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On the structural features of fiber suspensions in converging channel flow*

LIN Jian-zhong(林建忠), ZHANG Ling-xin(张凌新)†
(*Department of Mechanics, Zhejiang University, Hangzhou 310027, China*)
†E-mail: aling@zju.edu.cn
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Abstract: The structural features of fiber suspensions are dependent on the fiber alignment in the flows. In this work the orientation distribution function and orientation tensors for semi-concentrated fiber suspensions in converging channel flow were calculated, and the evolutions of the fiber alignment and the bulk effective viscosity were analyzed. The results showed that the bulk stress and the effective viscosity were functions of the rate-of-strain tensor and the fiber orientation state; and that the fiber suspensions evolved to steady alignment and tended to concentrate to some preferred directions close to but not same as the directions of local streamlines. The bulk effective viscosity depended on the product of Reynolds number and time. The decrease of effective viscosity near the boundary benefited the increase of the rate of flow. Finally when the fiber alignment went into steady state, the structural features of fiber suspensions were not dependent on the Reynolds number but on the converging channel angle.

Key words: Fiber suspensions, Orientation distribution, Effective viscosity, Converging channel flow
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INTRODUCTION

Modeling fiber-fiber interactions in non-dilute fiber suspensions and describing how these interactions affect the rheological properties of the suspensions in the flows are fundamental and important in multi-phase flows research. Ultimately we would like to be able to design and control the manufacture processing of these suspensions, thus to be able to improve the structural features of the composites as expected.

Research on the dominant structural features of fiber suspensions is based on a statistical conception, that is, the bulk quantities, such as bulk stress, are defined as averages over an ensemble of realizations; this average is equal to the integrals over a suitably chosen volume of ambient fluid and fibers together. The contribution to the bulk quantities due to the presence of the fibers is expressed as the integrals over the surfaces of fibers. Several fundamental researches which took into account the fluid's microscopic structure had been conducted. Prager (1957) considered a model of non-interacting dumbbell particle suspensions and derived a constitutive equation of stress for the dilute suspensions. Batchelor (1970) developed a general constitutive equation for the suspensions of particles of any shape at arbitrary concentrations; and proposed a "cell model" to make his theory applicable. Using this model Dinh *et al*. (1984) obtained an explicit constitutive equation for semi-concentrated fiber suspensions, which was expressed in terms of the integrals involving the fiber orientation distribution function.

The orientation distribution function of fibers and the integrals of it over the orientations, i.e. orientation tensors, can be used to describe the dynamical orientation state of the fiber suspensions.

Based on the local velocity gradients and fiber orientation state in the flows, we can determine the structural features such as the bulk stress and the bulk effective viscosity of the whole flows. Givler *et al*. (1983) calculated the cylindrical particle orientations and gave a single orientation for each point in the flows, but they did not give the statistical characteristic of large

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numbers of cylindrical particles. Folgar *et al*. (1984) proposed a statistical model for the orientation behavior of fibers in concentrated suspensions. Advani *et al*. (1987) used this model to provide theoretical groundwork for numerical calculations of orientation tensors. Jackson *et al*. (1986) predicted the orientation of short fibers in thin compression moldings. Altan *et al*. (1989) reported an application of Advani's groundwork in a homogeneous flow and obtained the fourth-order orientation tensors which were in good agreement with the experimental results. Shaqfeh *et al*. (1990) presented a theory to describe the momentum transport properties of suspensions containing randomly placed, slender fibers and studied the dilute and semi-dilute concentration regimes. Mackaplow *et al*. (1996) completed a set of numerical simulations of the volume-averaged stress tensor in a suspension of rigid, non-Brownian slender fibers at zero Reynolds number and predicted the rheological properties of suspensions with concentrations ranging from the dilute regime into the semi-dilute regime. Grosso *et al*. (2000) presented a new closure model, i.e., approximated the fourth rank order tensor in terms of lower rank tensors, which could be easily implemented; then considered a shear flow for nematic polymers as a test and obtained satisfactory results.

