on a set of pre-selected sampling points $x_i(i = 1, 2, \cdots, N)$. The quadrature rules simply express the values of function derivatives at these sampling points as the linear weighted sum of the function values $\Psi_i = \Psi(x_i)$. Thus, one may write the quadrature rule for an rth-order derivative as

$$\left.\frac{\mathrm{d}^r \Psi}{\mathrm{d}x^r}\right|_{x = x_i} \cong \sum_{k=1}^{N} D_{ik}^{(r)} \Psi_k, \quad i = 1, 2, \cdots, N \tag{1}$$

where $i, k = 1, 2, \cdots, N$ and summation on the repeated index is implied. In the above equation, $D_{ik}^{(r)}$ is the rth-order derivative weighting coefficient at the ith sampling point, and $r < N$.

There are many methods for determining the derivative weighting coefficients (Bellman et al., 1972; Quan et al., 1989). In this paper, the method for determining the weighting coefficients is introduced from Quan and Changs' work (1989), in which the weighting coefficients were obtained by considering the test functions introduced in a Lagrangian process as follows.

The off-diagonal terms of the weighting coefficient matrix of the first-order derivatives are given by

$$D_{ik}^{(1)} = \frac{\prod\limits_{v=1, v \neq i}^{N} (x_i - x_v)}{(x_i - x_k) \prod\limits_{v=1, v \neq k}^{N} (x_k - x_v)} \tag{2}$$

for $i, k = 1, 2, \cdots, N$ and $k \neq i$.

The off-diagonal terms of a weighting coefficient matrix of the second- and higher-order derivatives may be obtained through the following recurrence relationship

$$D_{ik}^{(r)} = r\left[D_{ii}^{(r-1)} D_{ik}^{(1)} - \frac{D_{ik}^{(r-1)}}{x_i - x_k}\right] \tag{3}$$

for $i, k = 1, 2, \cdots, N$ and $k \neq i$ and $2 \leqslant r \leqslant N - 1$.

The diagonal terms of a weighting coefficient matrix are given by

$$D_{ii}^{(r)} = -\sum_{v=1, v \neq i}^{N} D_{iv}^{(r)} \tag{4}$$

for $i = 1, 2, \cdots, N$.

EQUATIONS OF CONSOLIDATION

Adopting Xie's model (1994) shown in Fig. 1 indicating where the coordinates and the parameters of layered soil are. The basic equation is given by

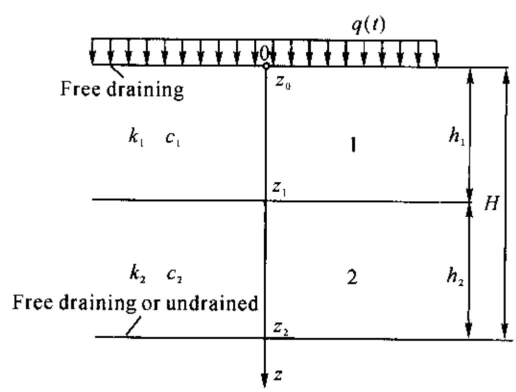

Fig.1 Double-layered ground

$$\frac{\partial u^i}{\partial t} = c_i \frac{\partial^2 u^i}{\partial z^2} + R^i(t) \tag{5}$$

for $h_{i-1} \leqslant z \leqslant h_i$, $i = 1, 2$.

The conditions of solution to Eq. (5) are

$$u^1|_{z=0} = 0 \tag{6}$$

$$u^1|_{z=h_1} = u^2|_{z=h_1} \tag{7}$$

$$k_1 \left.\frac{\partial u^1}{\partial z}\right|_{z=h_1} = k_2 \left.\frac{\partial u^2}{\partial z}\right|_{z=h_1} \tag{8}$$

$$\begin{cases} u^2|_{z=H} = 0 & \text{(Free draining)} \\ \left.\dfrac{\partial u^2}{\partial z}\right|_{z=H} = 0 & \text{(Undrained)} \end{cases} \tag{9}$$

$$u^i|_{t=0} = \sigma(z), \ (i = 1, 2) \tag{10}$$

Where u^i, c_i and k_i are the pore-water pressure, the coefficient of consolidation and the coefficient of permeability of the ith layer, respectively; $R^i(t) = \frac{\mathrm{d}q}{\mathrm{d}t}$ is the velocity of applied load.

