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A simple channel estimator for space-time coded OFDM systems in rapid fading channels*

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Abstract: A simple channel estimator for space-time coded orthogonal frequency division multiplexing (OFDM) systems in rapid fading channels is proposed. The channels at the training bauds are estimated using the EM (expectation-maximization) algorithm, while the channels at the data bauds are estimated based on the method for modelling the time-varying channel as the linear combination of several time-invariant "Doppler channels". Computer simulations showed that this estimator outperforms the decision-directed tracking in rapid fading channels and that the performance of this method can be improved by iteration.

Key words: Space-time coding, OFDM, Channel estimation, EM algorithm, Doppler channel
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INTRODUCTION

Transmission diversity is an efficient technique for combating fading in mobile wireless communications. Many space-time coding technology have been proposed for transmission diversity. But these space-time techniques are only effective in flat fading channels. Orthogonal frequency division multiplexing (OFDM) modulation with a cyclic prefix can transform the frequency selective fading channels into multiple flat subchannels, so space-time coding can be applied to OFDM systems. Space-time coded OFDM was studied by Agarwal *et al*. (1998). Space-time coding is a promising technique to improve the efficiency and performance of OFDM systems.

Decoding of space-time codes requires channel state information (CSI) between multiple transmit antennas and receive antennas, which is usually difficult to obtain, especially for time-varying dispersive fading channels since the received signal is the superposition of the signals transmitted from different transmit antennas simultaneously. In most channel estimation algorithms for space-time coded systems (Li *et al*., 1998; 1999; Gong and Letaief, 2001; Nilsson *et al*., 1997), the channels are estimated utilizing the training or pilot symbols based on some algorithms, e.g., LS (least squares) or MMSE (minimum mean squared error). A channel estimation algorithm based on the EM algorithm for space-time coded OFDM system was proposed by Xie and Georghiades (2001). In that paper, the channels at the data bauds are estimated using decision-directed (DD) channel tracking; which is not efficient in rapid fading channels. Since the time-varying channel can be modeled as the sum of multiple time-invariant "Doppler channels" (Thomas and Vook, 2000), a simple channel estimator based on this channel model is proposed in this paper for space-time coded OFDM systems. Simulation results showed that this channel estimator could track channel variation effectively in rapidly time-varying channels.

The paper is organized as follows. Section II describes the space-time coded OFDM system and the channel model. In Section III, we present the proposed channel estimator based on EM algorithm and Doppler channel model. In Section IV, we evaluate the estimator by computer simulations and compare the proposed estimator with the decision-directed tracking method. We conclude the paper in Section V.

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SYSTEM DESCRIPTION

Space-time coded OFDM systems

Consider a space-time trellis coded (Tarokh *et al*., 1998) OFDM system with t transmit antennas and r receive antennas. Every transmitted symbol is selected from a unit-energy complex signal constellation φ. The space-time trellis coded symbols transmitted at antenna i are then modulated by inverse discrete Fourier transform (IDFT). At baud n, the frequency domain sample of received data after discrete Fourier transform (DFT) at receive antenna j at subcarrier k is,

$$Y_{nk}^{j} = \sum_{i=1}^{t} H_{nk}^{ji} X_{nk}^{i} + \eta_{nk}^{j},$$
$$j = 1, 2, \cdots, r, k = 0, 1, \cdots, K-1 \quad (1)$$

where H_{nk}^{ji} is the frequency response of channel at subcarrier k from transmit antenna i to receive antenna j, η_{nk}^{j} is the additive noise which can be modeled as independent and identically distributed (i.i.d) zero-mean complex Gaussian random variables. K is the number of subcarriers. X_{nk}^{i} is the symbol transmitted from transmit antenna i at subcarrier k.

