

Analysis modeling for plate buckling load of vibration test

SUNG Wen-pei^{†1}, LIN Cheng-I², SHIH Ming-hsiang³, GO Cheer-germ⁴

¹Department of Landscape Design and Management, National Chin-Yi Institute of Technology, Taiwan 41111, China)

²Department of Fire Science, Wu-Feng Institute of Technology, Ming-Hsiung, Chiayi, Taiwan 621, China)

³Department of Construction Engineering, National Kaoshiang First University of Science and Technology, Taiwan 824, China)

⁴Department of Civil Engineering, National Chung Hsing University, Taiwan 40227, China)

[†]E-mail: sung809@chinyi.ncit.edu.tw

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Abstract: In view of the recent technological development, the pursuit of safe high-precision structural designs has been the goal of most structural designers. To bridge the gap between the construction theories and the actual construction techniques, safety factors are adopted for designing the strength loading of structural members. If safety factors are too conservative, the extra building materials necessary will result in high construction cost. Thus, there has been a tendency in the construction field to derive a precise buckling load analysis model of member in order to establish accurate safety factors. A numerical analysis model, using modal analysis to acquire the dynamic function calculated by dynamic parameter to get the buckling load of member, is proposed in this paper. The fixed and simple supports around the circular plate are analyzed by this proposed method. And then, the Monte Carlo method and the normal distribution method are used for random sampling and measuring errors of numerical simulation respectively. The analysis results indicated that this proposed method only needs to apply modal parameters of 7×7 test points to obtain a theoretical value of buckling load. Moreover, the analysis method of inequality-distant test points produces better analysis results than the other methods.

Key words: Energy equivalence, Buckling load, Monte Carlo method

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INTRODUCTION

Buckling occurs when the stress loading on a member reaches a certain critical point and the member is in a state of neutral balance after losing resistance to outside forces. A large amount of drifts are generated, leaving the member in an unstable state. Thus, buckling load is a critical factor for designers in designing members. The amount of buckling load is related to the size, material, and the boundary restraint conditions of the member. Size and material are not significant variables and easily controlled, but the boundary restraint conditions are not easily evaluated

accurately because connectors cannot be technically perfected. Although reduction factor can cover up the shortcoming, it is hard to examine if the buckling load of the member meets the design requirement during the construction. As a result, it is necessary to devise a method which can analyze the buckling load of the members and establish accurate safety factors without having to consider the boundary condition factor. Boundary connectors cannot be simulated, making it hard to estimate the boundary restraint conditions of the member or derive them from theories. Thus, in the past, scholars usually conducted experiments based on various static or dynamic theories, in order to obtain the value of the buckling load.

Experiments using dynamic theories include that of: (1) Lurie (1952), who conducted experiments to obtain the relation curve between axial loading and

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the square natural frequency, and get the structural members' bulking load value by using linear extrapolation at the point of frequency = 0; (2) Sweet and Genin (1971), Sweet *et al.* (1976; 1977), who used the natural frequency obtained from the results of motion experiments to estimate the buckling load of structural members; (3) Segall and Baruch (1980), who derived the buckling load of flexible poles by conducting motion experiments in conjunction with integral pole equation; (4) Segall and Springer (1986), who used integral equation and motion parameters to get the buckling load value of model members; (5) Go *et al.* (1997), who created a parameter based model to analyze the buckling load of the model members. Richard and Faires (1989) proposed the experiment's main static theory in which the slope of the W/P vs W curve is the buckling load value of the structural members (P being the measured structural member loading, W the deformation after buckling).

A dynamic analysis model is proposed to acquire buckling load of plate. We used the dynamic measured data from selected test points and by modal analysis got the modal parameters-mode shape and frequency; and then, derived a flexible matrix with the above model parameters. Force analysis was used to get the flexible matrix of equivalent force and the characteristic equation for determining the buckling load of the member. Under the equivalence, the buckling deformation function of the member was derived from the deformation tested at the test points and simulated by Lagrange's Interpolation Equations. In this paper, we took the circular model member of ideal margins and fixed restraints on each side as an example. First, we used the given frequency and the corresponding mode shape to conduct the verification of the analysis model and then discussed the number and location of the test points and the mode shapes composing a linear isolated equation with the 1st, 2nd, 3rd, 4th, 5th, ... values extracted from the vibration equation; and also discussed the impact of measured deviation caused by outside factors on buckling load. Monte Carlo Method was used to generate random scatter to simulate the values of measured deviations. Finally, we modeled the elements of the circular model member with simple restraint on each side based on the results of the above analysis, and used finite element analysis method to acquire modal parameters. Then, we verified the dynamic analysis

model with the modal parameters obtained from measured data. The proposed analysis model is applicable to all kinds of loading and boundary conditions.