In order to apply DQM to Eqs. (5) – (10), the layers must be discretized. The rule of discretization is one layer one differential quadrature element. The number of nodes in element is N^i of which superscript i represents the corresponding element and $i = 1, 2$. For using the DQM conveniently, the local coordinate ξ is introduced into every element. The relation between local coordinate and global coordinate adopted is expressed as

$$z^i = (0.5 - \xi) z_1^i + (0.5 + \xi) z_{N^i}^i \quad (11)$$

where z^i is global coordinate of ith element; z_1^i and $z_{N^i}^i$ are the global coordinates of the first node and N^ith node in ith element, $-0.5 \leqslant \xi \leqslant 0.5$.

In terms of Eq. (11), the differential of z^i can be expressed as

$$\mathrm{d}z^i = (z_{N^i}^i - z_1^i)\mathrm{d}\xi = h_i \mathrm{d}\xi \quad (12)$$

where h_i is the thickness of the ith element. From the above equation, the following relation can be obtained.

$$\begin{cases} \dfrac{\partial u^i}{\partial z^i} = \dfrac{1}{h_i}\dfrac{\partial u^i}{\partial \xi} \\ \dfrac{\partial^2 u^i}{\partial (z^i)^2} = \dfrac{1}{h_i^2}\dfrac{\partial^2 u^i}{\partial \xi^2} \end{cases} \quad (13)$$

Eq. (13) can be discretized by DQM into the following form

$$\begin{cases} \dfrac{\partial u_\alpha^i}{\partial z^i} = \dfrac{1}{h_i}\sum_{\beta=1}^{N^i} D_{\alpha\beta}^{i(1)} u_\beta^i \\ \dfrac{\partial^2 u_\alpha^i}{\partial (z^i)^2} = \dfrac{1}{h_i^2}\sum_{\beta=1}^{N^i} D_{\alpha\beta}^{i(2)} u_\beta^i \end{cases} \quad (14)$$

where u_α^i is the pore-water pressure at every node in the ith element; $D_{\alpha\beta}^{i(1)}$ and $D_{\alpha\beta}^{i(2)}$ are the weighting coefficient matrices of the first-order and second-order derivatives, respectively, where $\alpha = 1, 2, \cdots, N^i$.

According to the rule of discretization above, let $T = \frac{c_1 t}{h_1}$, $m_i = \frac{h_i}{h_1}$, $a_i = \frac{c_i}{c_1}$, $b_i = \frac{k_i}{k_1}$ and $\bar{R}_\alpha^i(T) = \frac{h_1^2}{c_1} R_\alpha^i(t)$, Eqs. (5) – (10) can be deduced to the following form

$$\begin{cases} \dfrac{\partial u_\alpha^1}{\partial T} = \dfrac{a_1}{m_1^2}\sum_{\beta=1}^{N^1} D_{\alpha\beta}^{1(2)} u_\beta^1 + \bar{R}_a^1(T) \\ (\alpha = 2, 3, \cdots, N^1 - 1) \\ \dfrac{\partial u_\alpha^2}{\partial T} = \dfrac{a_2}{m_2^2}\sum_{\beta=1}^{N^2} D_{\alpha\beta}^{2(2)} u_\beta^2 + \bar{R}_a^2(T) \\ (\alpha = 2, 3, \cdots, N^2 - 1) \end{cases} \quad (15)$$

$$u_1^1 = 0 \quad (16)$$

$$u_{N^1}^1 = u_1^2 \quad (17)$$

$$\frac{b_1}{m_1}\sum_{\beta=1}^{N^i} D_{N^1\beta}^{1(1)} u_\beta^1 = \frac{b_2}{m_2}\sum_{\beta=1}^{N^2} D_{1\beta}^{2(1)} u_\beta^2 \quad (18)$$

$$\begin{cases} u_{N^2}^2 = 0 \quad \text{(Free draining)} \\ \sum_{\beta=1}^{N^2} D_{N^2\beta}^{2(1)} u_\beta^2 = 0 \quad \text{(Undrained)} \end{cases} \quad (19)$$

$$u^i \,|_{T=0} = \sigma^i(\xi), \ (i = 1, 2) \quad (20)$$

Considering free draining in the lower boundary and substituting Eqs. (16), (17) and (19) into Eq. (18), yields

$$\begin{Bmatrix} u_{N^1}^1 \\ u_1^2 \end{Bmatrix} = \frac{b_1 m_2 \sum_{\beta=2}^{N^1-1} D_{N^1\beta}^{1(1)} u_\beta^1 - b_2 m_1 \sum_{\beta=2}^{N^2-1} D_{1\beta}^{2(1)} u_\beta^2}{b_2 m_1 D_{11}^{2(1)} - b_1 m_2 D_{N^1 N^1}^{1(1)}} \quad (21)$$

