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Sensitivity analyses of cables to suspen-dome structural system*

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Abstract: The construction of the cables is a key step for erecting suspen-dome structures. In practical engineering, it is difficult to ensure that the designed pre-stresses of cables have been exactly introduced into the structures in the site; so it is necessary to evaluate the influence of the variation of the pre-stresses on the structural behavior. In the present work, an orthogonal design method was employed to investigate the pre-stressed cables' sensitivity to the suspen-dome system. The investigation was concentrated on a Kiewitt suspen-dome. Parametric studies were carried out to study the sensitivity of the structure's static behavior, dynamic behavior, and buckling loads when the pre-stresses in the cables varied. The investigation indicated that suspen-dome structures are sensitive to the pre-stresses in all cables; and that the sensitivity depended on the location of the cables and the kind of structural behavior. Useful suggestions are given at the end of the paper.

Key words: Suspen-dome, Orthogonal design method, Cable, Sensitivity analysis
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INTRODUCTION

Suspen-dome system is a new hybrid spatial structure form. It applies the concept of tensegrity of cable dome to the single-layer dome system (Mamoru and Masaru, 1994; Mamoru *et al*., 1999; Kono *et al*., 1999; Qian, 1995; Liu, 2003), and combines the advantages of cable domes and single-layer domes. It can efficiently improve the global stability of single-layer domes and enable spanning the structure over longer space. Contrary to completely flexible structures, such as cable domes, suspen-domes have inherent structural stiffness, which greatly simplifies the design, construction and details of joints. On the other hand, the cable domes and single-layer domes can offset their effects on the supporting structures, which decreases the sensitivity of the structural behavior of suspen-domes of the supporting structures. Theoretical and experimental investigations of the suspen-domes were concentrated on the static behavior and seismic response (Kono *et al*., 1999; Yi and Liu, 2000; Cui and Guo, 2003; Kang *et al*., 2003). Technical problems encountered during the construction had been solved theoretically (Dong *et al*., 2003). However, comparisons between the design and measured initial pre-stresses in a practical project demonstrated that the problem was more complex than expected. The project located at Taizhou, Zhejiang Province shows that the error can be up to 22.7%, which obviously exceeds the tolerance specified by the design code (Gao *et al*., 2003; Gao and Weng, 2004). Possible reasons could be: (1) practical construction conditions were not identical with theoretical assumptions; for instance, it was impossible to properly simulate the structural behavior of the scaffolds theoretically; (2) factors such as working environments, pipe length and the accuracy of the oil jacks, can affect the

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precision of the constructions, all these facts cannot be taken into considerations simultaneously in theoretical analyses. The great discrepancy between the theoretical pre-stresses and the measured ones involves two issues: (1) whether it is necessary to rectify the pre-stresses and (2) how to rectify them. On the other hand, it is difficult to implement increment pre-stressing procedures, especially when a large number of cables are used in practical engineering, because it is difficult to properly put cables in place. In the present work, an orthogonal design method (Wu, 1997; Yang and Wang, 1998) was adopted to investigate the sensitivity of cables. Results of nonlinear static, dynamic, and buckling analyses led to the conclusions presented at the end of the paper.

ORTHOGONAL DESIGN METHOD

The general method of sensitivity analysis was, at a certain step, to vary a certain factor progressively within the range of the factor which influences the experimental result, and fix other factors, analyze the parameter value, obtain sensitive degree of factors. This kind of method has not enough coverage and is comparatively isolated and static.

The orthogonal experimental design method adopts a set of available orthogonal tables, regularly picks out a series of typical test points from the overall multifactor test, and then does a test under treatment combinations. Orthogonally calculated data from analysis of variance can offer detailed information. The actual method is as follows: select a suitable orthogonal table $L_n(t^m)$, where L is the form symbol; n is the times of experiment arranged in the form; t is the level number; m is the number of the form columns, and then calculate and analyze on the basis of definite calculation scheme for the level of every factor setting, to achieve the result finally.