ANALYSIS METHOD

Simulation function of buckling shape

Assume that the deformation function of the plate after buckling is $W(r, \theta)$; then, select test points along the vertical direction of plate where lateral displacement is D_{ij} ($i, j=1, 2, \dots, n$). Using Lagrange's Interpolation Function to tie in with lateral displacement D_{ij} to represent the deformation function of the plate after buckling $W(r, \theta)$ (Dym, 1974) shown as follows:

$$W(r, \theta) = \sum_{i=1}^n \sum_{j=1}^n L_{ij} D_{ij} \quad (1)$$

Where, $L_{ij} = L_i(r) \times L_j(\theta)$

$$= \frac{(r-r_1) \cdots (r-r_{i-1})(r-r_{i+1}) \cdots (r-r_n)}{(r_i-r_1) \cdots (r_i-r_{i-1})(r_i-r_{i+1}) \cdots (r_i-r_n)} \\ \times \frac{(\theta-\theta_1) \cdots (\theta-\theta_{i-1})(\theta-\theta_{i+1}) \cdots (\theta-\theta_n)}{(\theta_i-\theta_1) \cdots (\theta_i-\theta_{i-1})(\theta_i-\theta_{i+1}) \cdots (\theta_i-\theta_n)}$$

is Lagrange's interpolation function and D_{ij} is the test point (i, j), lateral displacement.

Energy equivalence

Assuming a homogeneous and isotropic plate, with radius a . When the plate is subjected to external forces N_{rr} , $N_{\theta\theta}$ and shear force $N_{r\theta}$, the energy equation can be expressed as follows (Timoshenko and Gere, 1997):

$$U = \frac{1}{2} \iint_A \left[N_{rr} \left(\frac{\partial W}{\partial r} \right)^2 + \frac{2}{r} N_{r\theta} \left(\frac{\partial W}{\partial r} \frac{\partial W}{\partial \theta} \right) + \frac{1}{r^2} N_{\theta\theta} \left(\frac{\partial W}{\partial \theta} \right)^2 \right] r dr d\theta \\ = \frac{1}{2} \iint_A (N_{rr} [D]_{1 \times n^2}^T [L_r]_{n^2 \times 1}^T [L_r]_{1 \times n^2} [D]_{n^2 \times 1}$$

$$\begin{aligned}
 & + \frac{1}{r} N_{r\theta} \left\{ [D]_{1 \times n^2}^T [L_r]_{n^2 \times 1}^T [L_\theta]_{1 \times n^2} [D]_{n^2 \times 1} \right. \\
 & \left. + [D]_{1 \times n^2}^T [L_\theta]_{n^2 \times 1}^T [L_r]_{1 \times n^2} [D]_{n^2 \times 1} \right\} \\
 & + \frac{1}{r^2} N_{\theta\theta} [D]_{1 \times n^2}^T [L_\theta]_{n^2 \times 1}^T [L_\theta]_{1 \times n^2} [D]_{n^2 \times 1} \} r dr d\theta \quad (2)
 \end{aligned}$$

Where, $L_r = \frac{\partial L}{\partial r}$; $L_\theta = \frac{\partial L}{\partial \theta}$.

From the concept of the energy method, the axial and shear forces can be transformed as action force acting on test points. Then substitute $N_{rr} = Nf_1(r, \theta)$; $N_{\theta\theta} = Nf_2(r, \theta)$; $N_{r\theta} = Nf_3(r, \theta)$ into Eq.(2). The equivalent force matrix $[F]_{n^2 \times 1}$ can be acquired through Castigliano's theorem, as follows:

$$[F]_{n^2 \times 1} = \frac{\partial U}{\partial [D]_{n^2 \times 1}} = N[B]_{n^2 \times n^2} [D]_{n^2 \times 1} \quad (3)$$

Where:

$$\begin{aligned}
 [B]_{n^2 \times n^2} = \iint_A \left\{ f_1 [L_r]_{n^2 \times 1}^T [L_r]_{1 \times n^2} + \frac{1}{a} f_3 \left([L_r]_{n^2 \times 1}^T [L_\theta]_{1 \times n^2} \right. \right. \\
 \left. \left. + [L_\theta]_{n^2 \times 1}^T [L_r]_{1 \times n^2} \right) + \frac{1}{a^2} f_2 [L_\theta]_{n^2 \times 1}^T [L_\theta]_{1 \times n^2} \right\} r dr d\theta,
 \end{aligned}$$

and $[D]$ is the displacement matrix.

