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Study on one-dimensional consolidation of soil under cyclic loading and with varied compressibility*

ZHUANG Ying-chun (庄迎春), XIE Kang-he (谢康和)†

(*Department of Civil Engineering, Zhejiang University, Hangzhou 310027, China*)

†E-mail: zdkhxie@zju.edu.cn

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Abstract: This paper presents a semi-analytical method to solve one dimensional consolidation problem by taking consideration of varied compressibility of soil under cyclic loading. In the method, soil stratum is divided equally into *n* layers while load and consolidation time are also divided into small parts and time intervals accordingly. The problem of one-dimensional consolidation of soil stratum under cyclic loading can then be dealt with at each time interval as one-dimensional linear consolidation of multi-layered soils under constant loading. The compression or rebounding of each soil layer can be judged by the effective stress of the layer. When the effective stress is larger than that in the last time interval, the soil layer is compressed, and when it is smaller, the soil layer rebounds. Thus, appropriate compressibility can be chosen and the consolidation of the layered system can be analyzed by the available analytical linear consolidation theory. Based on the semi-analytical method, a computer program was developed and the behavior of one-dimensional consolidation of soil with varied compressibility under cyclic loading was investigated, and compared with the available consolidation theory which takes no consideration of varied compressibility of soil under cyclic loading. The results showed that by taking the variable compressibility into account, the rate of consolidation of soil was greater than the one predicted by conventional consolidation theory.

Key words: Cyclic loading, One-dimensional consolidation, Semi-analytical solution, Varied compressibility of soil
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INTRODUCTION

In practical geotechnical engineering, soils beneath many structures, such as oil and water tanks, highway embankment, ocean bank, etc., undergo cyclic loading. Schiffman (1958) first obtained a general solution of soil consolidation considering loadings increase linearly with time. Wilson and Elgohary (1974) presented an analytical solution of one-dimensional consolidation of saturated soil subjected to cyclic loading based on Terzaghi's linear consolidation theory. Alonso and Krizek (1974) considered the settlement of elastic soft soil under stochastic loading. Olson (1977) analyzed the consolidation under time dependent loading. Baligh and Levadoux (1978) developed a simple prediction method for one-dimensional consolidation of clay layer subjected to cyclic rectangular loading with the superposition principle. More recently, Favaretti and Soranzo (1995), Chen *et al.*(1996), and Guan *et al.*(2003) derived some solutions for different types of cyclic loading.

It is obvious that soil stratum will be compressed and rebounds when it is subjected to cyclic loading. When soil stratum is loaded, the effective stress at any point will be increased and the soil thus compressed, but it is not just the reverse when soil stratum is unloaded. In the period of unloading, part of the soil in the upper layer near pervious boundary rebounds but the lower part of the soil near impervious boundary still compressed. Because soil has different compressibility when it is compressed or rebounds, it is

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necessary for analysis to distinguish reasonably which part of the soil stratum is compressed and which rebounds during consolidation so that suitable compressibility of soil can be used. However, because of the complexity involved, there is so far no consolidation theory available that takes into consideration the variations of compressibility of soil under cyclic loading.

In this paper, a semi-analytical method is proposed to solve the problem of one-dimensional consolidation under cyclic loading while taking consideration of varied compressibility of soil. A relevant computer program is then developed and used to investigate the consolidation behavior of soil with varied compressibility under cyclic loading. The result of this method is subsequently compared with the available analytical solution without considering varied compressibility.

MATHEMATICAL MODELING

The consolidation problem studied here is shown in Fig.1. A soil stratum with thickness H, vertical permeability coefficient k_v, volume compressibility coefficient m_v, consolidation coefficient C_v, and with a pervious upper boundary and an impervious lower boundary, is subjected to a vertical uniform trapezoidal cyclic loading shown in Fig.2a, in which q_0 is maximum load, t_0 is cyclic time, α and β are the coefficients of proportion. When α=0.5, the load becomes triangular one as shown in Fig.2b; and when