Now we arrange the calculation scheme based on $L_n(t^m)$, the result of the ith scheme marked as y_i, i=1,2,3,…,n, and $y_1,y_2,\ldots,y_n$ are independent of each other and obey normal distribution, namely, $y_i \sim N(\mu, \sigma^2)$ (i=1,2,…,n), and perform variance analysis of y_i; in other words, perform significance test on the hypothesis H_0: $\mu_1=\mu_2=\mu_3=\ldots=\mu_n$.

Applying the principle of hypothesis testing, create a statistic for F-test, the significance test procedure is described as follows:

$$T=\sum_{t=1}^{n} y_t,\ \bar{y}=\frac{T}{n},\ r=\frac{n}{t} \tag{1}$$

$$S_T=\sum_{i=1}^{n}(y_i-\bar{y})^2,\ S_j=r\sum_{i=1}^{t}(T_{ij}/r-\bar{y})^2 \quad (j=1,2,\ldots,m) \tag{2}$$

where T_{ij} is the sum of y_i of the ith level in the jth column of table $L_n(t^m)$, r is the replications times of the same level in a certain column, S_T is the sum of squares of $(y_i-\bar{y})$, S_j is the sum of squares of $(T_{ij}/r-\bar{y})$, which denotes the difference between the different level of the jth column in table $L_n(t^m)$.

The degrees of freedom of S_T, S_j are denoted by f_T, f_j respectively, and

$$f_T=n-1, f_j=t-1 \tag{3}$$

The sum of $(y_i-\bar{y})^2$ of all the void columns is denoted as S_e; accordingly the sum of the degree of freedom is denoted as f_e. When the calculated $\bar{S}_j=S_j/f_j$ of a certain column is less than $\bar{S}_e=S_e/f_e$, S_j is regarded as error term and is merged into S_e. So when all the error terms S_j are merged into S_e, we get a new one denoted as S_e^{Δ}, meanwhile, after degrees of freedom f_j are merged into f_e, we get a new one denoted as f_e^{Δ}; and then obtain

$$F_j=\frac{S_j/f_j}{S_e/f_e} \text{ or } F_j^{\Delta}=\frac{S_j/f_j}{S_e^{\Delta}/f_e^{\Delta}} \tag{4}$$

for testing. When H_0 is true, $F_j \sim F(f_j,f_e)$, j=1,2,…,m. So when F_j is more than $F_{1-a} \sim F(f_j,f_e)$, for the given significance level a, the factor is deduced to be significant, otherwise it is not significant.

STRUCTURAL MODEL

The model employed in this study is shown in Fig.1. The span and rise of the Kiewitt suspen-dome structure are 40 m and 5 m respectively (i.e. the rise to span ratio is 0.125). The Kiewitt dome uses $K_{6\times6}$ for gridding; the strut length of 1st to 3rd hoop is 2.5 m, and the 4th and 5th hoops are 2.0 m. Steel tubes 121 mm in diameter and 3.5 mm in thickness were used as ribs and latitudinal members, 114 mm in diameter and 3.5 mm in thickness were used as radial members in the single-layer steel truss (the upper part of the suspen-dome structure), and steel tubes 90 mm in diameter and 2.5 mm in thickness were used as the struts of the suspen-dome structure. Each cable was made of ten strands of 5 mm diameter steel wires. The Young's modulus of the tubes was 206 GPa and that of the cables was 180 GPa. The hoop and the radial cables were pre-stressed with the values being 50 kN and 20 kN respectively. All interior nodes of the single-layer dome were assumed to connect rigidly, while the perimeter members were pinned to the supporting members. Vertical load distributed uniformly at 1.0 kN/m^2 (precluding self-weight) on the top surface of the dome. Seven pre-stress levels were adopted to investigate the effects of pre-stresses variation on the maximum stress of the structure. Taking the designed pre-stresses as the basis, the pre-stresses were increased and decreased by 10%, 20% and 30%.