Buckling analysis

Considering the relation between action $[F]$ and displacement $[D]$ in the system of equivalent condition. Then use the force method to analyze the following expression:

$$[D] = [G][F] \quad (4)$$

Where, $[G]$ is flexible matrix.

$G(i, j)$ means that applying one unit loading at j point, induced the deflection at i point. Substituting $[F]$ in Eq.(3) into Eq.(4) to get the following expression:

$$[D] = [G][F] = N[G][B][D] \quad (5)$$

$$\lambda [D] = [G][B][D] \quad (6)$$

Where, $\lambda = 1/N$ is buckling load.

Solving the Eq.(6) to obtain the characteristic solution. The reciprocal of the characteristic solution maximum value is the buckling load.

$$N_{cr} = 1/\lambda_{\max} \quad (7)$$

The theoretical model shows that $[B]$ and $[G]$ are two factors affecting the analysis accuracy of buckling load. When the test point has been determined, we use dynamic measurement and modal analysis to get the modal shape φ and frequency ω of the plate to analyze and study the flexible matrix $[G]$ with regard to $[B]$ matrix, and their relations with test positions and number of test points.

Flexible matrix

The free vibration dynamic equation of circular plate is shown as follows (Meirovitch, 1967):

$$-D \nabla^4 w = \rho \frac{\partial^2 w}{\partial t^2} \quad (8)$$

where, $D = \frac{Eh^3}{12(1-\nu^2)}$ is the deflection rigidity;

$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$ is harmonic differential operator and $\nabla^2 \nabla^2 = \nabla^4$ is biharmonic differential operator.

Assuming deformation function is $w(r, \theta, t)$, and after using variable separation method to solve Eq.(8), the deformation function can be expressed as follows:

$$w(r, \theta, t) = \sum_{i=1}^n \varphi_i(r, \theta) T_i(t) \quad (9)$$

where, $\varphi_i(r, \theta)$ is the i th Modal Shape.

Then substitute Eq.(9) into Eq.(8) to derive the following expression:

$$\frac{D}{\rho} \cdot \frac{\nabla^4 \varphi_i}{\varphi_i} = -\frac{T_i''}{T_i} = \omega_i^2 \quad (10)$$

where, ω_i is the natural frequency for the i th modal shape and T_i is the time function of free vibration.

$$T_i'' + \omega_i^2 T_i = 0 \quad (11)$$

$$D \nabla^4 \varphi_i - \rho \omega_i^2 \varphi_i = 0 \quad (12)$$

The forced vibration dynamic equation of circular plate is shown as follows (Meirovitch, 1967):

$$D\nabla^4 w + \rho \frac{\partial^2 w}{\partial t^2} = f(r, \theta, t) \quad (13)$$

where, $f(r, \theta, t)$ is the external force applied on the circular plate.

Assuming the deformation function is $w_f(r, \theta, t)$ and given as:

$$w_f(r, \theta, t) = \sum_{i=1}^n \varphi_i(r, \theta) \psi_i(t) \quad (14)$$

where, $\psi_i(t)$ is time function of forced vibration.

Eq.(14) can be substituted into Eq.(13), then, multiplied by $\varphi_j(r, \theta)$ and integrated over the area of the circular plate to get the following equation:

$$\begin{aligned} \sum_{i=1}^{\infty} \psi_i'' \iint_A \varphi_i \varphi_j \rho r dr d\theta + \sum_{i=1}^{\infty} \psi_i \iint_A (D\nabla^4 \varphi_i) \varphi_j r dr d\theta \\ = \iint_A f(r, \theta, t) \varphi_j r dr d\theta \end{aligned} \quad (15)$$

Using the characteristics of Modal Orthogonal, Eq.(16) can be rearranged as follows:

$$\iint_A \varphi_i \varphi_j \rho r dr d\theta = M_i \delta_{ij} \quad (16)$$

where, $\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$ and $M_i = \iint_A \varphi_i^2 \rho r dr d\theta$ is generalized mass.