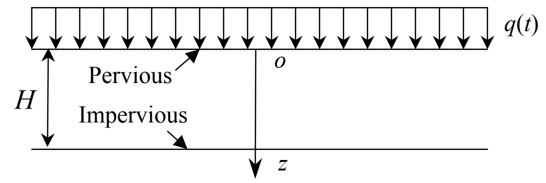

Fig.1 Model of clay layer

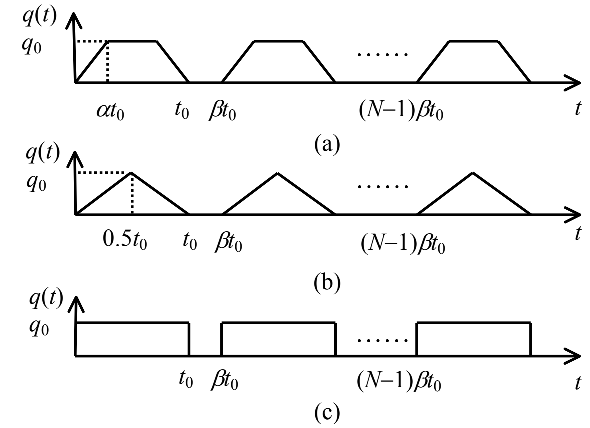

Fig.2 Loading types
(a) trapezoidal cyclic loading; (b) triangular cyclic loading; (c) rectangular cyclic loading

α=0, it changes into square cyclic loading, Fig.2c.

Based on the basic assumptions made in Terzaghi's 1-D linear consolidation theory except for loading condition and constant compressibility (i.e the value of m_v or C_v is constant) during consolidation, the differential equation governing the consolidation process of the clay layer can be expressed as follows:

$$C_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} - f(t) \quad (1)$$

in which u is excess porewater pressure; $f(t)$=dq/dt is the rate of loading.

The conditions for the solution of Eq.(1) are: (1) $u\big|_{z=0} = 0\ (t > 0)$; (2) $u\big|_{t=0} = 0$; (3) $\left.\frac{\partial u}{\partial z}\right|_{z=H} = 0$ (impervious); $u\big|_{z=H} = 0$ (pervious).

According to Terzaghi's effective stress principle, the effective stress σ' can be related to excess porewater pressure u and load q by

$$\sigma' + u = q(t) \quad (2)$$

For trapezoidal cyclic loading shown in Fig.2a, the expression of $q(t)$ can be given by

$$q(t) = \begin{cases} \dfrac{q_0}{\alpha t_0}[t-(N-1)\beta t_0] & (N-1)\beta t_0 \le t \le (N-1)\beta t_0 + \alpha t_0 \\ q_0 & (N-1)\beta t_0 + \alpha t_0 < t \le (N-1)\beta t_0 + (1-\alpha)t_0 \\ q_0\left\{1 - \dfrac{1}{\alpha t_0}[t-(N-1)\beta t_0 - (1-\alpha)t_0]\right\} & (N-1)\beta t_0 + (1-\alpha)t_0 < t \le (N-1)\beta t_0 + t_0 \\ 0 & (N-1)\beta t_0 + t_0 < t \le N\beta t_0 \end{cases} \quad (3)$$

where N=1,2,3,…, the number of cycles.

The expressions of other loading types shown in Fig.2b and Fig.2c can be obtained from Eq.(3) by setting the value of α at 0.5 and 0 respectively.

SEMI-ANALYTICAL METHOD

An analytical solution to the consolidation under cyclic loading can be obtained by Laplace transformation when the compressibility coefficient is constant during the loading. However, because soil stratum will be compressed and rebounds when it is subjected to cyclic loading, and because soil has different compressibility when it is compressed or rebounds, it is necessary for a reasonable analysis to take into consideration the varied compressibility of soil, that is to distinguish which part of the soil stratum is compressed and which part rebounds during consolidation so that suitable compressibility of soil can be used. Because of the complexity involved, a semi-analytical method similar to the one developed by Xie and Leo (1999) and Li *et al.*(1999) for nonlinear consolidation analysis is proposed.

First, soil stratum of thickness H is divided equally into n layers of equal thickness H/n (Fig.3). Let the origin of the coordinate z be at the top of the soil stratum, and denote the distance between the origin and the bottom of each layer as z_i. Thus, $z_0=0$, $z_i=iH/n$, i=1, 2, 3, …, n, and $z_n=H$.

