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On the dissipation of negative excess porewater pressure induced by excavation in soft soil*

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Abstract: Unloading induces negative excess porewater pressure in soil mass around a foundation pit during excavation. In this work, the dissipation rule of negative excess porewater pressure after excavation was studied. Analytical formulas for calculating the negative excess porewater pressures and the effective stresses were derived based on one-dimensional consolidation theory and Terzaghi's effective stress principle. The influence of the dissipation of negative excess porewater pressure on earth pressure inside and outside a foundation pit and the stability of the retaining structure were analyzed through a numerical example. It was indicated that the dissipation of negative excess porewater pressure is harmful to the stability of the retaining structure and that rapid construction can make full use of the negative porewater pressure.

Key words: Negative excess porewater pressure, Effective stress, Earth pressure, Excavation
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INTRODUCTION

Since Terzaghi founded his one-dimensional consolidation theory, the studies on consolidation of soft soil have been made by many researchers (Gray, 1945; Gibson *et al*., 1967; 1981; Schiffman and Stein, 1970; Xie, 1994; Xie *et al*., 2002a). However, although as we all know, negative excess porewater pressure will be induced by unloading in excavation, the dissipation of the negative pressure and its effects are rarely studied because of their complexity (Chen and Wen, 1999; Li, 2000; Wang *et al*., 2003), and current researches are mainly focused on steady seepage after excavation (Yang and Gong, 1997; Xie *et al*., 2002b). Since investigation of the dissipation of negative porewater pressure after excavation by analytical method is very difficult, it should be done first from one-dimensional case although the stress and strain conditions involved in excavation are three-dimensional or at least a plane-strain condition. In this paper based on one-dimensional consolidation theory and the principle of Terzaghi's effective stress, analytical formulas for calculating the negative excess porewater pressures and the effective stresses after excavation were obtained on the assumption that the dissipation process of negative excess porewater pressure is the reverse process of consolidation. The influence of the dissipation of negative excess porewater pressure on active and passive lateral pressures acting on the retaining structure was then investigated.

GOVERNING EQUATIONS AND THEIR SOLUTIONS

It is well known that excavation will change the initial stress field in the soil stratum. As a result, negative excess porewater pressure occurs in soil mass inside and outside a foundation pit during excavation, and thereafter dissipates gradually accom-

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panied with soil swelling. It is therefore acceptable to consider the dissipation process of negative excess porewater pressure as a reverse process of consolidation.

Fig.1 is sketch map of a foundation pit, where Δh, h_1 and h_2 are excavation depth, thickness of soil layer in active zone and that in passive zone respectively. Since the permeability coefficient of soft soil is very small, the water table around the pit can be assumed to keep constant on the ground surface outside the pit and on the pit base inside the pit. Adopting all the assumptions made in Terzaghi's one-dimensional consolidation theory, and assuming further that the resilient moduli of soil inside and outside the foundation pit are constant and that excavation is instantaneously completed, the governing equations for negative porewater pressure dissipation after excavation can be obtained as follows:

$$\frac{\partial u_1}{\partial t} = c_{r1}\frac{\partial^2 u_1}{\partial z^2} \quad \text{(in active zone)} \tag{1}$$

$$\frac{\partial u_2}{\partial t} = c_{r2}\frac{\partial^2 u_2}{\partial z^2} \quad \text{(in passive zone)} \tag{2}$$

where u_1 and u_2 are the excess porewater pressures in active and passive zones; $c_{ri}=k_{vi}E_{ri}/\gamma_w=k_{vi}/(m_{ri}\gamma_w)$ (i=1,2), is called swelling coefficient; k_{vi}, E_{ri}, $m_{ri}=1/E_{ri}$ (i=1,2), are vertical permeability coefficient, resilient modulus, volumetric resilient coefficient of soil in active and passive zones respectively; t and z are the variables of time and space; γ_w is unit weight of water.