Substituting Eq. (21) into Eq. (15) yields

$$\begin{cases} \dfrac{\partial u_\alpha^1}{\partial T} = \dfrac{a_1}{m_1^2}\sum_{\beta=2}^{N^1-1} D_{\alpha\beta}^{1(2)} u_\beta^1 + \dfrac{a_1}{m_1^2} D_{\alpha N^1}^{1(2)} \times \\ \dfrac{b_1 m_2 \sum_{\beta=2}^{N^1-1} D_{N^1\beta}^{1(1)} u_\beta^1 - b_2 m_1 \sum_{\beta=2}^{N^2-1} D_{1\beta}^{2(1)} u_\beta^2}{b_2 m_1 D_{11}^{2(1)} - b_1 m_2 D_{N^1 N^1}^{1(1)}} + \bar{R}_a^1(T) \\ \dfrac{\partial u_\alpha^2}{\partial T} = \dfrac{a_2}{m_2^2}\sum_{\beta=2}^{N^2-1} D_{\alpha\beta}^{2(2)} u_\beta^2 + \dfrac{a_2}{m_2^2} D_{\alpha 1}^{2(2)} \times \\ \dfrac{b_1 m_2 \sum_{\beta=2}^{N^1-1} D_{N^1\beta}^{1(1)} u_\beta^1 - b_2 m_1 \sum_{\beta=2}^{N^2-1} D_{1\beta}^{2(1)} u_\beta^2}{b_2 m_1 D_{11}^{2(1)} - b_1 m_2 D_{N^1 N^1}^{1(1)}} + \bar{R}_a^2(T) \end{cases} \quad (22)$$

Note that $\alpha = 2, 3, \cdots, N^1 - 1$ in the first equation of Eq. (22) and that $\alpha = 2, 3, \cdots, N^2 - 1$ in the second equation of Eq. (22).

Eq. (22) can be written in the matrix form

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}T} = \boldsymbol{A}\boldsymbol{u} + \boldsymbol{R}(T) \quad (23)$$

where

$$\boldsymbol{u} = \{\boldsymbol{u}^1 \quad \boldsymbol{u}^2\}^{\mathrm{T}} \quad \boldsymbol{u}^1 = \{u_2^1 \quad u_3^1 \cdots u_{N^1-1}^1\}$$
$$\boldsymbol{u}^2 = \{u_2^2 \quad u_3^2 \cdots u_{N^2-1}^2\} \quad \boldsymbol{R} = \{\boldsymbol{R}^1 \quad \boldsymbol{R}^2\}^{\mathrm{T}}$$
$$\boldsymbol{R}^1 = \{\bar{R}_2^1 \quad \bar{R}_3^1 \cdots \bar{R}_{N^1-1}^1\}$$
$$\boldsymbol{R}^2 = \{\bar{R}_2^2 \quad \bar{R}_3^2 \cdots \bar{R}_{N^2-1}^2\}$$

Let matrix $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}$, where $\boldsymbol{A}_{11}$, $\boldsymbol{A}_{12}$, $\boldsymbol{A}_{21}$ and $\boldsymbol{A}_{22}$ are the sub-matrixes of $\boldsymbol{A}$. Also let $A_{11}^{\alpha\beta}$, $A_{12}^{\alpha\beta}$, $A_{21}^{\alpha\beta}$ and $A_{22}^{\alpha\beta}$ represent the elements of $\boldsymbol{A}_{11}$, $\boldsymbol{A}_{12}$, $\boldsymbol{A}_{21}$ and $\boldsymbol{A}_{22}$, respectively.

$$A_{11}^{\alpha\beta} = \frac{a_1}{m_1^2}\left(D_{(\alpha+1)(\beta+1)}^{1(2)} + \frac{b_1 m_2 D_{(\alpha+1)N^1}^{1(2)} D_{N^1(\beta+1)}^{1(1)}}{b_2 m_1 D_{11}^{2(1)} - b_1 m_2 D_{N^1N^1}^{1(1)}} \right) \tag{24}$$

where $\alpha = 1, 2, \cdots, N^1 - 2$ and $\beta = 1, 2, \cdots, N^1 - 2$.