Assuming the external force $f(r, \theta, t)$ as a period centralized force applied at the test point.

$$f(r, \theta, t) = \delta(r - \xi) \delta(\theta - \eta) \times \sin(\Omega t) \quad (17)$$

where, $\delta(r - \xi)$ is the Dirac Delta Function.

The properties of Dirac Delta Function are shown as follows:

$$\delta(r - \xi) \begin{cases} 0 & |r - \xi| > \varepsilon \\ \frac{1}{2\varepsilon} & |r - \xi| < \varepsilon \end{cases}$$

$$\int_0^a \delta(r - \xi) dr = \begin{cases} 0 & \xi \geq a; \xi \leq 0 \\ 1 & a > \xi > 0 \end{cases}$$

Then, Eq.(17) can be integrated to get the following equation:

$$\begin{aligned} f_i(r, \theta, t) &= \iint_A \delta(r - \xi) \delta(\theta - \eta) \sin(\Omega t) r dr d\theta \\ &= \xi \sin(\Omega t) \end{aligned} \quad (18)$$

Substitution of Eqs.(12), (16) and (18) into Eq.(15), yields the following equation.

$$M_j \psi_j'' + M_j \omega_j^2 \psi_j = \varphi_j(\xi, \eta) \sin(\Omega t) \quad (19)$$

Substitution of initial conditions $\psi_j'(0) = \psi_j(0) = 0$ into Eq.(19) and use of Inverse Laplace Transform yield $\psi(t)$ as:

$$\psi(t) = \frac{r \varphi_j(\xi, \eta) \xi}{M_j \omega_j^2 \left[1 - \left(\frac{\Omega^2}{\omega_j^2} \right) \right]} \left[\sin(\Omega t) - \frac{\Omega}{\omega_j} \sin(\omega_j t) \right] \quad (20)$$

Substitute Eq.(20) into Eq.(14) to obtain $w(r, \theta, t)$ shown as follows:

$$\begin{aligned} w_f(r, \theta, t) &= \sum_{i=1}^{\infty} \varphi_i(r, \theta) \psi_i(t) \\ &= \frac{r \xi \varphi_i(r, \theta) \varphi_j(\xi, \eta)}{M_j \omega_j^2 \left[1 - \left(\frac{\Omega^2}{\omega_j^2} \right) \right]} \left[\sin(\Omega t) - \frac{\Omega}{\omega_j} \sin(\omega_j t) \right] \end{aligned} \quad (21)$$

If $\Omega \rightarrow 0$, the external force approaches static load and the influence function can be shown as follows:

$$\begin{aligned} G(r, \theta, \xi, \eta) &= \lim_{\Omega \rightarrow 0} \frac{w(r, \theta, t)}{f(t)} = \sum_{i=1}^{\infty} \frac{\varphi_i(r, \theta) \varphi_i(\xi, \eta) r}{M_i \omega_i^2} \\ &= \frac{r \varphi_i(r, \theta) \varphi_j(\xi, \eta)}{M_i \omega_i^2 \left[1 - \left(\frac{\Omega^2}{\omega_j^2} \right) \right]} \left[\sin(\Omega t) - \frac{\Omega}{\omega_j} \sin(\omega_j t) \right] \end{aligned} \quad (22)$$

where, Parameter $G(r, \theta, \xi, \eta)$ represents the induced deflection at test point (r, θ) , one unit load is applied at the test point (ξ, η) , which is the flexible matrix.

Use of the three-point method of Simpson's rule

yields the following expression for generalized mass M_i .

$$M_i = \iint_A \varphi_i^2 \rho r dr d\theta = \frac{h_r h_\theta}{9} \sum_{i=1}^m \sum_{j=1}^n f_{ij} C_i C_j \quad (23)$$

The used constant values of Eq.(23) are:

- $C_i=1$ For $i=1, m$
- $C_i=2$ For $i=\text{odd}; \neq 1, m$
- $C_i=4$ For $i=\text{even}$
- $C_j=1$ For $j=1, n$
- $C_j=2$ For $j=\text{odd}; \neq 1, n$
- $C_j=4$ For $j=\text{even}$

VERIFICATION OF THEORETICAL MODEL

Error of simulation

The measured error of numerical analysis test is used in the Monte Carlo method for sampling random variable, following the normal distribution model. The obtained error of simulation err_i is multiplied by dynamic parameters—natural frequency ω_i and modal shape $\varphi_i(r, \theta)$ and then substituted into the theoretical model before proceeding with the numerical analysis. The buckling load of circular plate with error of simulation can be acquired. The natural frequency and modal shape with error of simulation are as follows:

$$\omega_i^{err} = \omega_i \times err_1, \quad \varphi_i^{err} = \varphi_i \times err_2 \quad (24)$$

where, err_1 and err_2 are simulation errors.