Since seepage inside and outside a pit is continuous at the bottom of the retaining structure, the boundary conditions for Eqs.(1) and (2) can be expressed as:

$$\begin{cases} u_1(0,t)=0 \\ u_2(\Delta h,t)=0 \\ u_1(h_1,t)=u_2(h_1,t) \\ k_{v1}\dfrac{\partial u_1(h_1,t)}{\partial z}=-k_{v2}\dfrac{\partial u_2(h_1,t)}{\partial z} \end{cases} \tag{3}$$

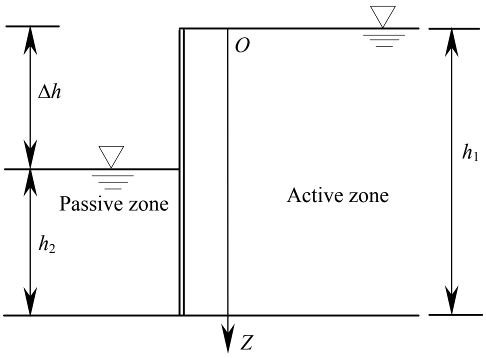

Fig.1 Sketch map of a foundation pit

Because negative excess porewater pressure is induced by unloading during excavation, after excavation its initial value at the pit base is just the original effective overburden pressure therein. Thus, the initial negative excess porewater pressure in the passive zone can be assumed as $u_2(z,0)=-\gamma_0'\Delta h$. The initial negative excess porewater pressure in the active zone is approximatively regarded as linear change from zero at the ground surface to $-\gamma_0'\Delta h$ at the bottom of the retaining structure. So the initial conditions for Eqs.(1) and (2) can be written as:

$$\begin{cases} u_1(z,0)=u_0 z/h_1=-\gamma_0'\Delta h z/h_1 \\ u_2(z,0)=u_0=-\gamma_0'\Delta h \end{cases} \tag{4}$$

where γ_0' is the effective unit weight of soil, and u_0 is the initial excess porewater pressure in passive zone.

In order to solve Eqs.(1) and (2) expediently, active zone and passive zone are to be a two-layered system as shown in Fig.2, considering that seepage inside and outside foundation pit is continuous at the

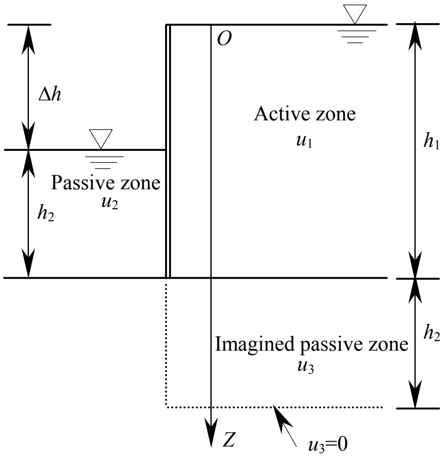

Fig.2 Sketch map for the analysis of negative excess porewater pressure dissipation

bottom of retaining structure. The excess porewater pressure in imagined passive zone is expressed by u_3, and the governing equation is given by:

$$\frac{\partial u_3}{\partial t} = c_{r2}\frac{\partial^2 u_3}{\partial z^2} \tag{5}$$

where u_2 and u_3 have the following relation:

$$u_2(z,t) = u_3(2h_1 - z, t) \tag{6}$$

Accordingly, the boundary and initial conditions should be transformed into:

$$\begin{cases} u_1(0,t) = 0 \\ u_3(h_1+h_2,t) = 0 \\ u_1(h_1,t) = u_3(h_1,t) \\ k_{v1}\dfrac{\partial u_1(h_1,t)}{\partial z} = k_{v2}\dfrac{\partial u_3(h_1,t)}{\partial z} \end{cases} \tag{7}$$

$$\begin{cases} u_1(z,0) = u_0 z/h_1 = -\gamma_0'\Delta h z/h_1 \\ u_3(z,0) = u_0 = -\gamma_0'\Delta h \end{cases} \tag{8}$$

From Eqs.(1) and (5), the negative excess porewater pressures in active and imagined passive zones satisfying boundary conditions expressed by Eq.(7) can be obtained as follows:

$$u_1(z,t) = \sum_{m=1}^{\infty} B_m \sin\left(\lambda_m \frac{z}{h_1}\right)\mathrm{e}^{-\beta_m t} \quad (0 \le z \le h_1) \tag{9}$$

$$u_3(z,t) = \sum_{m=1}^{\infty} A_m B_m \sin\left(\mu\lambda_m \frac{h_1+h_2-z}{h_1}\right)\mathrm{e}^{-\beta_m t} \quad (h_1 \le z \le h_1+h_2) \tag{10}$$

where $\beta_m = \dfrac{c_{r1}\lambda_m^2}{h_1^2}$, $\mu = \sqrt{\dfrac{c_{r1}}{c_{r2}}} = \sqrt{\dfrac{b}{K}}$, $b = \dfrac{m_{r2}}{m_{r1}}$,

$$A_m = \begin{cases} \dfrac{\sin\lambda_m}{\sin(\mu c\lambda_m)} & \sin(\mu c\lambda_m) \ne 0; \\ -\dfrac{\cos\lambda_m}{\mu K\cos(\mu c\lambda_m)} & \sin(\mu c\lambda_m) = 0, \end{cases}$$