THEORY OF SENSITIVITY ANALYSIS IN SUSPEN-DOME

Suspen-dome structural system belongs to hybrid cable-strut structure; its basic units consist of beams (members in the single-layer dome), struts (members in the tensegric system) and cables (hoop and radial cables). Equilibrium equations were derived on the basis of three models: beam element model and cable element model both based on nonlinear finite element theory (Kaid, 1999; Zhang and Shen, 2000); beam-column model based on beam-column theory. A general nonlinear finite element analysis software package ANSYS was employed for sensitivity analysis of the suspen-dome structure. Structural members–beams, struts and cables, were discretized with elements BEAM4, LINK8 and LINK10 respectively.

Orthogonal design method was applied for sensitivity analyses of cables classified into seven groups (a group is defined as a factor) in Fig.2. Seven pre-stress levels described in the previous section were involved.

The analysis procedures were signed according to orthogonal table $L_{49}(7^8)$, where seven factors occupy the first seven columns of the table, while the eighth column is left blank for further variance analysis of orthogonal experimental design. Totally 49 case studies were carried out, including static, dynamic, and buckling analyses. In the nonlinear static analyses, the structure was assumed to carry a uniformly distributed load of 1 kN/m^2, while for transient dynamic analyses, the EL-Centro earthquake records were adopted. The increment equilibrium equation for stability analyses can be expressed as follow (Hangai and Kawaguchi, 1990):

$$[\boldsymbol{K}]\{\Delta\boldsymbol{u}\}=\Delta\lambda\{\boldsymbol{p}\} \tag{5}$$

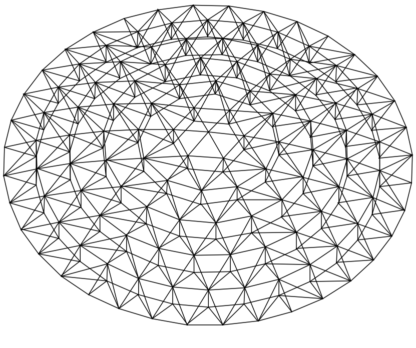

Fig.1 Components of the suspen-dome structure

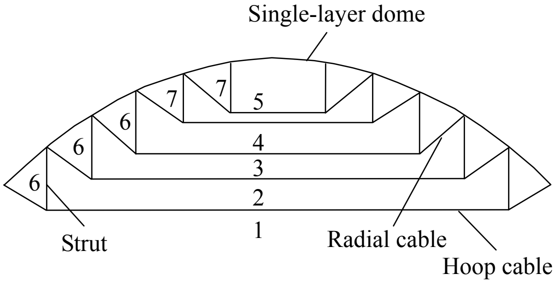

Fig.2 Numbering of cables

where [**K**] is the structural tangent stiffness matrix of U.L. (Updated Lagrange), $\{\Delta \boldsymbol{u}\}$ is the increment vector of displacement, $\Delta\lambda$ is the load increment parameter, $\{\boldsymbol{p}\}$ is the load reference vector. Before the critical load, $|\boldsymbol{K}|\neq 0$, thus, an exclusive solution of the linear Eq.(5) can be obtained by using LDLT. At the critical point, $|\boldsymbol{K}|\neq 0$, rank$(\boldsymbol{K})\leq n-1$, and the structural stiffness matrix is singular. On the basis of the generalized incremental algorithm (Hangai, 1981; Chen *et al.*, 2002), the Eq.(5) can be written as:

$$\boldsymbol{A}\begin{Bmatrix}\Delta\delta \\ \Delta\lambda\end{Bmatrix}=\{0\} \tag{6}$$

where $\boldsymbol{A}=[\boldsymbol{K}-f]$, is the augmented equilibrium matrix with $n\times(n+1)$ dimensions.

