



## A novel constant modulus array for multiuser detection<sup>\*</sup>

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**Abstract:** This paper proposes a new multitarget constant modulus array structure for code division multiple access (CDMA) systems. The new algorithm for the structure is called pre-despreading and wavelet denoising constant modulus algorithm (D-WD-CMA). In the new algorithm, the pre-despreading is applied to multitarget arrays to remove some multiple access interferences. After that the received signal is subjected to wavelet de-noising to reduce some noise, and used in CMA adaptive iteration for signal separation. Simulation results showed that the proposed algorithm performed better than the traditional CMA algorithm.

**Key words:** Constant modulus algorithm, Adaptive array, Wavelet de-noising, Multiuser detection, CDMA

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### INTRODUCTION

The constant modulus algorithm (CMA) is a very effective blind approach to remove multiple access interference (MAI) and had recently been applied to direct-sequence code division multiple access (DS-CDMA) systems with array antennas, and shown to have tremendous potential for increasing cellular system capacity (Zhu *et al.*, 2003). But the CMA algorithm lacks signal-selectivity and may capture an interference instead of the desired user. Pre-despreading is an effective method to alleviate the capture problem and make the CMA adaptive array be available in CDMA systems (Zhu *et al.*, 2003; Xu and Vu, 1998). After despreading, the signal obtained at the receiver still contains the interference caused by MAI from co-channel users and interference caused by the noise. If we use CMA algorithm to extract the desired signal at this stage, the system performance is

not improved much because of the interferences above. On the contrary, the signal after pre-despreading can be first processed by wavelet de-noising to reduce some noise according to the different frequency characteristics of the desired signal and noise, even if the signal has the relatively high data rate. Then the CMA algorithm forces the array output to unity (or to some other prespecified constant value) so as to extract the desired signal. By this method, the values used for adaptive iteration will result in small interference, and improve system performance.

Based on the idea above, a new multitarget constant modulus array for multiuser detection in CDMA systems is presented in this paper. The proposed algorithm is called pre-despreading and wavelet denoising CMA algorithm (D-WD-CMA). Simulation results showed that the D-WD-CMA algorithm can improve the system performance.

### SIGNAL MODEL

Consider a synchronous baseband DS-CDMA

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system with  $P$  users, and receiver with  $M$  antennas. The received signal vector from the  $M$  antennas is

$$\mathbf{r}(t)=[r^{(1)}(t), r^{(2)}(t), \dots, r^{(m)}(t)]^T \quad (1)$$

$$r^{(m)}(t) = \sum_{p=1}^P \sum_{i=1}^{l_p} a_{pi}^{(m)} e^{j\phi_{mpi}} b_p s_p(t - \tau_{pi}) + n(t), t \in [0, T] \quad (2)$$

where  $b_p, s_p(t)$  stands for transmitted symbols and normalized signaling waveform of the  $p$ th user, respectively.  $a_{pi}^{(m)}$  is the received amplitude of the signal through path  $i$  at the  $m$ th antenna for user  $p$ ,  $\tau_{pi}$  is the corresponding delay.  $n(t)$  is the additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma^2$ .  $\phi_{mpi} = 2\pi \frac{d}{\lambda} (m-1) \sin \theta_{pi}$  is the array response,  $d$  is the inter-sensor spacing,  $\lambda$  is the carrier wavelength,  $\theta_{pi}$  is the signal angle of arrival for the path  $i$  of the  $p$ th source. Assuming the signal  $s_p(t)$  is supported in the interval  $[0, T]$  and has unit energy, then

$$s_k(t) = \sum_{i=1}^{N-1} c_i^p u(t - iT_c) \quad (3)$$

where  $N$  is the processing gain,  $(c_0^p, c_1^p, \dots, c_{N-1}^p)$  is a signature sequence of  $\pm 1$ 's assigned to the  $p$ th user;  $u$  is a normalized chip pulse waveform of duration  $T_c$ , the symbol interval is  $T, NT_c=T$ .