There are only few studies on non-homogeneous flows, which, in fact, are often encountered in research programs and industrial practice. Shanker *et al*. (1991) studied the effect of quadratic flows on the orientation distribution and rheology. Chiba *et al*. (1998) investigated the two-dimensional fiber orientations in a Newtonian flow through a 1:4 backward-facing step channel. Lin *et al*. (2000) extended the case to converging channel flow, and presented the motion behavior of fiber suspensions. In this work we continue in planar converging channel flow, investigate the orientation distribution function and the orientation tensors, and give the description of the dominant structural features of semi-concentrated fiber suspension in terms of the bulk effective viscosity.

BASIC THEORY

Definition

We consider a suspension of n fibers per unit volume, with each fiber regarded as a rigid, cylindrical and uniform particle with length L and diameter D. There are no concentration gradients so that the particle number n per unit volume is a constant. The suspension to be semi-concentrated requires that (Dinh *et al*., 1984)

$$1/L^3 < n < 1/(DL^2) \tag{1}$$

The solvent is a Newtonian fluid which flows with a low Reynolds number. In this paper, we make the following assumptions. The analysis is focused on a single test fiber, shown in Fig. 1, that is immersed in a continuous medium. A distribution function $\varphi(\boldsymbol{r}_c, \boldsymbol{P}, t)$ is introduced to account for the probability that the test fiber selected has a specific location $\boldsymbol{r}_c$ and orientation $\boldsymbol{P}$ at time t. The function is defined such that the probability of having a fiber between angles $\delta\theta$ and $\delta\phi$ is

$$\varphi(\boldsymbol{r}_c, \boldsymbol{P}, t)\delta\boldsymbol{P} = \varphi(\boldsymbol{r}_c, \boldsymbol{P}, t)\sin\theta\delta\theta\delta\phi \tag{2}$$

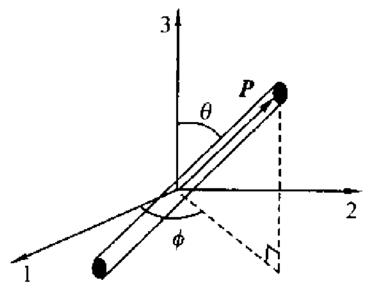

Fig. 1 A single fiber

Introduction of orientation tensors is a suitable and concise way for describing the orientation state of fibers. Because the distribution function is an even one, only the even-order tensors are of interest. The even-order orientation tensors are defined as:
the second-order tensor:

$$a_{ij} = \oint P_i P_j \varphi(\boldsymbol{P})\mathrm{d}\boldsymbol{P}, \tag{3}$$

the fourth-order tensor:

$$a_{ijkl} = \oint P_i P_j P_k P_l \varphi(\boldsymbol{P})\mathrm{d}\boldsymbol{P}, \tag{4}$$

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We limit our discussion to the second-order and the fourth-order tensors because they are sufficient for rheological uses.

Dinh-Armstrong model

The bulk stress in a suspension of particles in a Newtonian fluid obtained by Batchelor (1970) has two separate parts, one due to the viscous dissipation of the fluid and the other due to the presence of particles. This model provides a direct link between the micro-structural properties and the macroscopic rheological behavior of a suspension system. Dinh *et al*. (1984) applied this model to semi-concentrated fiber suspensions and obtained the rheological equation:

$$\boldsymbol{\tau} = \eta_s \dot{\boldsymbol{\gamma}} + m\eta_s \dot{\boldsymbol{\gamma}} : \int \boldsymbol{PPPP}\varphi \mathrm{d}\boldsymbol{P} \quad (5)$$

where $\boldsymbol{\tau}$ is the bulk stress tensor, η_s is the solvent viscosity, $\dot{\boldsymbol{\gamma}}$ is the rate-of-strain tensor, $\dot{\gamma}_{ij} = \partial v_j/\partial x_i + \partial v_i/\partial x_j$, m is a coefficient involving the fiber parameters, and $m = \pi n L^3/(12\ln(2H/D))$; H is the average distance from a given fiber to its nearest neighbor, here H is $(nL)^{-1/2}$ for the aligned systems and is $(nL^2)^{-1}$ for the random systems.