By using the generalized inverse theory, the solution of the Eq.(6) is:

$$\begin{Bmatrix}\Delta\delta \\ \Delta\lambda\end{Bmatrix}=[I_{n+1}-A^{+}A]\boldsymbol{a}=\alpha\ null(\boldsymbol{A})=\alpha\boldsymbol{a} \tag{7}$$

where $\boldsymbol{A}^{+}$ is the *M-P* generalized inverse of $\boldsymbol{A}$, $\boldsymbol{a}$ is the zero spatial radix and the orthogonal radix of $[I_{n+1}-\boldsymbol{A}^{+}\boldsymbol{A}]$. α is a number without dimensions, named the generalized increment. Regarding the physical significance, the conservative structure has an exclusive equilibrium route at non-critical points and limit points so that $\boldsymbol{a}$ is a vector of one-dimension; while the conservative structure has two equilibrium routes at least at the bifurcation point so that $\boldsymbol{a}$ is a multi-dimension vector. The results of nonlinear analyses showed that the suspen-dome presented in this paper has no bifurcation points.

RESULTS OF ANALYSES OF CABLES SENSITIVITY TO SUSPEN-DOME

Table 1 listed the maximum static and dynamic stresses, the maximum static and dynamic displacements, and buckling loads. The results listed in Table 1 can be used for evaluating the sensitivity degree of cables. Tables 2–4 give the evaluation results. The last column of the tables is the sensitivity of cables to static and dynamic stresses and buckling loads. The number of stars represents the significance. It is determined according to the significance level F_j as follows:

If $F_j>F_{0.99}$, it is of great significance, marked with three stars; If $F_{0.95}<F_j<F_{0.99}$, it is of average significance, marked with two stars; If $F_j<F_{0.95}$, it is of minor significance, marked with one star.

Figs.3–5 described the structural response when varying the pre-stresses of a certain group of cables. A group is defined as a factor. Number of factor has shown in Fig.2. The designed pre-stresses in the hoop and the radial cables are 50 kN and 20 kN respectively. Taking the designed pre-stresses as the basis, the pre-stresses were increased by 5%, 10%, 15%, 20%, 25% and 30%.

Analyses of cables static sensitivity to suspen-dome

Table 2 gives the results of the static sensitivity analyses. The table shows that the outermost hoop cable was the most sensitive; that its significance level was much larger than that of the other cables; that all cables, except for the innermost, were sensitive; that outer three rings were more sensitive than the inner rings. The sensitivity decreased inward gradually. The sensitivity of the radial cables was similar to that of the hoop cables.

The vertical axis in Fig.3 denotes the number of members whose static stresses variations beyond 5% among all structural members. The results are consistent with the cables sensitivity to static stresses shown in Table 2. The significance levels of Factor 1, 2, 3, and 4 are larger than that of Factor 5, and the significance level of Factor 6 are larger than that of Factor 7.

Analyses of cables dynamic sensitivity to suspen-dome

Table 3 on the results of the dynamic sensitivity analyses shows that the dynamic sensitivity was weaker than the corresponding static sensitivity. Among all cables, the outermost hoop cable was the most sensitive one, whose significance level