The normal random variable is produced through the uniform distribution random variation on the central limit theorem. The distribution situation is 99.7% of the weight factor plus/minus 100%. The standard deviation is 1/3. This diagram is shown in Fig.1. The normal random variable of RAN_n is shown as:

$$RAN_n = \frac{\sigma}{\sqrt{n/12}} \sum_{i=1}^n (RAN_u)_i + \left(\mu - \frac{n}{2} \frac{\sigma}{\sqrt{n/12}}\right) \quad (25)$$

where, n is the number of the selected distribution random variables. There are 1000 points for this paper; μ is the mean value of normal distribution and σ is the

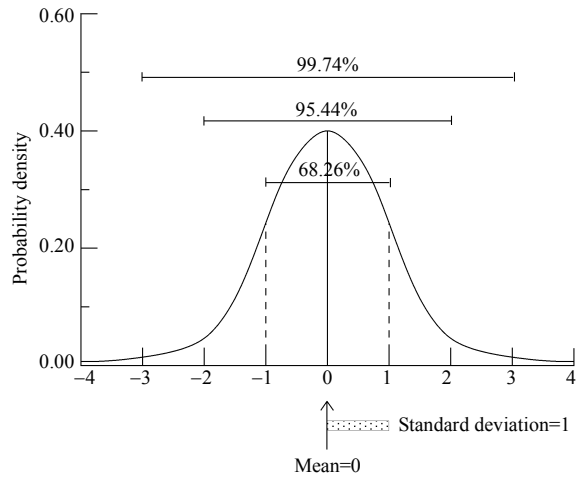


Fig.1 A diagram of standard normal distribution

standard deviation of normal distribution.

$$RAN_u = X_u / M \quad (26)$$

where, RAN_u is the uniform random variable between 0 and 1, X_u is the random variable and M is the unit length for sampling.

Influence parameters for buckling load of circular plate

In order to study the influence of position of selected test point, number of selected points and simulation of modal shape on the results of analysis of the buckling load of circular plate, using the given theoretical solution of buckling load, the constrain boundary conditions of a circular plate with uniform axial load N_{rr} , applied on it, as an example. The material properties of the circular plate are given in Table 1.

Table 1 Material properties of circular plate

Deflected rigidity D ($\text{kg}\cdot\text{cm}^3$)	Area of mass ρ (kg/cm^2)	Radius a (cm)
1	1	1

The feasibility study of this proposed theoretical analysis model of buckling loads used the given theoretical formula of natural frequency and modal shape where the boundary conditions of circular plate are fixed. The analysis results are shown in Tables 2 and 3. Table 2 is composed of five modal shapes for analysis of the simulation effects of modal shapes. The analysis results indicated that the comparison of

analysis solution and theoretical solution of buckling load had barely 1.5% error number of test points 7×7 using equidistant test point and not equidistant test point. Table 3 is the simulation results of 7×7 test points with equidistant test point. The analysis results show that more accurate buckling load of circular plate can be acquired by using simulation model, which buckling load is obtained by the second mode. The positions of test points of equidistant test point and inequality-distant test point are shown in Tables 4, 5 and 6.

The theoretical formulas of natural frequency and modal shape of circular plate with fixed support

are expressed as Eqs.(27) and (28) respectively.

$$\omega_{mn} = \left(\frac{m}{2} + n\right)^2 \frac{\pi^2}{\alpha^2} \sqrt{\frac{D}{\rho}} \quad \text{for large } n \quad (27)$$

where, a is the radius and $m, n=1, 2, \dots$

$$\left. \begin{aligned} W_{mnc} &= A_{mnc} \begin{bmatrix} I_m(\beta_{mn}a)J_m(\beta_{mn}r) \\ -J_m(\beta_{mn}a)I_m(\beta_{mn}r) \end{bmatrix} \cos(m\theta) \\ W_{mns} &= A_{mns} \begin{bmatrix} I_m(\beta_{mn}a)J_m(\beta_{mn}r) \\ -J_m(\beta_{mn}a)I_m(\beta_{mn}r) \end{bmatrix} \sin(m\theta) \end{aligned} \right\} m, n = 1, 2, \dots \quad (28)$$

Table 2 The buckling load of fixed support of circular plate bearing uniform unit axial load ($N_{rr}=1$)