In this way, the Newtonian fluid and fibers flow like an effective continuum, in which the effect of the other fibers on a test fiber is considered to be the continuum approximation.

Calculation equations

The theory of fiber motion in non-uniform deformation flows is not available. The assumption that the fiber orientation state and the velocity gradients do not change much over the length of fibers should be imposed. Thus, the behavior of the fibers at a particular point is determined only by the orientation state and the deformation conditions at that point.

According to the conservation of fiber orientation, the governing equation for the distribution function of fibers is given (Advani *et al*., 1987) as:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = -\frac{\partial}{\partial \phi}(\varphi \dot{\phi}), \quad (6)$$

where ϕ is the angle between the fiber axis and x direction, $\dot{\phi}$ represents the differential of ϕ with respect to time.

Using Eqs.(3), (4) and (6), we can get the governing equations of the second-order and the fourth-order tensors as follows (Advani *et al*., 1987).

$$\frac{\mathrm{d}a_{ij}}{\mathrm{d}t} = -\frac{1}{2}(\omega_{ik}a_{kj} - a_{ik}\omega_{kj}) + \frac{1}{2}(\dot{\gamma}_{ik}a_{kj} + a_{ik}\dot{\gamma}_{kj} - 2\dot{\gamma}_{kl}a_{ijkl}) + 2C_i\dot{\gamma}(\delta_{ij} - \Omega a_{ij}), \quad (7)$$

$$\frac{\mathrm{d}a_{ijkl}}{\mathrm{d}t} = -(\omega_{im}a_{mjkl} - a_{ijkm}\omega_{ml}) + (\dot{\gamma}_{im}a_{mjkl} + a_{ijkm}\dot{\gamma}_{ml} - 2\dot{\gamma}_{mn}a_{ijklmn}) + C_i\dot{\gamma}[-\beta a_{ijkl} + 2(a_{ij}\delta_{kl} + a_{ik}\delta_{jl}a_{il}\delta_{jk} + a_{jk}\delta_{il} + a_{jl}\delta_{ik} + a_{kl}\delta_{ij})], \quad (8)$$

where $\omega_{ij} = \partial v_j/\partial x_i - \partial v_i/\partial x_j$, and for planar orientation Ω is 2, β is 16. C_i is a phenomenological coefficient modeling the randomizing effect of interactions between particles.

Eq.(7) and Eq. (8) cannot be solved unless the closure approximations are adopted. Here we take the hybrid closure approximation for a_{ijklmn}; then a_{ij} and a_{ijkl} are calculated by combining Eq.(7) with Eq.(8). According to the property of the two-dimensional orientation tensors, it can be easily proved that only three second-order tensor components and five fourth-order tensor components are independent; these independent components can be concentrated into a_{11}, a_{12}, a_{1111} and a_{1112}, taking into account the normalization conditions.

We define an effective viscosity η'_s of fiber suspensions so that the general constitutive equation for the pure fluid is available. The effective viscosity is a function of the rate-of-strain tensor and the orientation tensor with its coefficient dependent on the fiber shape, the concentration and the interaction of the fibers. According to the Dinh-Armstrong model, the constitutive equations are written as

$$\tau_{ij} = \eta'_s \dot{\gamma}_{ij} = \eta_s(\dot{\gamma}_{ij} + m\dot{\gamma}_{kl}a_{ijkl}). \quad (9)$$

For planar flow, the above equations include four components. In the cases where the shear rate is dominant in the rate-of-strain, the dimensionless viscosity can be given as:

$$\bar{\eta} = \eta'_s/\eta_s - 1 = m(\dot{\gamma}_{11}a_{1112} + 2\dot{\gamma}_{12}a_{1122} + \dot{\gamma}_{22}a_{1222})/\dot{\gamma}_{12} \quad (10)$$

Here we consider the fiber suspensions as an aligned system.