Table 1 Orthogonal experimental design scheme and the corresponding results

Scheme	1	2	3	4	5	6	7	8	Static tresses (MPa)	Static displacement (mm)	Dynamic stresses (MPa)	Dynamic displacement (mm)	Buckling loads (kN/m^2)
1	1	1	1	1	1	1	1	1	246.46	14.59	217.65	12.67	6.28
2	1	2	2	2	2	2	2	2	253.78	15.21	224.81	14.53	6.28
3	1	3	3	3	3	3	3	3	261.10	15.83	231.98	16.38	6.31
4	1	4	4	4	4	4	4	4	268.42	16.45	239.14	18.23	6.34
5	1	5	5	5	5	5	5	5	275.74	17.14	246.31	20.08	6.37
6	1	6	6	6	6	6	6	6	283.07	18.16	263.63	21.92	6.42
7	1	7	7	7	7	7	7	7	290.39	19.22	284.17	23.76	6.46
8	2	1	2	3	4	5	6	7	288.80	16.41	260.04	17.66	6.35
9	2	2	3	4	5	6	7	1	296.12	17.03	267.20	19.51	6.40
10	2	3	4	5	6	7	1	2	303.23	17.27	274.14	18.42	6.42
11	2	4	5	6	7	1	2	3	279.57	16.73	250.40	19.79	6.35
12	2	5	6	7	1	2	3	4	286.81	17.27	257.48	18.17	6.40
13	2	6	7	1	2	3	4	5	293.78	17.51	264.23	16.88	6.43
14	2	7	1	2	3	4	5	6	297.54	17.40	268.29	16.85	6.46
15	3	1	3	5	7	2	4	6	299.95	16.69	271.30	19.22	6.40
16	3	2	4	6	1	3	5	7	307.19	17.22	278.38	17.60	6.42
17	3	3	5	7	2	4	6	1	314.51	17.84	285.54	19.44	6.44
18	3	4	6	1	3	5	7	2	321.47	17.42	292.28	18.17	6.48
19	3	5	7	2	4	6	1	3	328.59	17.67	299.22	17.03	6.46
20	3	6	1	3	5	7	2	4	332.34	17.19	302.86	17.04	6.53
21	3	7	2	4	6	1	3	5	308.78	17.09	279.40	18.41	6.46
22	4	1	4	7	3	6	2	5	342.02	18.04	313.38	17.71	6.46
23	4	2	5	1	4	7	3	6	348.98	17.62	320.12	16.47	6.46
24	4	3	6	2	5	1	4	7	325.76	17.07	296.99	17.85	6.44
25	4	4	7	3	6	2	5	1	332.72	17.70	303.80	19.69	6.46
26	4	5	1	4	7	3	6	2	336.37	15.45	307.18	19.69	6.52
27	4	6	2	5	1	4	7	3	343.62	16.40	314.26	18.09	6.54
28	4	7	3	6	2	5	1	4	350.73	17.30	321.20	16.92	6.58
29	5	1	5	2	6	3	7	4	353.13	17.65	324.72	19.13	6.48
30	5	2	6	3	7	4	1	5	360.13	17.90	331.38	18.06	6.53
31	5	3	7	4	1	5	2	6	367.37	18.43	338.45	16.38	6.55
32	5	4	1	5	2	6	3	7	371.12	15.17	342.10	16.35	6.58
33	5	5	2	6	3	7	4	1	378.44	16.21	349.26	18.20	6.64
34	5	6	3	7	4	1	5	2	355.51	16.02	326.43	19.56	6.54
35	5	7	4	1	5	2	6	3	362.11	16.88	332.81	18.32	6.58
36	6	1	6	4	2	7	5	3	395.10	19.01	366.53	17.66	6.61
37	6	2	7	5	3	1	6	4	372.51	18.48	344.04	19.02	6.54
38	6	3	1	6	4	2	7	5	375.92	16.16	347.35	18.99	6.58
39	6	4	2	7	5	3	1	6	382.67	15.45	353.92	17.88	6.58
40	6	5	3	1	6	4	2	7	389.27	15.24	360.31	16.68	6.64
41	6	6	4	2	7	5	3	1	396.54	16.22	367.26	18.52	6.66
42	6	7	5	3	1	6	4	2	403.78	17.27	374.34	16.87	6.70
43	7	1	7	6	5	4	3	2	406.64	19.29	378.35	19.17	6.63
44	7	2	1	7	6	5	4	3	410.05	16.03	381.66	19.16	6.64
45	7	3	2	1	7	6	5	4	416.96	15.61	388.18	17.96	6.65
46	7	4	3	2	1	7	6	5	424.19	16.14	395.25	16.30	6.73
47	7	5	4	3	2	1	7	6	401.90	15.60	373.05	17.66	6.64
48	7	6	5	4	3	2	1	7	408.65	15.84	379.63	16.52	6.65
49	7	7	6	5	4	3	2	1	415.60	16.78	386.43	18.37	6.73