$$A_{12}^{\alpha\beta} = -\frac{a_1}{m_1^2} \frac{b_2 m_1 D_{(\alpha+1)N^1}^{1(2)} D_{1(\beta+1)}^{2(1)}}{b_2 m_1 D_{11}^{2(1)} - b_1 m_2 D_{N^1N^1}^{1(1)}} \tag{25}$$

where $\alpha = 1, 2, \cdots, N^1 - 2$ and $\beta = 1, 2, \cdots, N^2 - 2$.

$$A_{21}^{\alpha\beta} = \frac{a_2}{m_2^2} \frac{b_1 m_2 D_{(\alpha+1)1}^{2(2)} D_{N^1(\beta+1)}^{1(1)}}{b_2 m_1 D_{11}^{2(1)} - b_1 m_2 D_{N^1N^1}^{1(1)}} \tag{26}$$

where $\alpha = 1, 2, \cdots, N^2 - 2$ and $\beta = 1, 2, \cdots, N^1 - 2$.

$$A_{22}^{\alpha\beta} = \frac{a_2}{m_2^2}\left(D_{(\alpha+1)(\beta+1)}^{2(2)} - \frac{b_2 m_1 D_{(\alpha+1)1}^{2(2)} D_{1(\beta+1)}^{2(1)}}{b_2 m_1 D_{11}^{2(1)} - b_1 m_2 D_{N^1N^1}^{1(1)}} \right) \tag{27}$$

where $\alpha = 1, 2, \cdots, N^2 - 2$ and $\beta = 1, 2, \cdots, N^2 - 2$.

The solution to Eq. (23) can now be given by

$$\boldsymbol{u}(T) = \mathrm{e}^{AT}\boldsymbol{u}(0) + \int_0^T \mathrm{e}^{-A(\tau - T)} \boldsymbol{R}(\tau)\mathrm{d}\tau \tag{28}$$

Where $\boldsymbol{u}(0)$ is the vector of initial pore-water pressure which can be determined by Eq. (20).

The solution for Eq. (28) can be used for considering the undrained lower boundary. However, the matrix $\boldsymbol{A}$ is different from that of Eq. (28), which can be given by

$$A_{11}^{\alpha\beta} = \frac{a_1}{m_1^2}\left(D_{(\alpha+1)(\beta+1)}^{1(2)} + \frac{b_1}{m_1} \frac{D_{(\alpha+1)N^1}^{1(2)} D_{N^1(\beta+1)}^{1(1)}}{\Gamma} \right) \tag{29}$$

where $\alpha = 1, 2, \cdots, N^1 - 2$ and $\beta = 1, 2, \cdots, N^1 - 2$.

$$A_{12}^{\alpha\beta} = -\frac{a_1}{m_1^2} \frac{b_2}{m_2} D_{(\alpha+1)N^1}^{1(2)} \left(D_{1(\beta+1)}^{2(2)} + \frac{D_{1N^2}^{2(1)}}{D_{N^2N^2}^{2(1)}} D_{N^2(\beta+1)}^{2(1)} \right) / \Gamma \tag{30}$$

where $\alpha = 1, 2, \cdots, N^1 - 2$ and $\beta = 1, 2, \cdots, N^2 - 2$.

$$A_{21}^{\alpha\beta} = \frac{a_2}{m_1^2} \frac{b_1}{m_1} (D_{(\alpha+1)1}^{2(2)} D_{N^2N^2}^{2(1)} - D_{(\alpha+1)N^2}^{2(2)} D_{N^21}^{2(1)}) \times D_{N^1(\beta+1)}^{1(1)} / (D_{N^2N^2}^{2(1)} \Gamma) \tag{31}$$

Where $\alpha = 1, 2, \cdots, N^2 - 2$ and $\beta = 1, 2, \cdots, N^1 - 2$.