Let $\boldsymbol{Y}_n^j = [Y_{n,0}^j \ Y_{n,1}^j \cdots \ Y_{n,K-1}^j]^{\mathrm{T}}$,

$\boldsymbol{H}_n^{ji} = [H_{n,0}^{ji} \ H_{n,1}^{ji} \cdots \ H_{n,K-1}^{ji}]^{\mathrm{T}}$,

$\boldsymbol{X}_n^i = \mathrm{diag}(X_{n,0}^i, \ X_{n,1}^i, \ \cdots \ X_{n,K-1}^i)$,

$\boldsymbol{\eta}_n^j = [\eta_{n0}^j \ \eta_{n1}^j \cdots \ \eta_{n,K-1}^j]^{\mathrm{T}}$, where superscript T denotes transpose of matrix. Thus Eq.(1) can be written in matrix form as

$$\boldsymbol{Y}_n^j = \sum_{i=1}^{t} \boldsymbol{X}_n^i \boldsymbol{H}_n^{ji} + \boldsymbol{\eta}_n^j, \ j = 1, 2, \cdots r \quad (2)$$

Channel model

The complex baseband representation of a mobile wireless channel impulse response can be described by (Proakis, 1995)

$$\bar{h}(t, \tau) = \sum_k \gamma_k(t) \delta(\tau - \tau_k) \quad (3)$$

where τ_k and $\gamma_k(t)$ are the time delay and complex amplitude of the kth path, respectively. $\gamma_k(t)$ is modeled as a wide-sense stationary (WSS) narrow-band complex Gaussian process, which are independent for different paths. The channel frequency response for OFDM system with proper cyclic extension and sample timing can be expressed as

$$H_{nk} = H(nT_f, k\Delta f) = \sum_{l=0}^{L-1} \bar{h}_{nl} \mathrm{e}^{-j2\pi lk/K} \quad (4)$$

where $\bar{h}_{nl} = \bar{h}(nT_f, lT_s)$, T_f and Δf are the block length and subcarrier spacing of the OFDM system, respectively. T_s is the sample interval, $T_s = 1/(K\Delta f)$. The average power and index of $\bar{h}_{hl}$ (σ_k^2 and L) depend on the delay profiles of the wireless channels.

In this paper, the channel frequency response matrix can be expressed as

$$\boldsymbol{H}_n^{ji} = \boldsymbol{W} \boldsymbol{h}_n^{ji} \quad (5)$$

where $\boldsymbol{W}$ is a matrix with $\boldsymbol{W}[k, l] = \mathrm{e}^{-j2\pi kl/K}$, $k = 0, 1, \cdots, K-1$, $l = 0, 1, \cdots, L-1$, $\boldsymbol{h}_n^{ji}$ is discrete channel impulse response vector between transmit antenna i and receive antenna j at baud n, and $\boldsymbol{h}_n^{ji} = [h_{n,0}^{ji} \cdots \ h_{n,L-1}^{ji}]^{\mathrm{T}}$.

The time-varying channel can also be modeled as the sum of multiple time-invariant Doppler channels. Thus define V and N_k similarly to Thomas and Vook (2000), we have

$$h_{nl}^{ji} = \sum_{v=-V}^{V} d_{lv}^{ji} \mathrm{e}^{j2\pi vn/N_k}, \ l = 0, 1, \cdots, L-1,$$
$$i = 1, 2, \cdots, t, j = 1, 2, \cdots, r \quad (6)$$

where d_{lv}^{ji} is the vth Doppler channel of the lth path of the time-domain channel between transmit antenna i and receive antenna j.

SIMPLE CHANNEL ESTIMATOR

In this section, we introduce the simple channel estimator proposed in this paper. This estimator performs its work in two steps. First, estimate the channels at training bauds using EM algorithm. Second, then estimate the channel at data bauds directly using the Doppler channels obtained from the channels at training bauds.