Test point arrangement	No. of test points	Analysis solution	Theoretical solution	Error ratio
Equidistant test point	5×5	35.270	14.684	+140.19 %
	7×7	14.895	14.684	+1.4376 %
	9×9	14.872	14.684	+1.1127 %
Not equidistant test point 1	5×5	18.021	14.684	+22.725 %
	7×7	14.879	14.684	+1.3279 %
	9×9	14.838	14.684	+1.0488 %
Not equidistant test point 2	5×5	17.898	14.684	+21.888 %
	7×7	14.869	14.684	+1.2599 %
	9×9	14.842	14.684	+1.0760 %

Table 3 The influence of number of modes shape for buckling load

No. of test points	No. of modal shape	Analytical solution	Theoretical solution	Error ratio
7×7	The first mode	15.196	14.684	3.483 %
	The second mode	14.911	14.684	1.548 %
	The third mode	14.911	14.684	1.544 %
	The forth mode	14.902	14.684	1.485 %
	The fifth mode	14.895	14.684	1.438 %

Table 4 Coordinates positions of equidistant test points of circular plate

No. of test points	Aspect	Position
5×5	r direction	0, $a/4$, $a/2$, $3a/4$, a
	θ direction	0, $\pi/2$, π , $3\pi/2$, 2π
7×7	r direction	0, $a/6$, $a/3$, $a/2$, $2a/3$, $5a/6$, a
	θ direction	0, $\pi/3$, $2\pi/3$, π , $4\pi/3$, $5\pi/3$, 2π
9×9	r direction	0, $a/8$, $a/4$, $3a/8$, $a/2$, $5a/8$, $3a/4$, $7a/8$, a
	θ direction	0, $\pi/4$, $\pi/2$, $3\pi/4$, π , $5\pi/4$, $3\pi/2$, $7\pi/4$, 2π

Table 5 Coordinates positions of not equally test point 1 of circular plate

No. of test points	Aspect	Position
5×5	r direction	0, $a/5$, $a/2$, $4a/5$, a
	θ direction	0, $\pi/2$, π , $3\pi/2$, 2π
7×7	r direction	0, $a/8$, $a/4$, $a/2$, $3a/4$, $7a/8$, a
	θ direction	0, $\pi/3$, $2\pi/3$, π , $4\pi/3$, $5\pi/3$, 2π
9×9	r direction	0, $a/10$, $a/5$, $3a/10$, $a/2$, $7a/10$, $4a/5$, $9a/10$, a
	θ direction	0, $\pi/4$, $\pi/2$, $3\pi/4$, π , $5\pi/4$, $3\pi/2$, $7\pi/4$, 2π

Then, the circular plate with simple support is used as an example. According to the analysis results of Table 2, the 7×7 equidistant and not equidistant test points of the finite element method were selected to calculate modal parameters of natural frequency and modal shape. The theoretical buckling load with simple support of circular plate bearing uniform unit axial load N_{rr} and composed of five modal shapes, was derived and used for simulating the analysis modal shape. The analysis solutions were close to the theoretical solutions, shown in Table 7. These results indicate that the analysis method employing vibration experiment conjunction with modal analysis method to obtain the buckling load of a structural member is certainly practicable.

OUTCOME OF ERROR ANALYSIS

The error produced in the experimental measurement and having bearing on the theoretical analysis model for obtaining the buckling load of structural members was investigated using 7×7 equidistant test points with cyclic fixed support of circular plate as an example. As regards five sets with maximum measuring error of 2%, 4%, 6%, 8% and 10%, it is noteworthy that one thousand simulations experiments were set up for each of the simulations. The analysis results are shown in Tables 8 and 9. These analysis results revealed that the error scope of dynamic measuring theoretical model of buckling load accorded with the increase of the maximum measuring

error.

CONCLUSION

The proposed theoretical analysis method using dynamic measurement to acquire the buckling load of the structural member is truly feasible. The advantages of this proposed analysis model are: (1) it is not constrained by the boundary conditions; (2) there is no need to apply axial load; (3) there is no need to know the detailed material properties. Only the modal parameters—natural frequency and mode shape are required by this analysis model to figure out the buckling load.

Take a circular plate using fixed cyclic support as an example for using this proposed analysis model to analyze its buckling load with equidistant and not equidistant test points. The analysis results revealed that this proposed method only needs to apply the modal parameters of 7×7 test points for acquiring the theoretical approximate values of buckling load. Moreover, the analysis method using not equidistant test points produces better analysis results, mainly because that the test points of this method gather around the inflection point of the deformation curve.