## DENOISING BASED ON WAELET TRANSFORM

### Theory of wavelet transform

Let  $L^2$  denote the space of finite-energy functions in the continuous domain  $(-\infty, +\infty)$ . Signal decomposition using wavelets requires calculation of two sequences  $\{a_n\}, \{b_n\}$  to obtain the coefficients of the following identity (Cetin et al., 1994)

$$\phi(2x-l) = \sum_{n=-\infty}^{\infty} \{a_{l-2n}\phi(x-n) + b_{l-2n}\psi(x-n)\} \quad (4)$$

where  $L=0, \pm 1, \pm 2, \dots$ . Let  $\phi_{k,j}=\phi(2^k x-j), \psi_{k,j}(x)=\psi(2^k x-j)$ , any signal  $f(x)$  in  $L^2$  can be decomposed into a wavelet series

$$f(x) = \sum_{k=-\infty}^{\infty} g_k(x), g_k(x) \in W_k, \quad (5)$$

where  $W_k$  is the orthogonal complementary subspace of  $V_{k+1}$  relative to  $V_k$ , and  $\{V_k\}$  is a nested sequence of closed subspaces of  $L^2$  for multiresolution analysis of  $L^2$ . In the  $V_k$ , considering the partial sum  $f_k(x):=g_{k-1}(x)+g_{k-2}(x)+\dots$ , then

$$f_k(x) = \sum_{j=-\infty}^{\infty} c_{k,j} \phi_{k,j}(x) \quad (6)$$

$$g_k(x) = \sum_{j=-\infty}^{\infty} d_{k,j} \psi_{k,j}(x) \quad (7)$$

where the subscript  $k$  indicates the level of decomposition. The wavelet coefficients  $\{c_{k,n}\}, \{d_{k,n}\}$  are (Cetin et al., 1994; Chui, 1992)

$$c_{k,n} = \sum_{j=-\infty}^{\infty} a_{j-2n} c_{k+1,j} \quad (8)$$

$$d_{k,n} = \sum_{j=-\infty}^{\infty} b_{j-2n} c_{k+1,j} \quad (9)$$

For signal reconstruction, two sequences  $\{p_n\}, \{q_n\}$  must be calculated, then

$$\phi(x) = \sum_{n=-\infty}^{\infty} p_n \phi(2x-n) \quad (10)$$

$$\psi(x) = \sum_{n=-\infty}^{\infty} q_n \phi(2x-n) \quad (11)$$

$$c_{k+1,n} = \sum_{j=-\infty}^{\infty} (p_{n-2j} c_{k,j} + q_{n-2j} d_{k,j}) \quad (12)$$

For more details, the reader is referred to (Chui, 1992).

### Wavelet de-noising

The use of discrete wavelet transform (DWT) to remove noise from signals is an important application of wavelet analysis. The DWT analyzes a finite length signal at different frequency bands with different resolutions by successive decomposition into coarse approximation and information details.

Let  $W(\cdot)$  and  $W(\cdot)^{-1}$  denote the forward and inverse DWT operators respectively;  $T(\cdot, \lambda)$  is the

thresholding operator with threshold  $\lambda$ . Let  $X$  denote the noisy signal vector, with  $v$  samples. Then the wavelet de-noising processing steps are:

$$\begin{aligned} \mathbf{w} &= W(X) \\ \hat{\mathbf{w}} &= T(\mathbf{w}, \lambda) \\ \hat{X} &= W^{-1}(\hat{\mathbf{w}}) \end{aligned} \quad (13)$$

where  $\mathbf{w}$  is the wavelet coefficients vector comprised of approximation coefficient  $\mathbf{a}^l$  and the detail coefficients  $\mathbf{d}^l$  at the level  $l=1, \dots, L$ ;  $\hat{\mathbf{w}}$  is the wavelet coefficients vector after thresholding of  $\mathbf{d}^l$ ;  $\hat{X}$  is the estimation of  $X$  by wavelet de-noising.

**Practical considerations of wavelet decomposition level  $L$**

We know from the previous section that appropriate choice of the decomposition level is fundamental to the effectiveness of the described wavelet de-noising procedure. Too large levels will remove some parts of the desired signal whereas too small levels result in retaining noise in the reconstruction. Practically, the optimal level  $L$  applied in the wavelet transform depends on many factors and theoretical analysis for the optimal  $L$  is almost impossible so that many researches the problem is not considered (Paraschiv-Ionescu et al., 2002; Kazubek, 2003; Ranta et al., 2003; Sardy et al., 2001). Generally, suitable  $L$  is always obtained by the computer simulation in different communication environment. The simulation in this paper yielded the optimal value of  $L$  for the D-WD-CMA algorithm.