CONVERGING CHANNEL FLOW

The flow is considered a plane flow caused by a convergence at the origin and confined by two walls with angles $\pm \beta_0$. The coordinate origin is oriented at the point of intersection between two flats, so a polar coordinate is shown in Fig. 2. For the absence of the velocity in the direction of β, i.e., $u_\beta = 0$, the radial velocity is obtained finally as (Lin *et al.*, 2000):

$$u_r = Re\nu f(\varepsilon)/(r\beta_0), \qquad (11)$$

where ν equals η_s/ρ, pole-angle coefficient is $\varepsilon = \beta/\beta_0$, the function $f(\varepsilon)$ satisfies the equation:

$$f'' + Re\beta_0 f^2 + 4\beta_0^2 f = \text{const}. \qquad (12)$$

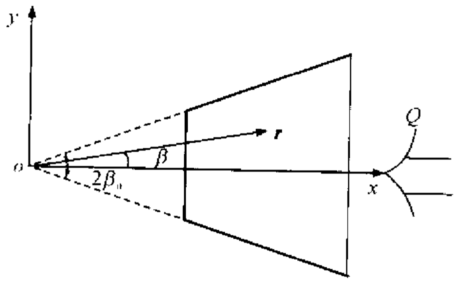

Fig.2 A converging channel flow field

From the above equation, we can see that Re and β_0 are two factors determining the converging channel flow. We discuss the cases of β_0 from π/36 to π/12 with Reynolds number changing. Because of the symmetry of the flow field, the calculation is carried out for half of the flow only. As shown in Fig. 3, the velocity of the flow decreases with r or ε increasing, and the scalar magnitude of the rate-of-strain decreases along a certain direction.

RESULTS AND DISCUSSION

Eq. (6) is solved by using conventional finite-difference scheme with random initial and boundary conditions:

$$\varphi(\phi, t)|_{t=0} = 1/\pi$$

$$\varphi(\phi + n\pi, t)|_{\phi=0} = 1/\pi.$$

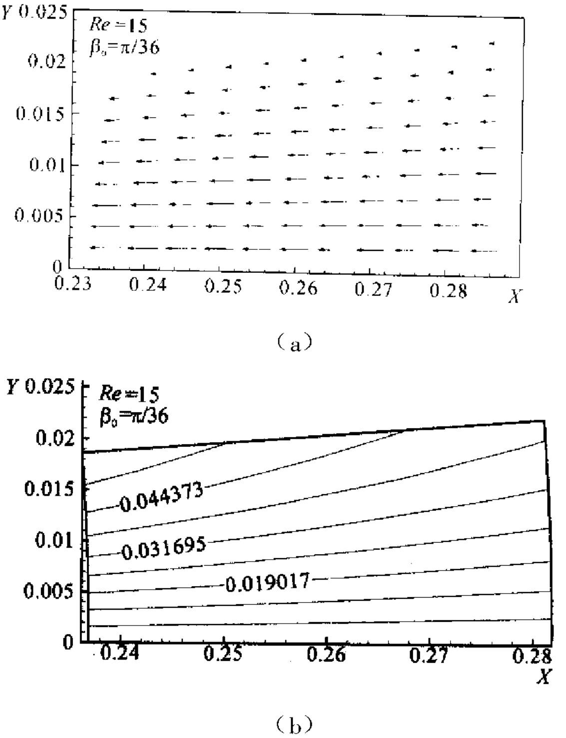

Fig.3 The velocity field (a) and the contour map of $|\gamma_{12}|$ (b) of converging channel flow

Fig.4(a) shows the preferred alignments of fibers in the whole field at different time. Fibers are at a random state initially, then quickly align in the directions around 45°, and in the later evolution, tend to align in directions close to those of the local streamlines. The fibers at the sites where the scalar magnitude of the rate-of-strain tensor is larger rotate strongly. Even though there are no great differences in the fiber alignment between the former and the latter state in Fig.4(a), the values of fiber distribution function in the whole field change significantly as shown in Fig. 4(b). With the increase of time, the fiber orientations tend to concentrate to some preferred angles, which shows that the probability of fiber aligning in the preferred orientation increases.