$K=k_{v2}/k_{v1}$, $c=h_2/h_1$, λ_m is the positive eigenvalue of the following eigen-equation:

$$\mu K\sin\lambda_m\cos(\mu c\lambda_m) + \cos\lambda_m\sin(\mu c\lambda_m) = 0 .$$

Using initial conditions expressed by Eq.(8) and the following orthogonal relation:

$$\mu^2 K\int_{h_1}^{h_1+h_2} A_m A_n \sin\left(\mu\lambda_m\frac{h_1+h_2-z}{h_1}\right)\cdot \sin\left(\mu\lambda_n\frac{h_1+h_2-z}{h_1}\right)\mathrm{d}z + \int_0^{h_1}\sin\left(\lambda_m\frac{z}{h_1}\right)\sin\left(\lambda_n\frac{z}{h_1}\right)\mathrm{d}z$$

$$= \begin{cases} 0 & (m \ne n) \\ \dfrac{h_1}{2}(1+\mu^2 KcA_m^2) = L & (m = n) \end{cases}$$

Let

$$Q = \sin\left(\mu\lambda_m\frac{h_1+h_2-z}{h_1}\right)$$

$$M = \int_0^{h_1} u_1(z,0)\sin\left(\lambda_m\frac{z}{h_1}\right)\mathrm{d}z$$

$$N = \mu^2 K\int_{h_1}^{h_1+h_2} A_m u_3(z,0)Q\mathrm{d}z$$

B_m can be given by:

$$B_m = \frac{M+N}{L} = \frac{2u_0(\mu KA_m\lambda_m + \sin\lambda_m)}{\lambda_m^2(1+\mu^2 KcA_m^2)}$$

Finally, the negative excess porewater pressure in actual passive zone can be obtained from Eqs.(6) and (10):

$$u_2(z,t) = u_3(2h_1 - z, t) = \sum_{m=1}^{\infty} A_m B_m \sin\left(\mu\lambda_m\frac{z-\Delta h}{h_1}\right)\mathrm{e}^{-\beta_m t} \quad (\Delta h \le z \le h_1) \tag{11}$$

DISTRIBUTIONS OF NEGATIVE EXCESS POREWATER PRESSURE

Based on the above solutions, the distributions of negative excess porewater pressure in active and passive zone are studied. Fig.3 shows the variation of the excess porewater pressures with soil stiffness represented by b. It can be seen that excess porewater pressures dissipate slower when soil volumetric resilient coefficient in passive zone is greater. The influence of soil permeability on the distribution of excess porewater pressure is shown in Fig.4. It indicates that excess porewater pressures dissipate faster when soil permeability in passive zone is greater.

LATERAL PRESSURES ACTING ON RETAINING STRUCTURE

Based on Terzaghi's effective stress principle, the effective stresses in active and passive zones at any time can be given by:

$$\sigma_1'(z,t)=\sigma_1-u_1(z,t)-\gamma_w z=\gamma_{\text{sat}1}z-\gamma_w z-u_1(z,t)$$
$$=\gamma_1' z-u_1(z,t)\quad (0\le z\le h_1)\qquad(12)$$

$$\sigma_2'(z,t)=\sigma_2-\gamma_w(z-\Delta h)-u_2(z,t)$$
$$=\gamma_{\text{sat}2}(z-\Delta h)-\gamma_w(z-\Delta h)-u_2(z,t)$$
$$=\gamma_2'(z-\Delta h)-u_2(z,t)\ (\Delta h\le z\le h_1)\qquad(13)$$

where σ_i', σ_i, γ_i', $\gamma_{\text{sat}i}$ (i =1,2) are effective vertical stress, total vertical stress, effective unit weight of soil, unit weight of saturated soil in active zone and passive zone, respectively.

