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A robust tolerance design method based on process capability*

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Abstract: This paper presents a method for robust tolerance design in terms of Process Capability Indices (PCI). The component tolerance and the suitable manufacturing processes can be selected based on the real manufacturing context. The robustness of design feasibility under the effect of uncertainties is also discussed. A comparison between the results obtained by the proposed model and other methods indicates that robust and reliable tolerance can be obtained.

Key words: Tolerance design, Robust design, Process capability
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INTRODUCTION

Tolerance design is a bridge to link the requirement of product performance and the manufacturing process; and affects not only the product performance, but also the cost of the product and the manufacturing process. Therefore Computer Aided Tolerancing (CAT) becomes the key technique for integrating CAD/CAPP/CAM (Wu and Yang, 1999). Traditionally, the tolerance is assigned to the component dimensions according to the minimum cost. The restriction of the manufacturing ability of tools that the factory owns is neglected (Ji *et al.*, 1999). As a result, it is difficult to obtain economical manufacturing performance, and to get suitable machines that satisfy the designed tolerance.

This paper presents a method for robust tolerance design in terms of Process Capability Indices (PCIs) (Huang and Yang, 2000; Lee and Wei, 1998). The method can make full use of knowledge from the real manufacturing context. When producing the robust tolerance, the corresponding manufacturing processes are selected at the same time. The manufacturability of a product and the robustness of the tolerances can be ensured.

PROCESS CAPABILITY INDICES

Understanding processes and quantifying process performance are essential for any successful quality improvement initiative. The relationship between the actual process performance and the specification limits or tolerance may be quantified using appropriate process capability indices. Many capability indices providing numerical measures of whether a production process meets predetermined specification limits have been proposed. One of the commonly used in manufacturing industries for centered process, called C_p, is defined as (Huang and Yang, 2000)

$$C_p = \frac{T_u - T_l}{6\sigma} = \frac{T}{6\sigma} \quad (1)$$

where T_u, T_l are the upper and lower specification limits, respectively, T the specification tolerance, and σ the process standard deviation.

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The manufacturability and the satisfaction degree to the technology requirement of the process capacity can be evaluated by the index C_p. In order to reflect the impact of the deviation of the process mean μ from the midpoint y_0 of the specification limits on the process capability, another PCI called C_{pk} is defined as

$$C_{pk} = \min\left\{\frac{\mu - T_l}{3\sigma}, \frac{T_u - \mu}{3\sigma}\right\} \quad (2)$$

C_{pk} is sometimes written as

$$C_{pk} = (1 - k)\frac{T}{6\sigma} \quad (3)$$

where $k = 2\left|y_0 - \mu\right|/(T_u - T_l)$, and $y_0 = \frac{T_u + T_l}{2}$.

THE ROBUST TOLERANCE DESIGN METHOD BASED ON PCI

1. Objective function

(1) Manufacturing cost

Here, the manufacturing cost contains the cost for defects. The scrap rate of the i-th process can be computed by the following equation (Zhang, 1999).

$$P_i = 1 - \Phi[3C_{pki}] + \Phi\left[-3C_{pki}\frac{1 + k_i}{1 - k_i}\right] \quad (4)$$

Assuming that the manufacturing cost of the i-th method is C_i, the corresponding total manufacturing cost is expressed by the following equation (supposing that the unqualified products are wasters and the corresponding repair cost is $\psi_i C_i$).

$$C_i' = C_i(1 + \psi_i P_i) \quad (5)$$

where ψ_i is a scale coefficient.

(2) Quality loss

Variability in the production process is unavoidable owing to inconsistency in tool, workpiece, material and process parameters. Taguchi suggested that the quality of a product is the loss incurred due to the deviations of the products' characteristics from their target values. And the quality loss is (Wu and Yang, 1999)

$$Q_i' = K_i[\sigma_i^2 + (\mu_i - y_{0i})^2] \quad (6)$$

where K_i is the quality loss coefficient. Assuming that the loss due to the defect is $\lambda_i C_i$ (λ_i is a scale coefficient), then $K_i = \frac{\lambda_i C_i}{(T_i/2)^2}$.