Table 2 Results of analyses of cables sensitivity to static stresses

Factor	Stress S_j	Degree of freedom f_j	F_j	Significance
1	11.2224	6	619509.1804	***
2	0.1044	6	5761.8683	***
3	0.0055	6	304.6813	***
4	0.0001	6	5.9562	***
5	0.0001	6	4.0849	**
6	0.3695	6	20394.8520	***
7	0.0001	6	4.3805	**
Error	0.0000	6		

Note: $F_{0.95}(6,6)=4.28$, $F_{0.99}(6,6)=8.47$

Table 3 Results of analyses of cables Sensitivity to dynamic stresses

Factor	Stress S_j	Degree of freedom f_j	F_j	Significance
1	10.8272	6	1543.5753	***
2	0.1386	6	19.7566	***
3	0.0219	6	3.1173	***
4	0.0084	6	1.2004	*
5	0.0063			
6	0.4496	6	64.1020	***
7	0.0087	6	1.2345	*
Error	0.0140	12		

Note: $F_{0.95}(6,12)=3.00$, $F_{0.99}(6,12)=4.82$

Table 4 Results of analyses of cables sensitivity to buckling loads

Factor	Stress S_j	Degree of freedom f_j	F_j	Significance
1	5512.4108	6	404.1399	***
2	663.9368	6	48.6762	***
3	28.1993	6	2.0674	*
4	15.2098	6	1.1151	*
5	17.5869	6	1.2894	*
6	347.2871	6	25.4612	***
7	13.7096	6	1.0051	*
Error	13.6399	6		