According to Rankine's earth pressure theory, the lateral pressures acting on retaining structure can be obtained as follows:

$$p_{\text{a}}=K_{\text{a}}\sigma_1'(z,t)-2c_1'\sqrt{K_{\text{a}}}+u_1(z,t)+\gamma_w z$$
$$=K_{\text{a}}\gamma_1' z-2c_1'\sqrt{K_{\text{a}}}+(1-K_{\text{a}})u_1(z,t)+\gamma_w z$$
$$(0\le z\le h_1)\qquad(14)$$

$$p_{\text{p}}=K_{\text{p}}\sigma_2'(z,t)+2c_2'\sqrt{K_{\text{p}}}+u_2(z,t)+\gamma_w(z-\Delta h)$$
$$=K_{\text{p}}\gamma_2'(z-\Delta h)+2c_2'\sqrt{K_{\text{p}}}+\gamma_w(z-\Delta h)$$
$$+(1-K_{\text{p}})u_2(z,t)\ (\Delta h\le z\le h_1)\qquad(15)$$

where $K_{\text{a}}=\tan^2(45^{\circ}-\varphi_1'/2)$, $K_{\text{p}}=\tan^2(45^{\circ}+\varphi_2'/2)$,

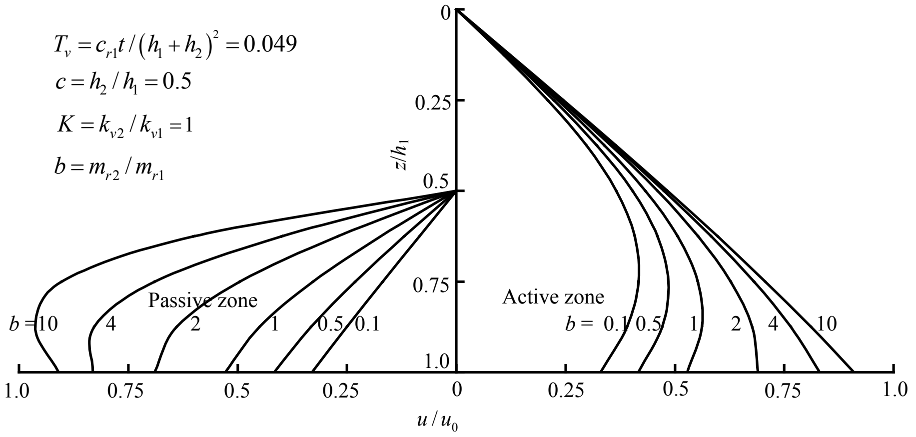

Fig.3 Influence of soil stiffness on excess porewater pressure distribution

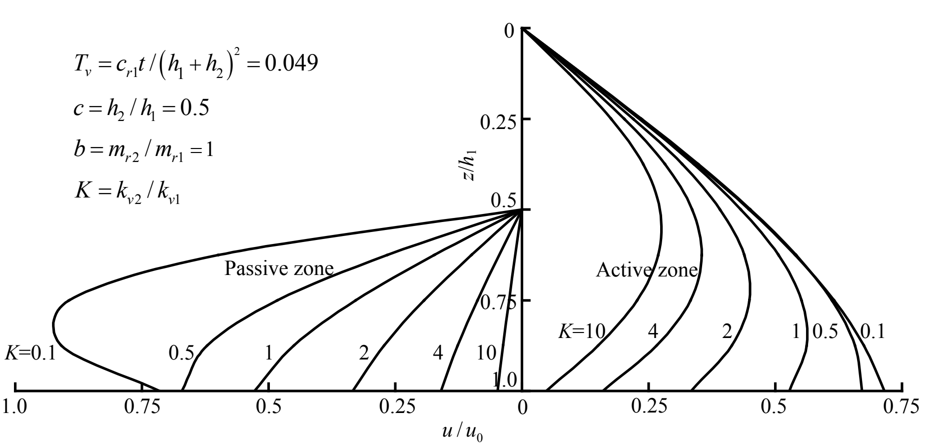

Fig.4 Influence of soil permeability on excess porewater pressure distribution

p_a and p_p are the lateral pressures acting on retaining structure in active and passive zones; c_i' and φ_i' (i=1,2) are the effective cohesion and the effective internal friction angle of soil in active and passive zones respectively.

It can be seen from Eqs.(12) and (13) that the effective stresses both in active and passive zones decrease gradually with the dissipation of negative excess porewater pressure. As a result, the lateral pressure acting on the retaining structure in passive zone (p_p) will decrease with the dissipation of negative excess porewater pressure, whereas the one in active zone (p_a) will increase. This means that the foundation pit gets more and more unsafe with the dissipation of negative porewater pressure. So installing brace in time and casting concrete as fast as possible can make full use of the negative excess porewater pressure and is thus beneficial to safety, which is completely in accordance with practical experience.

NUMERICAL EXAMPLE

In order to analyze the influence of the dissipation of negative excess porewater pressure on effective vertical stresses in soil and lateral pressures acting on retaining structure in active and passive zones, a numerical example is given below.