The i-th tolerance can be obtained from Eq. (3).

$$T_i = \frac{6\sigma_i C_{pki}}{1 - k_i} \quad (7)$$

And Eq. (6) can be rewritten as

$$Q_i' = K_i[\sigma_i^2 + (k_i T_i/2)^2] = \left(k_i^2 + \frac{1 - k_i}{9C_{pki}^2}\right)\lambda_i C_i \quad (8)$$

As mentioned before, there is a need to adjust the design tolerances to reach an economic balance between quality loss and manufacturing costs for product tolerance design. The total cost function can be defined as

$$Min\ C = \sum_{i=1}^{n}\sum_{j=1}^{m_i} x_{ij}(C_{ij}' + Q_{ij}') = \sum_{i=1}^{n}\sum_{j=1}^{m_i} x_{ij}\left(1 + \psi_{ij}P_{ij} + \lambda_{ij}k_{ij}^2 + \frac{\lambda_{ij}(1 - k_{ij})}{9C_{pkij}^2}\right)C_{ij} \quad (9)$$

where n is the total quantity of the dimensions in an assembly, m_i the total quantity of machining methods for the i-th dimension, C_{ij}', C_{ij}, Q_{ij}' the total manufacturing cost (containing the defect cost), manufacturing cost and quality loss of the i-th dimension using the j-th method respectively, x_{ij} is a discrete(0 − 1)variable. If the j-th method is chosen for the i-th dimension, then $x_{ij} = 1$, otherwise $x_{ij} = 0$.

2. Constraint functions

There are three types of constraints for the optimization model.

(1) Process capacity constraint

$$C_{pij}^L \leqslant C_{pij} \leqslant C_{pij}^U \quad (10)$$

where C_{pij}^U, C_{pij}^L are the upper and lower boundary of C_{pij} respectively.

(2) Constraints on machining methods selection

Only one machining method could be chosen for each dimension.

$$\sum_{j=1}^{m_i} x_{ij} = 1 \quad (11)$$

(3) Assembly requirements

The assembly requirements embody quality characteristic required by the product. It should satisfy:

a) Minimize the mean shift

The mean shift of the dimension is

$$\Delta_{\lim ij} = \mu_{ij} - y_{0ij} = k_{ij}\frac{T_{ij}}{2} = \frac{3k_{ij}C_{pkij}\sigma_{ij}}{1-k_{ij}} \quad (12)$$

Assuming that the maximal shift is Δ, and adopting a statistical model, then

$$g_1 = \Delta_{\lim\sum} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m_i}\left|\frac{\partial f}{\partial T_i}x_{ij}\Delta_{\lim ij}\right|^2} \leqslant \Delta \quad (13)$$

where f is an assembly function, $\frac{\partial f}{\partial T_i}$ the partial derivative of assembly function to the tolerance T_i.

b) Minimize the deviation

The standard deviation of each dimension, σ_{ij}, is the standard deviation of the chosen machining method. The standard deviation of the functional tolerance should satisfy

$$g_2 = \sigma_{\sum} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m_i}\left(\frac{\partial f}{\partial T_i}x_{ij}\sigma_{ij}\right)^2} \leqslant \sigma_{\lim} \quad (14)$$

where $\sigma_{\lim}$ is the upper limit of $\sigma_{\sum}$ given by designer.

It is generally recognized that there always exist uncertainties in engineering systems due to variations in design conditions. When we optimize a problem, the optimal solution is often at one or more constraint boundaries and fluctuations can cause the design to violate a binding constraint. Many methods can be adopted to achieve the feasibility robustness. The worst-case model is used in the design. Using a Taylor series, a first order approximation of the transmitted variation is given as (Du and Chen, 2000)

$$\Delta g_i = \Delta g_i(X) = \sum_{j=1}^{n}\left|\frac{\partial g_i}{\partial x_j}\Delta x_j\right| \quad (15)$$

When the variations Δg_1, Δg_2 obtained from the above formulation, the Eqs. (13) and (14) will translate into

$$g_1' = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m_i}\left|\frac{\alpha f}{\alpha T_i}x_{ij}\Delta_{\lim ij}\right|^2} + \Delta g_1 \leqslant \Delta \quad (16)$$

$$g_2' = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m_i}\left(\frac{\alpha f}{\alpha T_i}x_{ij}\sigma_{ij}\right)^2} + \Delta g_2 \leqslant \sigma_{\lim} \quad (17)$$

Thus the parameter that should determine in the design is μ_{ij} (to determine the methods for each dimension) and C_{pkij} (to optimize the process capacity of each machining method).