$$A_{22}^{\alpha\beta} = \frac{a_2}{m_2^2} \left\{ \begin{aligned} & D_{(\alpha+1)(\beta+1)}^{2(2)} - \frac{b_2}{m_2} \times \\ & (D_{(\alpha+1)1}^{2(2)} D_{N^2N^2}^{2(1)} - D_{(\alpha+1)N^2}^{2(2)} D_{N^21}^{2(1)}) \times \\ & \left(D_{1(\beta+1)}^{2(1)} + \frac{D_{1N^2}^{2(1)} D_{N^2(\beta+1)}^{2(1)}}{D_{N^2N^2}^{2(1)}} \right) \Big/ \\ & (D_{N^2N^2}^{2(1)} \Gamma) + D_{(\alpha+1)N^2}^{2(2)} D_{N^2(\beta+1)}^{2(1)} / \Gamma \end{aligned} \right\} \tag{32}$$

Where $\alpha = 1, 2, \cdots, N^2 - 2$ and $\beta = 1, 2, \cdots, N^2 - 2$.

and

$$\Gamma = \frac{b_2}{m_2}\left(D_{11}^{2(1)} - \frac{D_{1N^2}^{2(1)}}{D_{N^2N^2}^{2(1)}} D_{N^21}^{2(1)} \right) - \frac{b_1}{m_1} D_{N^1N^1}^{1(1)}$$

THE KIND OF APPLIED LOAD AND ITS CORESPONDING SOLUTION

1. Linear load

The linear load is shown in Fig. 2, $q(T)$ and $\bar{R}^i(T)$ are given by

$$q(T) = \begin{cases} \dfrac{T}{T_c} q_u & (T_c \geqslant T \geqslant 0) \\ q_u & (T \geqslant T_c) \end{cases} \tag{33}$$

$$\bar{R}^i(T) = \begin{cases} \dfrac{q_u}{T_c} & (T_c > T \geqslant 0) \\ 0 & (T > T_c) \end{cases} \tag{34}$$

Substituting Eq. (34) into Eq. (28), one obtains

$$\boldsymbol{u}(T) = \mathrm{e}^{AT}\boldsymbol{u}(0) + \frac{q_u}{t_c}\begin{bmatrix} (\boldsymbol{I} - \boldsymbol{A}^{-1})T \\ + \boldsymbol{A}^{-1}\mathrm{e}^{AT} \end{bmatrix}\boldsymbol{U},$$

$$(T_c \geqslant T \geqslant 0) \tag{35}$$

$$\boldsymbol{u}(T) = e^{A(T-T_c)}\boldsymbol{u}(T_c), (T > T_c) \tag{36}$$

where $\boldsymbol{I}$ is the identity matrix and $\boldsymbol{U}$ the unit vector.

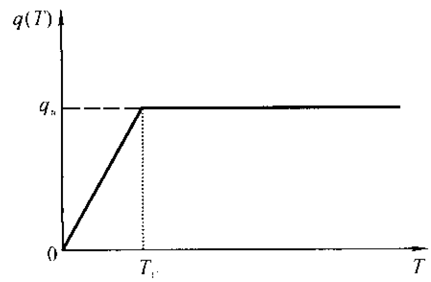

Fig.2 Linear load

2. Constant load

The case of constant load is shown in Fig.3, where $\bar{R}^i(T) = \delta(T)$ and δ is Dirac function. Substituting $\bar{R}^i(T) = \delta(T)$ into Eq. (28), one obtains

$$\boldsymbol{u}(T) = e^{AT}[\boldsymbol{u}(0) + q_u\boldsymbol{U}] \tag{37}$$

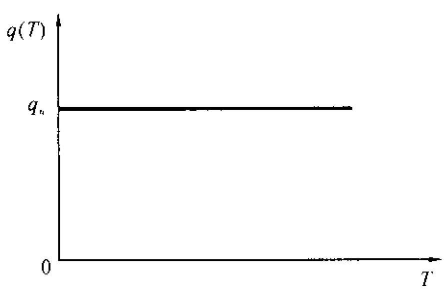

Fig.3 Constant load

3. Cycling square load

The case of cycling square load is shown in Fig.4, where T_p is the period of the applied load. $q(T)$ and $\bar{R}^i(T)$ can be expressed as

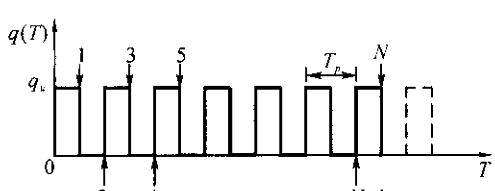

Fig.4 Cycling square load

$$q(T) = q_u \sum_{m=1}^{N} (-1)^{m-1} H\left[T - \frac{m-1}{2}T_p\right] \tag{38}$$

$$\bar{R}^i(T) = q_u \sum_{m=1}^{N} (-1)^{m-1} \delta\left[T - \frac{m-1}{2}T_p\right] \tag{39}$$

where H and δ are Heaviside and Dirac function, respectively.