The results of error simulation experiment indicated (1) the error range of the buckling load enlarges with the increase of the maximum measured error; (2) in order to obtain more exact analysis solution of the buckling load with minimum error, doing the calculation several times and getting the mean value of them is suggested.

Table 6 Coordinates positions of not equally test point 2 of circular plate

No. of test points	Aspect	Position
5×5	r direction	0, $a/5$, $2a/5$, $7a/10$, a
	θ direction	0, $\pi/2$, π , $3\pi/2$, 2π
7×7	r direction	0, $a/8$, $a/4$, $3a/8$, $7a/12$, $19a/24$, a
	θ direction	0, $\pi/3$, $2\pi/3$, π , $4\pi/3$, $5\pi/3$, 2π
9×9	r direction	0, $a/10$, $a/5$, $3a/10$, $2a/5$, $3a/5$, $4a/5$, $9a/10$, a
	θ direction	0, $\pi/4$, $\pi/2$, $3\pi/4$, π , $5\pi/4$, $3\pi/2$, $7\pi/4$, 2π

Table 7 The buckling load with simple support of circular plate bearing uniform unit axial load ($N_{rr}=1$)

No. of test points	Style of test point	Analytical solution	Theoretical solution	Error ratio
7×7	Equidistant test point	4.263	4.204	1.483%
	Not equidistant point	4.248	4.204	1.047%

Table 8 The theoretical buckling load influenced by 2%, 4% and 6% maximum measuring error

2% maximum measuring error		4% maximum measuring error				6% maximum measuring error					
Error ratio (%)	Times	Error ratio (%)	Times	Error ratio (%)	Times	Error ratio (%)	Times	Error ratio (%)	Times	Error ratio (%)	Times
-0.115	0	-0.225	0	0.015	47	-0.365	0	-0.125	30	0.115	8
-0.105	1	-0.215	2	0.025	34	-0.355	1	-0.115	25	0.125	5
-0.095	3	-0.205	4	0.035	29	-0.345	0	-0.105	28	0.135	5
-0.085	13	-0.195	5	0.045	30	-0.335	1	-0.095	30	0.145	4
-0.075	25	-0.185	14	0.055	22	-0.325	0	-0.085	32	0.155	2
-0.065	38	-0.175	18	0.065	18	-0.315	3	-0.075	26	0.165	3
-0.055	63	-0.165	23	0.075	19	-0.305	12	-0.065	41	0.175	1
-0.045	59	-0.155	39	0.085	12	-0.295	7	-0.055	36	0.185	0
-0.035	74	-0.145	29	0.095	17	-0.285	11	-0.045	31	0.195	1
-0.025	74	-0.135	31	0.105	8	-0.275	19	-0.035	38	0.205	0
-0.015	92	-0.125	35	0.115	4	-0.265	23	-0.025	35	0.215	1
-0.005	99	-0.115	31	0.125	4	-0.255	25	-0.015	27	0.225	0
0.005	96	-0.105	28	0.135	3	-0.245	22	-0.005	28		
0.015	113	-0.095	44	0.145	1	-0.235	21	0.005	30		
0.025	74	-0.085	29	0.155	1	-0.225	18	0.015	28		
0.035	62	-0.075	35	0.165	0	-0.215	22	0.025	20		
0.045	46	-0.065	39	0.175	1	-0.205	30	0.035	19		
0.055	39	-0.055	42	0.185	0	-0.195	26	0.045	16		
0.065	13	-0.045	58			-0.185	19	0.055	20		
0.075	7	-0.035	38			-0.175	24	0.065	14		
0.085	5	-0.025	50			-0.165	22	0.075	8		
0.095	2	-0.015	46			-0.155	20	0.085	13		
0.105	2	-0.005	61			-0.145	21	0.095	12		
0.115	0	0.005	49			-0.135	27	0.105	9		

Error=(analysis results-theoretical solution)/theoretical solution, 0.105=0.11~0.10

Table 9 The theoretical buckling load influenced by 8% and 10% the maximum measuring error