The excavated depth of the foundation pit is 8 m and retaining structure is embedded to 16 m deep. The water table outside the pit keeps on the ground surface. The parameters of soil are: $\gamma_1' = \gamma_2' = 8.9$ kN/m^3, $k_{v1} = k_{v2} = 8.7\times10^{-7}$ cm/s, $E_{r1} = E_{r2} = 7.2$ MPa, $c_1' = c_2' = 0$, $\varphi_1' = \varphi_2' = 20^\circ$.

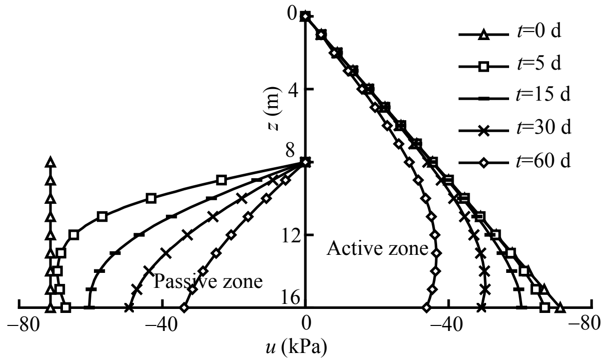

Fig.5 Curves of negative excess porewater pressure dissipation with time

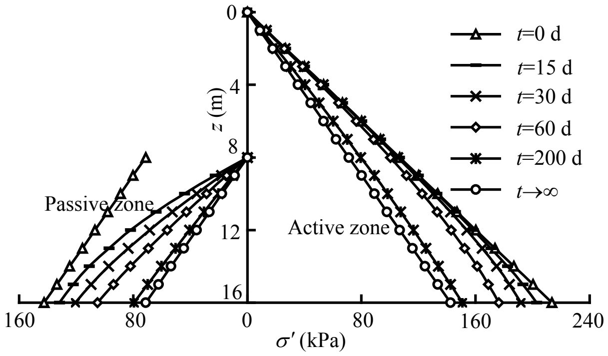

Fig.6 Variations of effective vertical stress with time

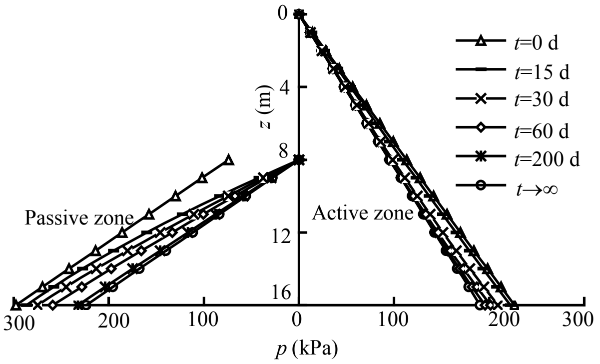

Fig.7 Variations of lateral pressure acting on retaining structure with time

Fig.5 to Fig.7 show the dissipation curves of porewater pressure, the variations of effective vertical stress and the changes of lateral pressure with exposure time after excavation, respectively. It can be seen that the negative excess porewater pressures in active and passive zones both dissipate gradually with exposure time. The effective vertical stresses in both zones are greater than the ones without considering negative excess porewater pressure, and both decrease gradually with exposure time. The lateral pressure acting on the retaining structure decreases in the passive zone with the dissipation of negative porewater pressure; but increases in the active zone. Therefore the dissipation of negative excess porewater pressure is harmful to the stability of the retaining structure and rapid construction can make full use of the negative porewater pressure.

CONCLUSION

Based on one-dimensional consolidation theory, analytical formulas for calculating the negative excess porewater pressures in active and passive zones were obtained on the assumption that the dissipation process of negative excess porewater pressure is the reverse of consolidation. Study on the influences of negative excess porewater pressure dissipation on effective vertical stresses and lateral pressures acting on the retaining structure led the following conclusions:

1. When soil permeability in passive zone is greater, the excess porewater pressures dissipate faster; and when soil volumetric resilient coefficient in passive zone is greater, the excess porewater pressures dissipate slower.

2. The effective stresses in active and passive zones both get less gradually with the dissipation of negative excess porewater pressure.

3. With the dissipation of negative porewater pressure, lateral pressure acting on the retaining structure gets greater and greater in active zone, but smaller and smaller in passive zone, which is harmful to the stability of the retaining structure. Rapid construction after excavation can make full use of the negative excess porewater pressure induced by excavation, and is thus beneficial to the safety of a foundation pit in soft soil.

4. As a first stage work, the study reported in this paper is based on one-dimensional case. However, excavation problems are naturally in two-or-three dimensional conditions. Therefore, further studies should be made both from theoretical investigation and from field measurement.

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