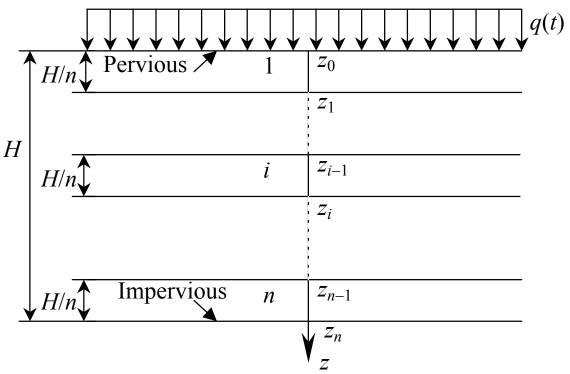

Fig.3 The divided soil layers in semi-analytical method

Because the thickness of each layer can be very small, at any time, the mean value of each soil parameter of each layer can stand for the value of the soil parameter of this layer, and the error caused by this approximation can be ignored. That is, the soil parameters of layer i can be respectively denoted as permeability coefficient k_{vi}, volume compressibility coefficient m_{vi}, consolidation coefficient C_{vi} (i=1, 2, 3, …, n).

Then taking into consideration the varied compressibility of soil during the process of consolidation, consolidation time is also divided into m small intervals and loading divided into m small loads accordingly. When the time interval is small enough, both the coefficient of consolidation at each small layer and the load in each small time interval can be taken as constants. The load q_k corresponding to time interval Δt_k ($\Delta t_k=t_k-t_{k-1}$, k=1, 2, …, m, t_0=0) can be given by

$$\Delta q_k=q(t_k)q(t_{k-1}) \quad (t_{k-1}\le t\le t_k;\ k=1,2,3,\ldots,m) \tag{4}$$

For trapezoidal cyclic loading, the expression of $q(t)$ is known, so q_k can then be obtained by substituting Eq.(3) into Eq.(4).

The differential equation governing the consolidation of each small layer can now be written as:

$$C_{vi}\frac{\partial^2 u_i}{\partial z^2}=\frac{\partial u_i}{\partial t} \quad (z_{i-1}\le z\le z_i,\ i=1,2,3,\ldots,n) \tag{5}$$

where, $u_i=u_i(z,t)$ and $C_{vi}=k_{vi}/(m_{vi}/\gamma_w)$, and γ_w is the unit weight of water, the excess pore water pressure and consolidation coefficient of small layer i respectively ($t_{k-1}\le t\le t_k$; k=1, 2, 3, …, m). C_{vi} changes with time t and depth z, and can be deduced from mean excess porewater pressure at time t_{k-1}.

The boundary conditions for Eq.(5) are:

$$z=0:\ u_1=0$$

$$z=H:\ \left.\frac{\partial u_n}{\partial z}\right|_{z=H}=0 \quad \text{(impervious)};$$

$$\left.u_n\right|_{z=H}=0 \quad \text{(pervious)} \tag{6a}$$

$$z=z_i:\ u_i=u_{i+1};$$

$$k_{vi}\frac{\partial u_i}{\partial z}=k_{v(i+1)}\frac{\partial u_{i+1}}{\partial z},\ i=1, 2, 3, \ldots, n-1 \tag{6b}$$

and the initial condition for Eq.(5) is given by

$$u_i=u_i'+q_k-q_{k-1} \tag{7}$$

where, u_i' is the excess porewater pressure at time t_{k-1}.

Thus, the problem of one-dimensional consolidation of soil with varied compressibility under cyclic loading has been reduced to the problem of one-dimensional linear consolidation of layered soils. According to Lee *et al*.(1992) and Xie and Pan (1995), the corresponding solution of Eq.(5) can be given as

$$u_i=\sum_{m=1}^{\infty}C_m g_{mi}(z)\mathrm{e}^{-\beta_m t},\ i=1, 2, 3\ldots, n \tag{8}$$

where,

$$\beta_m=\lambda_m^{\ 2}C_{\mathrm{v}1}\big/H^2;$$

$$g_{mi}(z)=A_{mi}\sin(\mu_i\lambda_m\frac{z}{H})+B_{mi}\cos(\mu_i\lambda_m\frac{z}{H})$$

$$C_m=\frac{\sum_{i=1}^{n}b_i\int_{z_{i-1}}^{z_i}[u_i'+(q_k-q_{k-1})]g_{mi}(z)\mathrm{d}z}{\sum_{i=1}^{n}b_i\int_{z_{i-1}}^{z_i}g_{mi}^{\ 2}(z)\mathrm{d}z}$$