Channel estimation for training bauds

In this paper, the channels at training bauds are estimated using the EM algorithm proposed by Xie and Georghiades (2001). Consider the received data $\boldsymbol{Y}_n^j$ as incomplete data, and define the complete data $\boldsymbol{R}_n^{ji}$ as

$$\boldsymbol{R}_n^{ji} = \boldsymbol{X}_n^i \boldsymbol{W} \boldsymbol{h}_n^{ji} + \boldsymbol{\omega}_n^{ji}, \quad i = 1, 2, \cdots, t, j = 1, 2, \cdots, r \tag{7}$$

where，$\boldsymbol{Y}_n^j = \sum_{i=1}^{t} \boldsymbol{R}_n^{ji}$，$\boldsymbol{\eta}_n^j = \sum_{i=1}^{t} \boldsymbol{\omega}_n^{ji}$. The iterative process is as follows.

E-Step：For $i = 1, 2, \cdots, t, j = 1, 2, \cdots, r$，compute

$$\widetilde{\boldsymbol{R}}_n^{ji}(k) = \boldsymbol{X}_n^i \boldsymbol{W} \widetilde{\boldsymbol{h}}_n^{ji}(k) + \beta_{ij}\left[\boldsymbol{Y}_n^j - \sum_{m=1}^{t} \boldsymbol{X}_n^m \boldsymbol{W} \widetilde{\boldsymbol{h}}_n^{jm}(k)\right] \tag{8}$$

M-Step：For $i = 1, 2, \cdots, t, j = 1, 2, \cdots, r$，compute

$$\widetilde{\boldsymbol{h}}_n^{ji}(k+1) = \min_{\boldsymbol{h}_n^{ji}}\{\| \widetilde{\boldsymbol{R}}_n^{ji}(k) - \boldsymbol{X}_n^i \boldsymbol{W} \boldsymbol{h}_n^{ji} \|^2\} = \frac{1}{K} \boldsymbol{W}^H (\boldsymbol{X}_n^i)^{-1} \widetilde{\boldsymbol{R}}_n^{ji}(k) \tag{9}$$

where k denotes the kth iteration and superscript H denotes conjugate transpose of matrix. β_{ij} satisfies $\sum_{i=1}^{t} \beta_{ij} = 1$.

The initial estimate of the channel for the EM iteration can be obtained using the following Eq.（10）.

$$\widetilde{\boldsymbol{h}}_n^{ji}(0) = \frac{1}{K} \boldsymbol{W}^H (\boldsymbol{X}_n^i)^{-1} \boldsymbol{Y}_n^j, \quad i = 1, 2, \cdots, t, j = 1, 2, \cdots, r \tag{10}$$

Channel estimation for data bauds

In Xie and Georghiades（2001），the channels at data bauds are estimated using DD tracking algorithm；which is not efficient in rapid fading channels because the channel changes much between OFDM symbols. So we present a channel estimator for the data bauds suitable for transmission in rapid fading channels in this section.

Using the Doppler channel model aforementioned，we define

$$\boldsymbol{q}_n = \begin{bmatrix} \mathrm{e}^{-j2\pi Vn/N_k} \\ \vdots \\ \mathrm{e}^{j2\pi Vn/N_k} \end{bmatrix},$$

$$\boldsymbol{d}^{ji} = \begin{bmatrix} d_{0,-V}^{ji} & \cdots & d_{0,V}^{ji} \\ \vdots & \ddots & \vdots \\ d_{L-1,-V}^{ji} & \cdots & d_{L-1,V}^{ji} \end{bmatrix},$$