8% maximum measuring error					10% maximum measuring error						
Error ratio (%)	Times	Error ratio (%)	Times	Error ratio (%)	Times	Error ratio (%)	Times	Error ratio (%)	Times	Error ratio (%)	Times
-0.485	0	-0.205	19	0.075	5	-0.585	0	-0.305	15	-0.025	16
-0.475	1	-0.195	20	0.085	5	-0.575	1	-0.295	13	-0.015	19
-0.465	1	-0.185	29	0.095	7	-0.565	1	-0.285	13	-0.005	14
-0.455	0	-0.175	17	0.105	5	-0.555	1	-0.275	23	0.005	13
-0.445	1	-0.165	21	0.115	9	-0.545	0	-0.265	16	0.015	6
-0.435	2	-0.155	26	0.125	3	-0.535	3	-0.255	15	0.025	9
-0.425	5	-0.145	24	0.135	4	-0.525	5	-0.245	19	0.035	9
-0.415	5	-0.135	19	0.145	3	-0.515	2	-0.235	28	0.045	8
-0.405	5	-0.125	28	0.155	4	-0.505	4	-0.225	13	0.055	5
-0.395	9	-0.115	24	0.165	3	-0.495	6	-0.215	20	0.065	5
-0.385	4	-0.105	30	0.175	2	-0.485	4	-0.205	25	0.075	5
-0.375	18	-0.095	28	0.185	1	-0.475	15	-0.195	19	0.085	4
-0.365	14	-0.085	28	0.195	2	-0.465	12	-0.185	23	0.095	3
-0.355	20	-0.075	23	0.205	0	-0.455	6	-0.175	20	0.105	3
-0.345	16	-0.065	26	0.215	1	-0.445	14	-0.165	24	0.115	4
-0.335	21	-0.055	29	0.225	0	-0.435	12	-0.155	21	0.125	6
-0.325	17	-0.045	21	0.235	1	-0.425	19	-0.145	21	0.135	2
-0.315	25	-0.035	22	0.245	1	-0.415	17	-0.135	23	0.145	4
-0.305	13	-0.025	19	0.255	0	-0.405	16	-0.125	28	0.155	1

-0.295	21	-0.015	29	-0.395	24	-0.115	21	0.165	2
-0.285	18	-0.005	18	-0.385	18	-0.105	25	0.175	3
-0.275	19	0.005	16	-0.375	10	-0.095	25	0.185	2
-0.265	26	0.015	20	-0.365	14	-0.085	16	0.195	2
-0.255	18	0.025	11	-0.355	17	-0.075	17	0.205	2
-0.245	16	0.035	13	-0.345	18	-0.065	21	0.215	1
-0.235	14	0.045	12	-0.335	20	-0.055	17	0.225	1
-0.225	16	0.055	16	-0.325	18	-0.045	16	0.235	1
-0.215	20	0.065	11	-0.315	20	-0.035	15	0.245	1

Error=(analysis results-theoretical solution)/theoretical solution, 0.105=0.11~0.10

References

- Dym, C.L., 1974. Stability Theory and Its Applications to Structural Mechanics. Noordhoff Publishing Company, Groningen.
- Go, C.G., Lin, Y.S., Khor, E.H., 1997. Experimental determination of the buckling load of a straight structural member by using dynamic parameters. *Journal of Sound and Vibration*, **205**(3):257-267.
- Lurie, R., 1952. Lateral vibrations as related to structural stability. *Journal of Applied Mechanics*, **19**(2):195-203.
- Meirovitch, L., 1967. Analytical Methods In Vibrations. The Macmillan Company.
- Richard, L.B., Faires, J.D., 1989. Numerical Analysis, Fourth Edition. PWS-KENT Publishing Company, Boston, U.S.A.
- Segall, A., Baruch, M., 1980. A nondestructive dynamic method for determination of the critical load of elastic columns. *Experimental Mechanics*, **20**(8):285-288.
- Segall, A., Springer, G.S., 1986. A dynamic method for measuring the critical loads of elastic flat plate. *Experimental Mechanics*, **26**:354-359.
- Sweet, A.L., Genin, J., 1971. Identification of a model for predicting elastic buckling. *Journal Sound Vibration and Vibration*, **14**(3):317-324.
- Sweet, A.L., Genin, J., Mlakar, P.F., 1976. Vibratory identification of beam boundary condition. *Journal of Dynamic Systems, Measurement, and Control*, **98**: 387-394.
- Sweet, A.L., Genin, J., Mlakar, P.F., 1977. Determination of column buckling criteria using vibration data. *Experimental Mechanics*, **17**:385-391.
- Timoshenko, S.P., Gere, J.M., 1997. Mechanics of Material. Fourth Edition, PWS Publishing Company, Boston, U.S.A.

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