APPLICATION EXAMPLES

In order to illustrate the proposed methodology, a simple component is used (He, 1996). The assembly drawing of parts, which is usually used in tools, is shown in Fig. 1a and the dimension chain is shown in Fig. 1b. The gap between the revolution axes 1 and sleeve 3 is

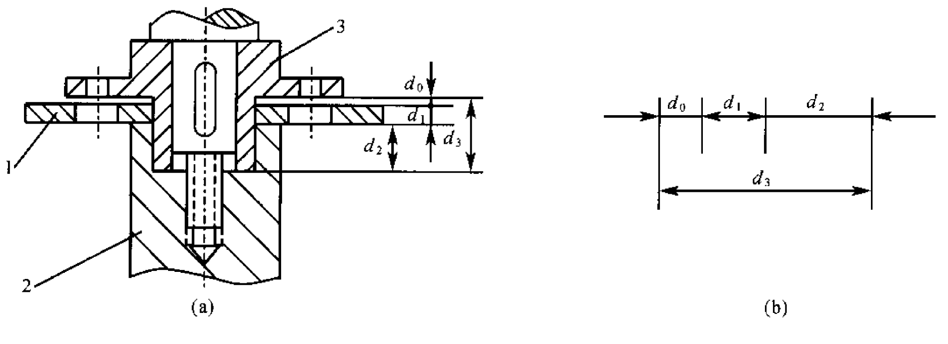

Fig. 1 The assembly drawing and dimension chain
(a) the assembly drawing; (b) the dimension chain

Table 1 Available machining methods and corresponding parameters

Dimension	Method 1			Method 2			Method 3		
	σ(mm)	k(%)	C*	σ(mm)	k(%)	C	σ(mm)	k(%)	C
d_1	0.018	0.03	1.7347	0.020	0.03	1.4089	0.022	0.04	1.1678
d_2	0.022	0.03	1.4548	0.023	0.03	1.3327	0.025	0.04	1.1311
d_3	0.023	0.03	1.5953	0.028	0.04	1.0829	0.033	0.04	0.7852

* The cost involved in the example is relative cost

required to be 0.1 – 0.4 mm and $k \leqslant 0.04$. The nominal dimensions are $d_1 = 38$ mm, $d_2 = 42$ mm, $d_3 = 83$ mm. According to the real manufacturing context and the economical machining precision, different machining method chosen are listed in Table 1.

In this example, $n = 3$, $m_1 = m_2 = m_3 = 3$. The limit of the process capacity index of each machining method is $C_{pij}^{U} = 1.67$, $C_{pij}^{L} = 1.0$ (for all i, j). Here $\Delta g_1 = 0.05\Delta$, $\Delta g_2 = 0.05\sigma_{\lim}$. The results optimized according to the model represented are shown in Table 2 (The minimum of the total cost is 4.3393).

Table 2 The design results

Dimension	d_1	d_2	d_3
Machining method	3	2	3
σ(mm)	0.022	0.023	0.033
k(%)	0.04	0.03	0.04
C_{pk}	1.0617	1.2354	1.0000
Tolerance(mm)	0.1401	0.1705	0.1980
Manufacturing cost	1.1678	1.3327	0.7852
Scrap rate(%)	0.0010	1.4672e-4	0.0019
Defect cost	0.0012	1.9553e-4	0.0015
Quality loss cost	0.1124	0.0953	0.0850

Comparison with other methods is given in Table 3. Analysis of the results in the table indicated that, as compared to the WS model, wider tolerances are assigned and the manufacturing cost is reduced. Moreover, there are no suitable machining methods in real manufacturing context for the tolerance given by the WS model. As compared to the statistical model, though its tolerance is wider and the cost is less (the manufacturing cost is 2.5431). However, the cost obtained using the machining method chosen within the manufacturing context, is 3.3818 and more than the cost of the proposed method.

Therefore the method presented is feasible and effective. The real manufacturing context is fully considered in the design.

CONCLUSIONS

The traditional tolerance design methods ignore the process capacity of the equipments. Therefore, the allocated tolerance are usually overestimated or underestimated and may not even remain economic for manufacturing. A robust tolerance design method based on the manufacturing context is proposed in this paper. Making full use of the real manufacturing context, the component tolerance and the suitable manufacturing processes can be selected. One can obtain higher quality and robustness. At the same time the reliability of the designed tolerance can be ensured.

Table 3 Comparison with other methods

	T_1		T_2		T_3		C
	Tolerance (mm)	Machining method	Tolerance (mm)	Machining method	Tolerance (mm)	Machining method	
Worst-case model	0.0931	–	0.1003	–	0.1066	–	–
Statistical model	0.1641	3	0.1735	3	0.1816	2	3.3818
Proposed	0.1401	3	0.1705	2	0.1980	3	3.2857

Notes: In the worst-case method and the statistical method the quality loss is not considered and assumes $C_{pk} = 1$ and $k = 0$

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