$$=\frac{2\sum_{i=1}^{n}[u_i'+(q_k-q_{k-1})]\sqrt{a_ib_i}\left[A_{mi}(C_i-D_{i+1})\right.}{\sum_{i=1}^{n}\sqrt{a_ib_i}\left[\frac{\mu_i\lambda_m}{n}(A_{mi}^{\ 2}+B_{mi}^{\ 2})+(B_{mi}^{\ 2}-A_{mi}^{\ 2})\right.}\rightarrow$$

$$\leftarrow\frac{\left.B_{mi}(B_{i+1}-A_i)\right]}{\left.(D_{i+1}B_{i+1}-C_iA_i)+2A_{mi}B_{mi}(C_i^2-D_{i+1}^{\ 2})\right]} \tag{9}$$

the definitions of parameters λ_m, μ_i, a_i, b_i, A_{mi}, B_{mi}, C_i, D_i can be found in the paper by Xie and Pan (1995).

From the above solution of excess porewater pressure, the average degree of consolidation of layer i defined in terms of settlement can be given by:

$$U_i=\frac{1}{q_0h_i}\int_{z_{i-1}}^{z_i}[q(t)-u_i]\mathrm{d}z$$

$$=\frac{1}{q_0}\left[q(t)-\sum_{m=1}^{\infty}\frac{A_{mi}(C_i-D_{i+1})+B_{mi}(B_{i+1}-A_i)}{\mu_i\lambda_m}\right.\rightarrow$$

$$\left.\leftarrow C_m\mathrm{e}^{-\beta_m t}\int_0^t\frac{\mathrm{d}q}{\mathrm{d}t}\mathrm{e}^{\beta_m t}\mathrm{d}t\right] \tag{10}$$

The total average degree of consolidation of the whole clay stratum defined in terms of settlement, U_s, can be given by:

$$U_\mathrm{s}=\frac{S_t}{S_\infty}=\frac{1}{q_0\sum_{i=1}^{n}m_{\mathrm{v}i}h_i}\left[\sum_{i=1}^{n}m_{\mathrm{v}i}\int_{z_{i-1}}^{z_i}(q(t)-u_i)\mathrm{d}z\right]$$

$$=\frac{\sum_{i=1}^{n}b_i\rho_iU_i}{\sum_{i=1}^{n}b_i\rho_i} \tag{11}$$

Meanwhile, the total average degree of consolidation defined in terms of effective stress, U_p, can be derived as follows:

$$U_\mathrm{p}=\frac{q(t)-\bar{u}}{q_0}$$

$$=\frac{1}{q_0}[q(t)-\frac{1}{H}\sum_{i=1}^{n}\int_{z_{i-1}}^{z_i}u_i\mathrm{d}z]=\sum_{i=1}^{n}\rho_iU_i \tag{12}$$

where, $b_i=\frac{m_{\mathrm{v}i}}{m_{\mathrm{v}1}}$; $\rho_i=\frac{h_i}{H}$.

As can be seen from Eq.(11) and Eq.(12), in which h_i=z_i, unlike that indicated by Terzaghi's 1-D consolidation theory, the total average degree of consolidation defined in terms of settlement and that defined in terms of effective stress is not the same, i.e., $U_\mathrm{s}\neq U_\mathrm{p}$, and will be the same only if the value of b_i=1, i.e., soil compressibility at each layer is equal. However, as mentioned above, soil compressibility of a layer is always changing under cyclic loading. It is therefore necessary to take into consideration the varied compressibility of soil.

The changes of compressibility of soil under cyclic loading can be illustrated by Fig.4, where e is the void ratio and p is the effective stress. It is obvious that soil will repeatedly undergo loading, unloading and reloading when it is subjected to cyclic loading. Accordingly, soil compresses, rebounds and recompresses again, and the compressibility will be changed

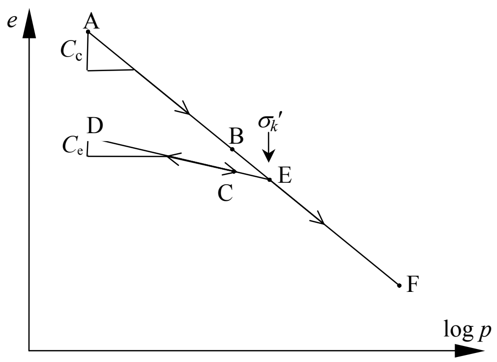

Fig.4 Compression curve of soil (*e*-log *p*)

frequently. The condition of soil at some elevation can be judged by comparison of the effective stress σ_k' at time t_k with σ_{k+1}' at time t_{k+1}. For example, when $\sigma_{k+1}' \geq \sigma_k'$, it means the soil is loaded (along straight line AB, see Fig.4), and when it rebounds (along CD), σ_{k+1}' must be smaller than σ_k' (i.e. $\sigma_{k+1}' \leq \sigma_k'$). The actual compressibility of soil can thus be chosen by this judgment.