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{d}^{11} \\ \vdots \\ \boldsymbol{d}^{1t} \\ \vdots \\ \boldsymbol{d}^{rt} \end{bmatrix}, \boldsymbol{h}_n = \begin{bmatrix} \boldsymbol{h}_n^{11} \\ \vdots \\ \boldsymbol{h}_n^{1t} \\ \vdots \\ \boldsymbol{h}_n^{rt} \end{bmatrix}, \text{then}$$

$$\boldsymbol{h}_n \doteq \boldsymbol{D} \boldsymbol{q}_n \tag{11}$$

In Eq.（11），where Doppler channel matrix $\boldsymbol{D}$ is a constant，we can obtain it from training symbols. Assuming there are p training symbols located at bauds n_1，n_2，$\cdots$，n_p in an OFDM frame and the remaining b data symbols are located at bauds $m_1, m_2, \cdots, m_b$；we have

$$\underline{\widetilde{\boldsymbol{h}}}_p = \boldsymbol{D} \boldsymbol{Q}_p \tag{12}$$

where $\underline{\widetilde{\boldsymbol{h}}}_p = [\widetilde{\boldsymbol{h}}_{n_1} \; \widetilde{\boldsymbol{h}}_{n_2} \cdots \widetilde{\boldsymbol{h}}_{n_p}]$，$\boldsymbol{Q}_p = [\boldsymbol{q}_{n_1} \; \boldsymbol{q}_{n_2} \cdots \boldsymbol{q}_{n_p}]$. The LS estimates of Doppler channels are

$$\widetilde{\boldsymbol{D}} = \underline{\widetilde{\boldsymbol{h}}}_p \boldsymbol{Q}_p^H (\boldsymbol{Q}_p \boldsymbol{Q}_p^H)^{-1} \tag{13}$$

thus the channel estimates for data bauds are

$$\underline{\widetilde{\boldsymbol{h}}}_b = \widetilde{\boldsymbol{D}} \boldsymbol{Q}_b \tag{14}$$

where $\underline{\widetilde{\boldsymbol{h}}}_b = [\widetilde{\boldsymbol{h}}_{m_1} \; \widetilde{\boldsymbol{h}}_{m_2} \cdots \widetilde{\boldsymbol{h}}_{m_b}]$，$\boldsymbol{Q}_b = [\boldsymbol{q}_{m_1} \; \boldsymbol{q}_{m_2} \cdots \boldsymbol{q}_{m_b}]$. Thus after the time-domain channels at data bauds are all estimated，the channel frequency response can be obtained using Eq.（5）.

We can view every frame as the first frame，so the locations of training or data symbols are invariant from frame to frame. $\boldsymbol{Q}_p$，$\boldsymbol{Q}_b$ and $\boldsymbol{Q}_p^H \cdot (\boldsymbol{Q}_p \boldsymbol{Q}_p^H)^{-1}$，can be calculated at initialization and need not to be computed for every frame. Compared to the decision-directed tracking estimator，this method is very simple and its computation complexity is very low.

SIMULATION RESULTS

In this section，we evaluate the performance of the proposed channel estimator by computer simulations for two-path mobile radio channels. Each path with equal average power experiences independent Rayleigh-fading. The entire bandwidth is 800 kHz and is divided into 128 subcarriers. The duration of one OFDM symbol is 200 μs，including 160 μs effective symbol period and 40 μs guard interval. The rms（root mean square）delay spread is 2.5 μs. The receiver and the terminal were all equipped with two antennas. Space-time trellis codes of QPSK with 16 states were used in simulations. For the EM algorithm, $\beta_{ij} = 0.5$ for all i and j. For the Doppler channel estimates, $V = 1$ and $N_k = 500$.

We used an eleven bauds OFDM system with the first, sixth and eleventh bauds being training symbols and then some channel estimation methods were implemented on the remaining data bauds. The eleventh baud was considered as both the ending of the current frame and the beginning of the next frame. The following four algorithms were used to test the channel estimator presented in this paper (their label in the figures is in parenthesis).

1. Decision-directed tracking (EM + DD) similar to the method proposed by Xie and Georghiades (2001). The channels at the training baud were estimated using the EM algorithm with 20 iterations, then channel estimates of baud n were used to decode the data at baud $n+1$. The decoded bits were re-encoded into symbols as the training symbols. The channel at baud n was used as the initial channel at baud $n+1$ and the channel at $n+1$ was still estimated using EM algorithm with 5 iterations.