Substituting Eq.(39) into Eq.(28), one obtains

$$\boldsymbol{u}(T) = e^{AT}\boldsymbol{u}(0) + q_u \sum_{m=1}^{N} (-1)^{m-1} e^{A\left[T - \frac{m-1}{2}T_p\right]} \boldsymbol{U} \tag{40}$$

CASE STUDIES

In order to check the precision of the DQM solution to consolidation of double-layered ground, we will compare the DQM solution with Xie's solution (1994), where constant load is applied and the initial pore-water pressure equals zero. The comparisons include two study cases with boundary conditions as listed in Table 1 and the descretization and parameters of soils as shown in Fig.5. The discretizations of every element adopted equally spaced nodes.

Table 1 Boundary conditions

Study cases		1	2
Boundaries	Upper	Free draining	Free draining
	Lower	Free draining	Undrained

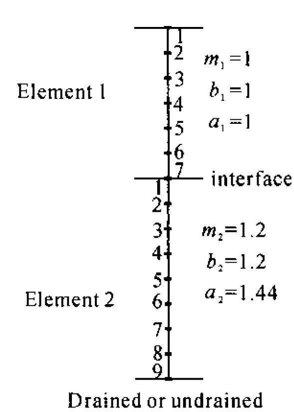

Fig.5 Discretization and parameters of soils

The results of comparison between the analytical solution and the DQM solution for calculating the pore water pressure in case 1 are shown in Fig.6 to Fig.9 and in case 2 in Fig.10 to Fig.13.

It can been seen from Fig. 6 to Fig. 13 that the DQM computational results for the pore-water pressure in the soil layer agreed with the analytical solution results even for the case of undrained boundary; and also agreed with the results for the soil layer interface, except for a short time small discrepancy at the beginning of computation. This discrepancy in the interface could be due to the fact that Eq. (21) cannot be applied directly for computing the pore-water pressure at initial time because the load acting on surface is