Note: $F_{0.95}(6,6)=4.28$, $F_{0.99}(6,6)=8.47$

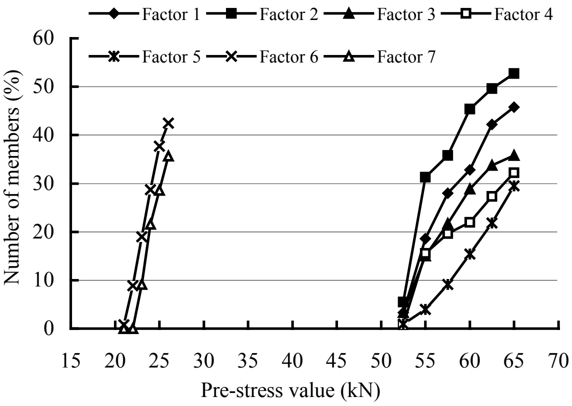

Fig.3 Effect of pre-stress of cables on static stresses

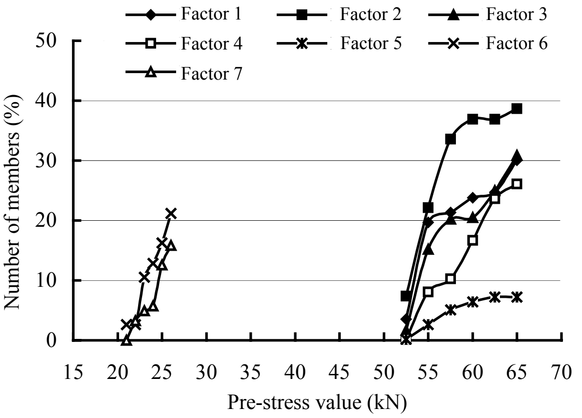

Fig.4 Effect of pre-stress of cables on dynamic stresses

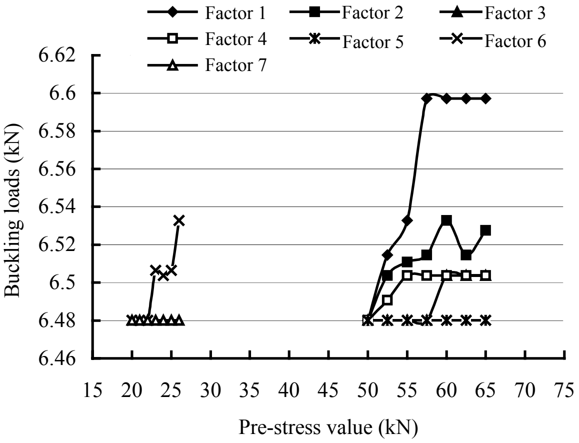

Fig.5 Effect of pre-stress of cables on buckling loads

F_j was at least more than 100 times that of the others; and the outer two rings hoop cables had very sensitive behavior, while the 4th−5th rings hoop cables and radial cables only had minor sensitivity. The table also shows that the sensitivity of the hoop cables and radial cables were weaker from outer ring to inner ring.

The vertical axis in Fig.4 denotes the number of members whose dynamic stresses variations beyond 5% among all structural members. The results are consistent with the cables sensitivity to dynamic stresses shown in Table 3. The significance levels of Factor 1, 2, and 3 are larger than that of Factor 4 and 5, and the significance level of Factor 6 are larger than that of Factor 7.

Analyses of cables stability sensitivity to suspen-dome

Compared with the static and dynamic properties, the pre-stress condition of the cables had less impact on stability properties (Table 4). The outer two rings hoop cables had very sensitive behavior, and the outermost hoop cable was the most sensitive one. The 3rd to 5th rings hoop cables and the 4th−5th radial cables had minor sensitivity. The sensitivity of hoop and radial cables was weaker from outer ring to inner ring.

The results in Fig.5 are consistent with the cables stability sensitivity shown in Table 4. Factor 1, 2, and 6 had very strong sensitivity, and Factor 3, 4, 5, and 7 had little influence to buckling loads.

Briefly, the results in Tables 2–4 show that the static, dynamic, and anti-buckling properties of the outermost hoop cable had the strongest sensitivity; and that the sensitivity was weaker from outer to inner hoop. The radial cables had sensitivity similar to that of the hoop cables, with the outer radial cable being more sensitive than the inner radial cable. Obviously, the outermost hoop cable had the greatest influence on this structural model. Minor deviations of the pre-stresses could greatly affect the structural behavior, including response to static, dynamic and buckling loads. Consequently much attention should be paid to the control of the stress states of the outermost hoop during construction. Furthermore, according to the quantitative analyses method of Figs.3–4, when the number of members whose stresses variations beyond 5% exceeds a certain value (assumed as 30%), it can be determined that the pre-stresses in this group of cables should be rectified. It is instructive for practical construction.

CONCLUSION

In the present work using orthogonal design method, we obtained a quantitative understanding of every factor influencing on the structural model. The results on static, dynamics and stability had instructive value for practical design and construction. Parametric studies showed that:

1. The suspen-dome system that combines two single systems: a stiff single-layer dome and flexible tension-only cable structure, is superior to the corresponding single systems. The response to buckling loads is greatly improved, while the static deflections under uniform gravity loads are reduced significantly;
2. On the other hand, the suspen-dome is sensitive to cables with pre-stresses. The outermost cables have great sensitivity to the structural behavior, while the innermost cables only have minor sensitivity. The sensitivity is weaker from outer to inner cables. The pre-stresses of the cables can improve the structural behavior, but they also make the structures sensitive to the variation of the pre-stresses;
3. The sensitivity of pre-stressed cables, static and dynamic behavior and buckling loads are different. It has the strongest influence on the static behavior and the weakest influence on the buckling loads.

This study indicated that strict measures should be taken to control the quality during pre-stressing of the cables, especially the outermost cables.

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