There are two ways to calculate the tensor components: 1. solving Eq. (6) to obtain the distribution function, then according to Eq. (3) and Eq. (4), integrating it numerically to find the values for the second-order and the fourth-order tensors; 2. solving the tensor equations directly by using the hybrid closure approximations for the sixth-order tensor. In this paper, the two ways are used to calculate the second-order and the fourth-order tensor components over whole flow fields. Fig.5 is one of the evolutions of the fiber orientation tensor components at the center position of the flow field. In the figure, dash lines are the results calculated with the first way mentioned above, and solid lines correspond to the second way. The two results agreed well. All these tensor components finally reach steady values after some time, which means that the fiber suspensions had gone into steady alignment. $a_{11} > a_{12}$ and $a_{1111} > a_{1112}$ indicate a preferred orientation close to the local streamline, while the non-zero a_{12} and a_{1112} indicate that the preferred orientation does not coincide with the streamline.

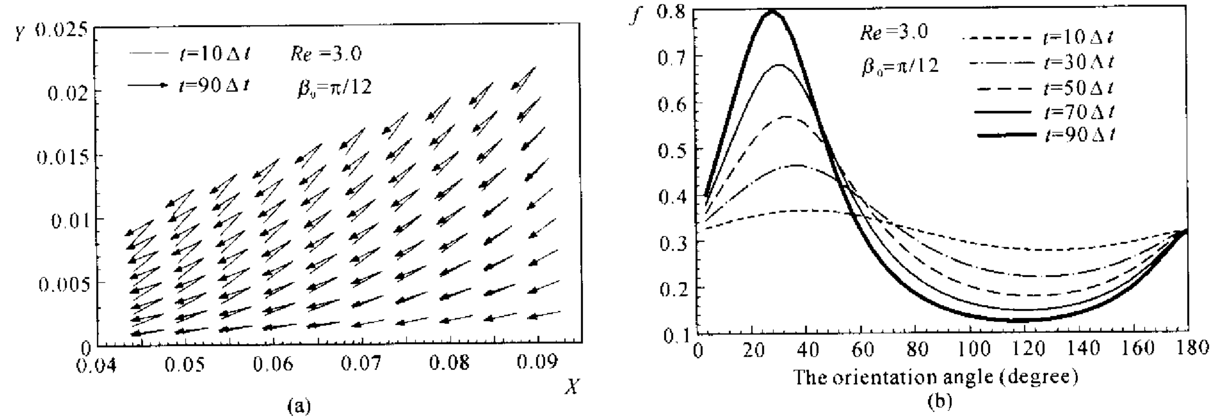

Fig.4 (a) The preferred alignments of fibers at different time; and (b) the evolution of the fiber distribution function at the center position of flow field

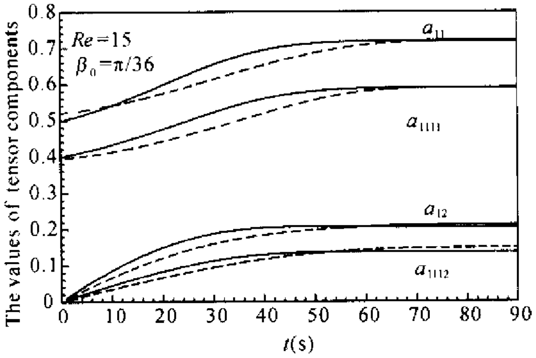

Fig.5 The evolution of the orientation tensor components at the center position of flow field. are solid lines

--- the results calculated with the first way; —— correspond to the second way.