**D-WD-CMA ALGORITHM**

**D-WD-CMA algorithm (Fig.1)**

The least-squares constant modulus algorithm minimizes a cost function  $J(\mathbf{w}_{\text{cma}})$  (Agee, 1986)

$$J(\mathbf{w}_{\text{cma}}) = E \left[ \left| \mathbf{w}_{\text{cma}}^H \mathbf{r} \right| - 1 \right]^2 \quad (14)$$

where  $\mathbf{w}_{\text{cma}} = [w_1, w_2, \dots, w_M]^T$  is the weight vector of the beamformer. At time instant  $k$ , the output of the beamformer is

$$y(k) = \mathbf{w}_{\text{cma}}(k)^H \mathbf{r}(k) \quad (15)$$

the updating of weight vector at time instant  $k+1$  is

$$\mathbf{w}_{\text{cma}}(k+1) = \mathbf{w}_{\text{cma}}(k) - \mathbf{R}_{rr}^{-1}(k) \mathbf{r}(k) e(k) \quad (16)$$

where  $\mathbf{R}_{rr}$  is the auto-correlation matrix for the input vector  $\mathbf{r}$  (Liu et al., 2003),

$$\mathbf{R}_{rr}(k) = \mathbf{R}_{rr}(k-1) + \mathbf{r}_k \mathbf{r}_k^H \quad (17)$$

$$e(k) = y(k) / |y(k)|.$$

Assume the desired user is  $p$ . In the D-WD-CMA algorithm, the received signal at the  $m$ th antenna is first despread, i.e.

$$r_{\text{pre-despreading}}^{(m)}(k) = \sum_{j=0}^{N-1} r^{(m)}(kT + jT_c) c_j^p \quad (18)$$

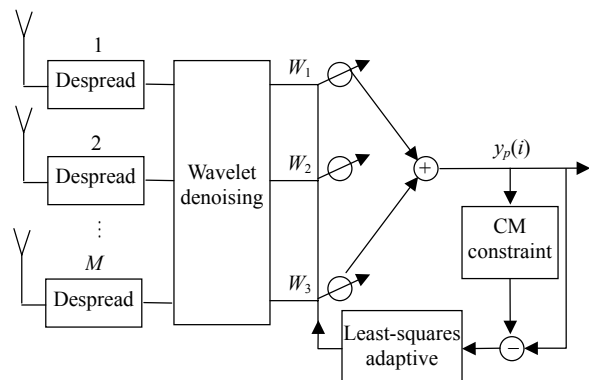
Then Eq.(1) is rewritten as

$$\begin{aligned} \mathbf{r}(t) &= [r_{\text{pre-despreading}}^{(1)}(t), r_{\text{pre-despreading}}^{(2)}(t), \\ &\dots, r_{\text{pre-despreading}}^{(M)}(t)]^T \end{aligned} \quad (19)$$

After wavelet denoising, Eq.(19) is changed to

$$\begin{aligned} \tilde{\mathbf{r}}(t) &= W(\mathbf{r}(t)) = [\tilde{r}_{\text{pre-despreading}}^{(1)}(t), \tilde{r}_{\text{pre-despreading}}^{(2)}(t) \\ &\dots, \tilde{r}_{\text{pre-despreading}}^{(M)}(t)]^T \end{aligned} \quad (20)$$

Substituting Eq.(9) into Eqs.(5)–(7), we can obtain the D-WD-CMA algorithm.



**Fig.1 Block diagram of D-WD-CMA algorithm**

### Complexity analysis of D-WD-CMA algorithm

The D-WD-CMA algorithm is composed of three procedures: pre-despreading, wavelet denoising and least-squares adaptive iteration based on CMA.

The pre-despreading involves  $M \cdot N^2$  multiplications and  $M \cdot (N-1)$  additions.

According to the theory of wavelet de-noising, the complexity of wavelet de-noising mainly depends on the wavelet decomposition and reconstruction because the selection of threshold and thresholding the detail coefficients require little calculation, compared with the wavelet decomposition and reconstruction. The D-WD-CMA algorithm's wavelet decomposition needs at most  $4(J+1)(M+LJ-2J+L)$  arithmetic operations for  $L$  levels wavelet de-noising and the reconstruction needs  $4L(J+1)(2J+\max(r,s)+1)$  arithmetic operations (Cetin *et al.*, 1994), where  $J$  denotes the length of the sequences  $\{a_n\}$ ,  $\{b_n\}$  relevant to the type of wavelet;  $r$  and  $s$  indicate the length of  $\{p_n\}$ ,  $\{q_n\}$ , respectively.