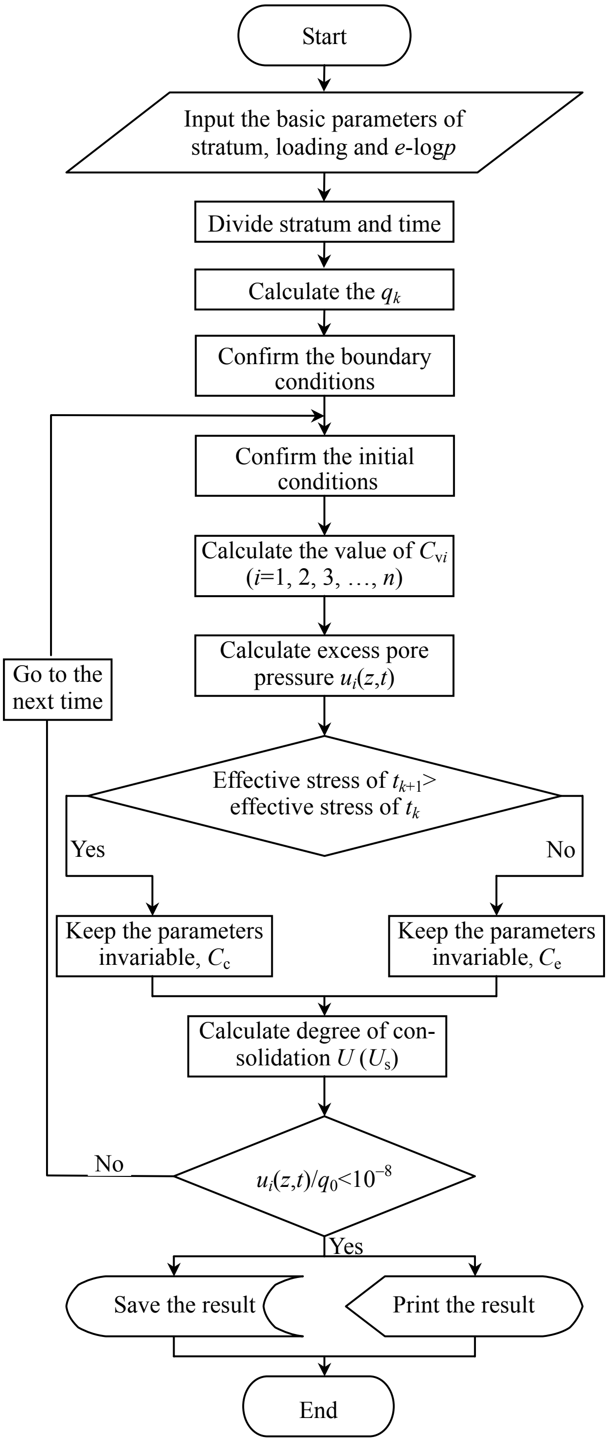

Fig.5 The computation procedure

Using the solution shown above, a computer program was developed for this semi-analytical method with FORTRAN language. Fig.5 is the flow chart of this program.

In the computations with the program, soil stratum can be divided into 40 small layers, and the time interval is controlled by non-dimensional time factor $\Delta T = C_{v0}\Delta t_k/H^2$, which can be 0.001. When the soil consolidation has occurred for a given time, and the ratio of the soil parameters has become smaller, the time step can be made larger to speed up the program calculation.

NUMERICAL ANALYSIS AND DISCUSSIONS

Consolidation behavior

There are three dimensionless variables, i.e. $T_{v0}=C_v t_0/H^2$, α and β, that govern the consolidation process in one cycle. T_{v0} reflects the influence of construction time t_0; α and β reflect the properties of loading. Therefore, the one dimensional consolidation behavior of the soil can be investigated by giving one of the variables different values while fixing the values of the other variables, as shown by the following discussions.