2. Channels at data bauds estimated using the Doppler channel model (EM + Dop). The Doppler channels were obtained using Eq. (13) from the channels at training bauds estimated based on the EM algorithm with 20 iterations; then the channels at data bauds could be obtained using Eq. (14) directly.

3. Channels at data bauds estimated using the Doppler channel model with iteration (EM + Dop + iter). This method is the same as the second method except that the decoded bits were re-encoded into symbols as training symbols and then the EM algorithm was used to estimate the channel again. The channels at data bauds estimated using method 2 were used as the initial estimation of the iteration and only 5 iterations were needed in the EM algorithm. Then the new Doppler channels were obtained from the channels at all bauds in a frame. The final channel estimates based on the new Doppler channels were used to demodulate the symbols again.

4. The channel information is known at the receiver (Optimal).

The performance of the above methods were evaluated in time-varying multipath channels under different Doppler frequency. Fig. 1 shows the BER performance when Doppler frequency was 50 Hz. The DD tracking method had nearly the same performance as the channel estimator based on Doppler channel model since the channel fading was slow. The third estimator had better performance than the second one because some errors could be corrected by iteration.

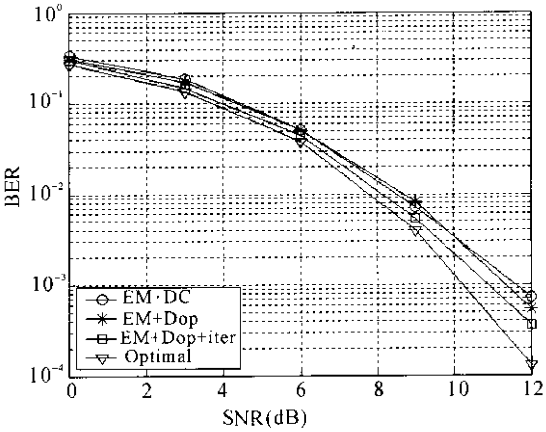

Fig. 1 The performance for 50 Hz Doppler frequency

Fig. 2 shows the BER performance of the four estimators when Doppler frequency was 300 Hz. For this simulation, we can see that the performances of these channel estimators were quite different. The worst performance of these channel estimators was that of the DD tracking method because it could not track the rapid channel variations perfectly. The best performance of the first three methods was also the one based on Doppler channel model with iteration. Comparing the curves in Fig. 1 and Fig. 2, we

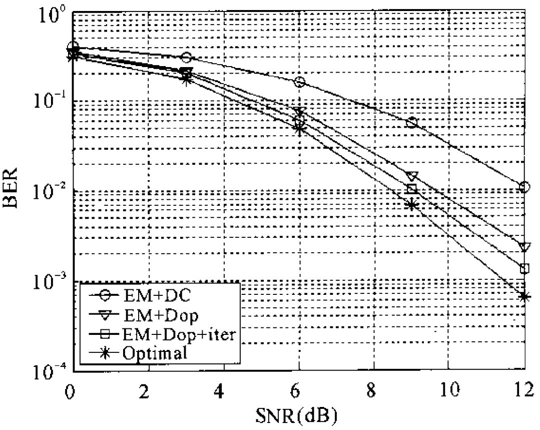

Fig. 2 The performance for 300 Hz Doppler frequency

can see that the channel estimator proposed in this paper is more efficient than the DD tracking method in rapid fading channels.

CONCLUSIONS

This paper presents a simple channel estimator based on the EM algorithm and Doppler channel model for space-time coded OFDM system in rapid fading channel. Compared with the decision-directed tracking method，this estimator can more effectively track rapid channel variations. The performance of the proposed estimator can be improved by iteration. But the additional iteration increases the computational complexity of the algorithm. Thus there is a tradeoff between the performance and complexity of the algorithm. This channel estimator can also be used in other OFDM systems with transmit diversity.

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