Calculations also showed that the fiber orientation state depended on the product of Reynolds number and time in the case of given constant angle β_0, not Reynolds number or time alone. For example, when the Reynolds number is small, the time of the orientation alignment into a steady state is long. Fig.6 indicates the evolution of bulk effective viscosity with $Re \times t$ increasing for $\beta_0 = \pi/36$. The dark color stands for effective viscosity with higher value, the shallow color for lower value. With increase of time, the effective viscosity over the flow field changes; the effective viscosity in the left and upper part decreases, while the one in the right and lower parts increases.

The steady dimensionless effective viscosity for the different Reynolds number over whole flow field was calculated. Fig.7 gives one of the results at the central position for different flow fields. The results indicated that the steady effective viscosity depended slightly on the Reynolds number but obviously on the angle β_0. Fig.8 gives the contour maps of the bulk effective viscosity at different converging channel angles. Similar to the map of $\beta_0 = \pi/36$, there are two parts for the case of $\beta_0 = \pi/18$, i.e., effective viscosity decreasing part and effective viscosity increasing part. However, for the case of $\beta_0 = \pi/12$, the two parts mentioned above move right, and another viscosity increasing part appears at the left and upper position of the field.

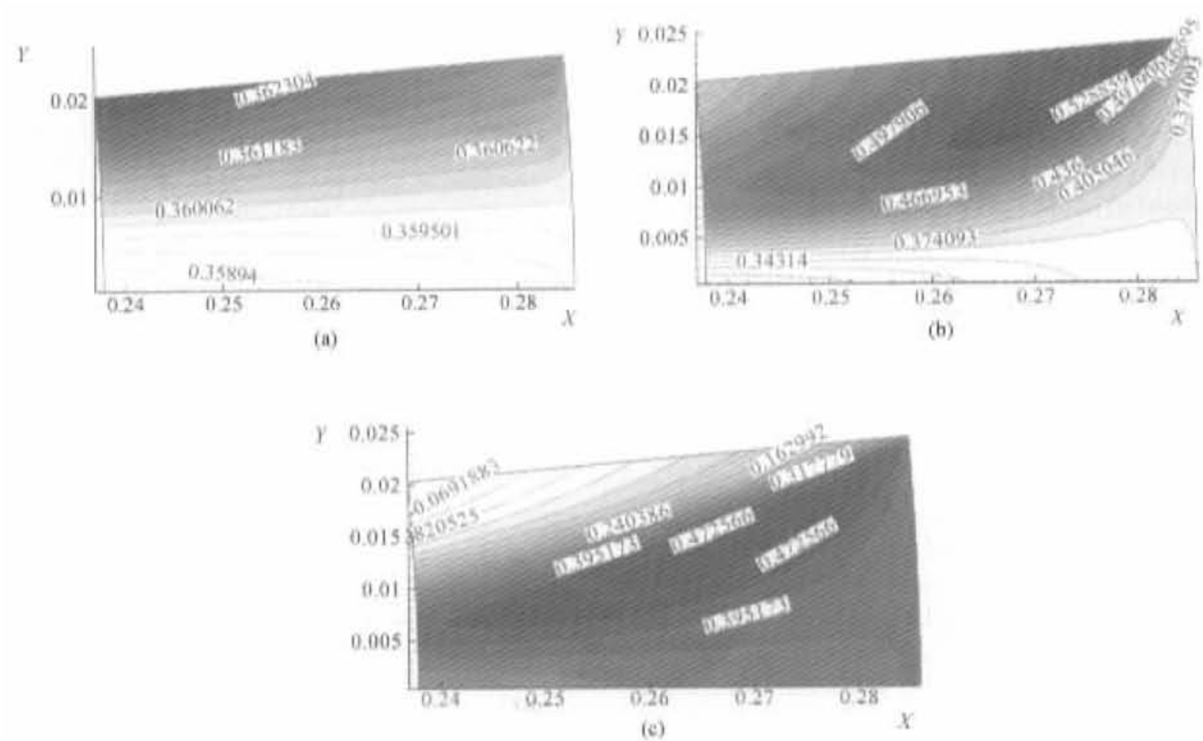