Fig.6 shows the influence of construction time t_0 on consolidation, in which, α=0.3, β=1.5, and T_{v0} is varying. It can be seen that the longer the construction time t_0 (i.e. the greater T_{v0}), the slower the rate of consolidation.

In Fig.7, T_{v0}=0.1 and β=1.5, α is varying. So the figure shows the influence of α, i.e. the rate of loading, on the consolidation of soil, from which it can be seen that the influence is more and more significant as the value of α increases.

Fig.8 was plotted to investigate the influence of β, in which α=0.3, and T_{v0}= 0.1. As can be seen from the figure, with the decrease of β (i.e. the time of one cycle) the consolidation degree is increased. Additionally, a steady-state condition is reached after a sufficiently large number of cycles (e.g. N=40 and $N\to\infty$).

It can also be seen from these figures that, in a given cycle, the consolidation degree of each cycle

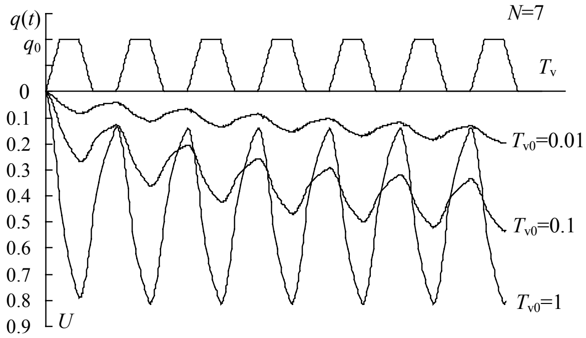

Fig.6 The influence of T_{v0} on U

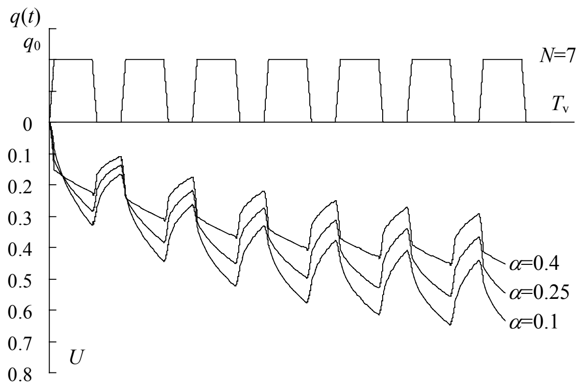

Fig.7 Influence of α to U

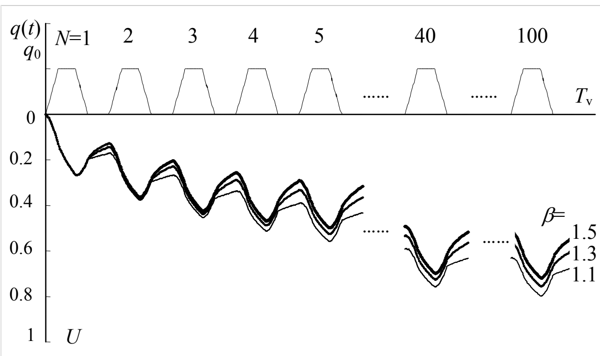

Fig.8 Influence of β to U

reaches the maximum value at the end of the constant loading phase and the minimum value at the beginning of the next cycle (i.e. at the end of the unloading).

Fig.9 presents the excess pore pressure distribution with depth at the end of constant loading per cycle, in which T_{v0}=0.1, α=0.3, β=1.5. It shows that the excess pore pressure decreases with increasing value of N.

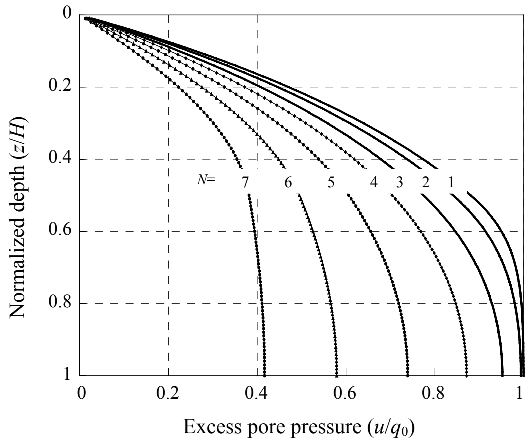

Fig.9 Excess pore pressure isochrone

Comparison with conventional theory

For the problem of one-dimensional consolidation under trapezoidal cyclic loading without considering varied compressibility of soil, the analytical solutions can be obtained by the method of Guan *et al.*(2003) with Laplace transformation, Laplace inverse transformation and the time-delay characteristics. The solution when $(N-1)\beta t_0 \le t \le (N-1)\beta t_0+\alpha t_0$ (loading phase) is