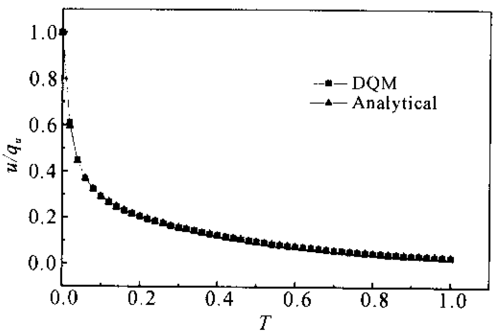

Fig.6 The pore-water pressure of 2nd node of the 1st layer in case 1

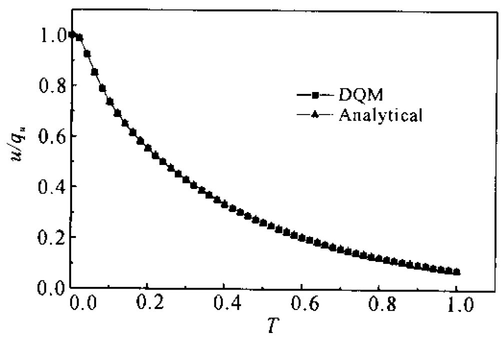

Fig.7 The pore-water pressure of 4th node of the 1st layer in case 1

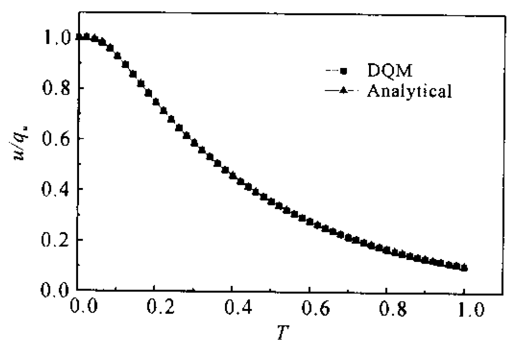

Fig.8 The pore-water pressure of 6th node of the 1st layer in case 1

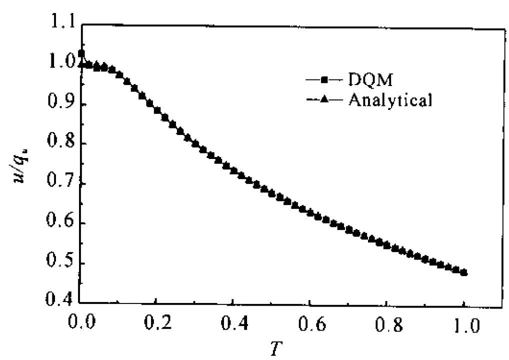

Fig.9 The pore-water pressure of 7th node of the 1st layer in case 1

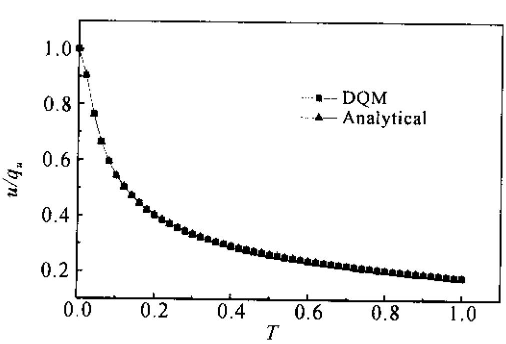

Fig.10 The pore-water pressure of 3rd node of the 1st layer in case 2

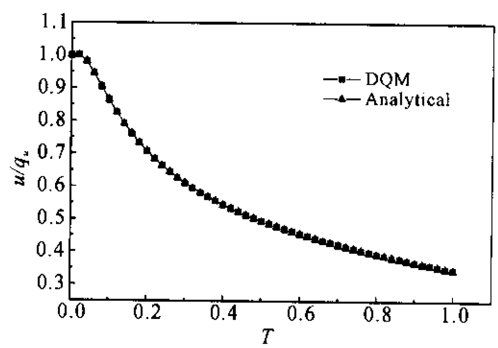

Fig.11 The pore-water pressure of 5th node of the 1st layer in case 2

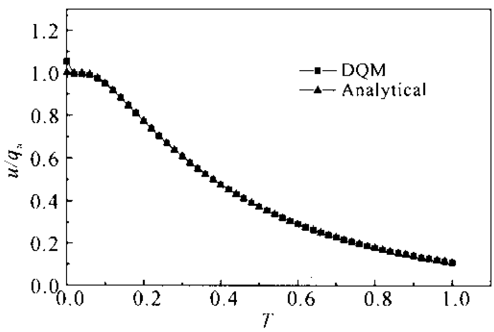

Fig.12 The pore-water pressure of 7th node of the 1st layer in case 2

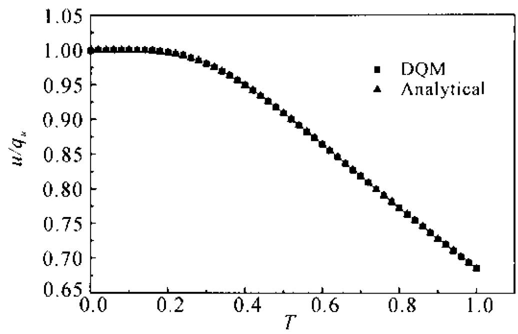

Fig.13 The pore-water pressure of 9th node of the 2nd layer in case 2

Heaviside function. Actually, we may assume that at the initial time, all the external loads are carried by the pore-water, thus, the calcution of the pore-water pressure is not necessary.

CONCLUSIONS

The authors derived the DQM solution to the model of consolidation of double-layered ground based on Xie's work (1994). Comparisons between the DQM solution and the analytical solution led to the following conclusions.

1. This paper's formulas for calculating one-dimensional linear consolidation by DQM yield high precision results. DQM's matrix form makes it convenient for designing computational programs for engineering practice.

2. The advantages of the DQM solution are that, compared with the analytical solution, it can avoid the cumbersome work in solving the eigen-equation for eigen values; and with the Laplace-transformed solution, because that it is an explicit formula in the time domain, it is possible that it can solve the precision problem induced by numerical solution of long time inverse Laplace transform.

3. As to multi-layered soils, we can also establish the relation (in matrix form) between the nodes at the boundary or interface and the internal nodes in every DQM's element by the same method as described above. And then, the DQ solutions to double-layered ground can be easily extended to multi-layered soils.

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