The complexity of the CMA algorithm is somewhat reduced because the iteration is based on the symbol, not chip signal. According to Eqs.(15)–(17), it is known that they individually need  $M$ ,  $M^3+M^2$  and  $M^2$  multiplications and  $M-1$ ,  $M^2$  and  $M$  additions.

To sum up, the higher cost of the D-WD-CMA algorithm is due to its increased complexity, compared with the LSCMA algorithm. When applying the D-WD-CMA algorithm, we can select some wavelets of requiring few calculations, decrease the decomposition levels and the array elements to reduce the complexity, and lead to numerical difficulties.

### SIMULATIONS

In this section, we provide the simulations to illustrate the performance of the D-WD-CMA algorithm. In the simulation, it is assumed a synchronous CDMA system with processing gain  $N=31$ . User 1 is the desired user. The antenna array  $M=4$ . The DOAs of signals are randomly distributed between  $5^\circ$  and  $185^\circ$ . The multiple access interferers are defined as  $MAI(k) = 10 \lg(A_k^2 / A_1^2)$ ,  $k=2 \sim P$ , where  $P$  is the number of active users in the system. The measure of performance is bit error rate (BER), it is calculated by

averaging 50 independent random run's instantaneous output.

Fig.2 is the plot of BER performance vs different signal-to-noise ratio (SNR), where  $MAI=15$  dB,  $P=10$ . The symlets wavelet filter is considered with the order of the symlets wavelet being 8.  $L$  stands for the level of wavelet decomposition; the threshold is selected according to the heursure principle (Hu *et al.*, 2004). It can be seen that the BER performance of the D-WD-CMA algorithm is better than that of the LSCMA algorithm when a suitable value is selected for  $L$ . From the simulation results in Fig.3,  $L=3$  is the best choice because it can reduce the noise better, while the wavelet denoising ( $L=2$ ,  $L=4$ ) cannot get the low BER because the wavelet de-noising ( $L=2$ ) cannot remove much noise and too large decomposition level ( $L=4$ ) might eliminate some desired signals. This accords with the analysis of Section 3.3.

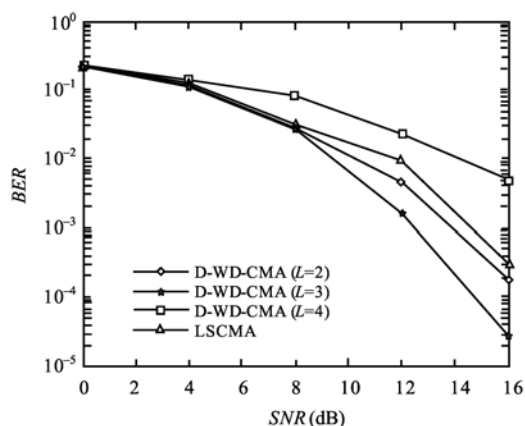


Fig.2 BER performance versus SNR ( $MAI=15$  dB)

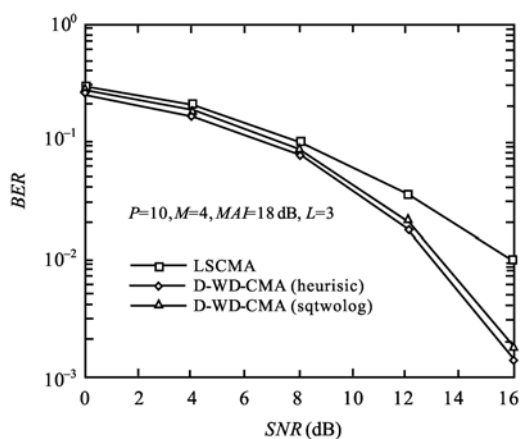


Fig.3 BER performance versus different threshold

BER performance vs different thresholds is illustrated in Fig.3, where the principles of threshold are (Hu et al., 2004): (1) Sqrtwolog principle, a fixed threshold whose value is the square root of 2 times the log of length  $X$ , where  $X$  is the input signal containing the noise; (2) Heuristic principle, which means heuristic selection of the threshold. Simulation showed that the heuristic principle is the best because it can achieve the best predicted threshold value (Hu et al., 2004).

## CONCLUSION

A new blind adaptive approach for MAI cancellation in DS-CDMA systems is presented, termed D-WD-CMA, in which the values for CMA adaptive iteration is processed by wavelet transform to suppress the noise. In addition, the pre-despreading method is used to alleviate the capture problem and improve the system performance further. Simulations showed that the D-WD-CMA algorithm's BER performance is better than that of the traditional LSCMA.

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