$$u=\sum_{m=1}^{\infty}\frac{2q_0}{\alpha M^3 T_0}\sin\left(\frac{Mz}{H}\right)(1-B_N \mathrm{e}^{-M^2T'}) \tag{13}$$

when $(N-1)\beta t_0+\alpha t_0 \le t \le (N-1)\beta t_0+(1-\alpha)t_0$ (constant loading phase) is

$$u=\sum_{m=1}^{\infty}\frac{2q_0}{\alpha M^3 T_0}\sin\left(\frac{Mz}{H}\right)\left[(\mathrm{e}^{\alpha M^2T_0}-B_N)\mathrm{e}^{-M^2T'}\right] \tag{14}$$

when $(N-1)\beta t_0+(1-\alpha)t_0 \le t \le (N-1)\beta t_0+t_0$ (unloading phase) is

$$u=-\sum_{m=1}^{\infty}\frac{2q_0}{\alpha M^3 T_0}\sin\left(\frac{Mz}{H}\right)\cdot \left[1-(\mathrm{e}^{\alpha M^2T_0}+\mathrm{e}^{(1-\alpha)M^2T_0}-B_N)\mathrm{e}^{-M^2T'}\right] \tag{15}$$

when $(N-1)\beta t_0+t_0 \le t \le N\beta t_0$ (no-loading phase) is

$$u=\sum_{m=1}^{\infty}\frac{2q_0}{\alpha M^3 T_0}\sin\left(\frac{Mz}{H}\right)\cdot$$

$$\left[(e^{\alpha M^2 T_0} - e^{M^2 T_0} + e^{(1-\alpha)M^2 T_0} - B_N)e^{-M^2 T'}\right] \quad (16)$$

where,

$$B_N = 1 + \frac{(e^{\alpha M^2 T_0} - 1)(e^{\alpha M^2 T_0} - e^{M^2 T_0})(1 - e^{-(N-1)\beta M^2 T_0})}{e^{\alpha M^2 T_0}(1 - e^{\beta M^2 T_0})}$$

$$T_0 = \frac{C_v t_0}{H^2},\ T' = \frac{C_v t'}{H^2},\ M = \frac{2m-1}{2}\pi,\ m=1, 2, 3, \ldots.$$

The difference between the conventional analytical solution and the semi-analytical solution developed here can be seen from Fig.10, where α=0.3, β=1.5, T_{v0}=0.1.

As indicated by Fig.10, the consolidation behaviors in the first cycle of these two solutions are almost same, which can prove the correctness of the semi-analytical solution, and with the increasing number of cycles, the difference between these two solutions becomes more and more significant although the rate of consolidation increases slowly with the number of cycles.

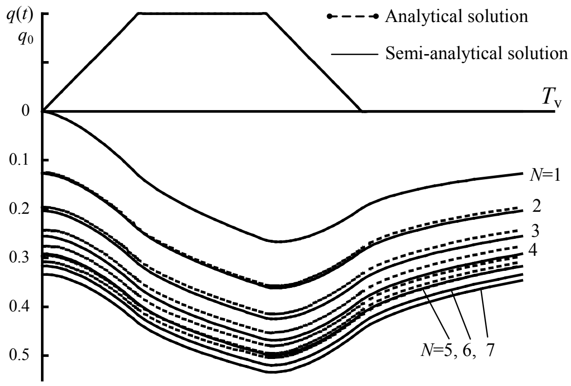

Fig.10 Comparison of conventional analytical solution and semi-analytical solution

CONCLUSION

The following conclusions may be drawn from this study:

1. The solution of the semi-analytical solution developed here for one-dimensional consolidation of soil under cyclic loading and with varied compressibility shows that the consolidation behavior is mainly governed by the three parameters α, β, T_{v0}.

2. For cyclic trapezoidal loading, the consolidation rate in each cycle maximizes at the end of the constant loading phase and minimizes at the beginning of the next cycle.

3. Varied compressibility of soil can affect the consolidation behavior of soil under cyclic loading. The rate of soil consolidation will be faster than in the conventional solution, when varied compressibility is considered.

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