Fig.6 The evolution of the bulk effective viscosity with the increase of $Re \times t$ with $\beta_0 = \pi/36$
(a) $Re \times t = 13.5$s; (b) $Re \times t = 225$s; (c) $Re \times t = 405$s

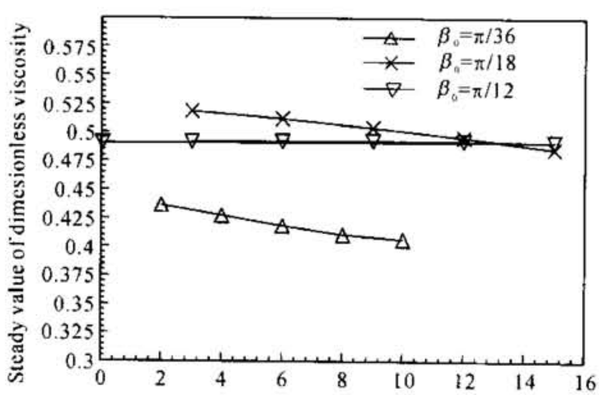

Fig.7 The relation between the steady effective viscosity and *Re* number at the center positions of different flows

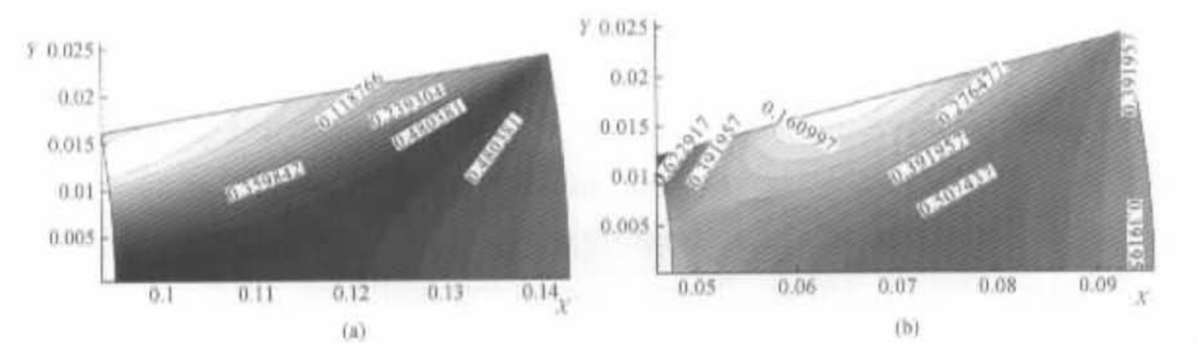

Fig.8 The bulk effective viscosity for different converging channel flows
(a) $\beta_0 = \pi/18$; (b) $\beta_0 = \pi/12$

CONCLUSIONS

The bulk stress and the effective viscosity depend on the rate-of-strain tensor and the fiber orientation state. The orientation state can be described by the orientation distribution function and the orientation tensors. In converging channel flows, the fiber suspensions evolve to be a steady alignment and tend to concentrate to some preferred directions which are close to but not the same as the directions of local streamlines. The fiber orientation state and the bulk effective viscosity are dependent on the product of Reynolds number and time in the case of given constant converging channel angle. The change of bulk effective viscosity will affect the structural features of the flows. The decrease of effective viscosity near the boundary benefits the increase of the rate of flow. Finally when the fiber alignment goes into steady state, the structural features of the fiber suspension are not dependent on the Reynolds number but on the converging channel angle.

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