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Transient response of a spherical cavity with a partially sealed shell embedded in viscoelastic saturated soil

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Abstract: Based on Biot's wave equation, this paper discusses the transient response of a spherical cavity with a partially sealed shell embedded in viscoelastic saturated soil. The analytical solution is derived for the transient response to an axisymmetric surface load and fluid pressure in Laplace transform domain. Numerical results are obtained by inverting the Laplace transform presented by Durbin, and are used to analyze the influences of the partial permeable property of boundary and relative rigidity of shell and soil on the transient response of the spherical cavity. It is shown that the influence of these two parameters is remarkable. The available solutions of permeable and impermeable boundary without shell are only two extreme cases of this paper.

Key words: Viscoelasticity, Partial sealing, Spherical shell, Transient response
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INTRODUCTION

The propagation of stress-waves in an elastic medium containing a cavity that are due to arbitrary dynamic loading applied on the cavity is of great importance in the fields of seismology, geophysical prospecting, underground tunnels and deeply buried pipelines, particularly as a model of an earthquake source. Ben-Menahem and Cisternas (1963) developed the theory for the dynamic response of an elastic half-space medium to an explosion spherical cavity. Norwood and Miklowitz (1967) investigated the diffraction of transient elastic waves by a spherical cavity; two cases of a suddenly applied normal point load and the impingement of a plane transient dilatational pulse on the cavity were considered. Akkas and Zakout (1997) obtained the analytical solution for the transient response to an axisymmetric and non-torsional load of an infinite, isotropic, elastic medium containing a spherical cavity with and without thin elastic shell embedment by using the Residual Variable Method.

The propagation theory of elastic waves and the general solutions for fluid-saturated porous elastic and viscoelastic medium were first presented by Biot (1956; 1962). To date, however, rather limited work was focused on the dynamic response of spherical cavity in fluid saturated porous medium. Xu and Wu (1998) and Xu and Cai (2001) investigated spherical wave propagation in saturated soil and, by employing a viscoelastic model presented by Erigen (1980), obtained the dynamic response of the spherical cavity in viscoelastic soil subjected to axisymmetric surface load, but just considered two extreme cases of flow boundary condition: permeable and impermeable. In fact, the case of a thin elastic shell bonded to the surface of a cavity can represent most practical situations. Assuming that the shell or liner is porous material, Li (1999) presented the concept of a partially sealed boundary of a circular tunnel excavated in saturated soil.

The transient response of a spherical cavity with a partially sealed shell embedded in viscoelastic saturated soil was investigated in this work. The analytical solutions of stresses, displacements and pore pressure induced by axisymmetric surface load and fluid pressure are derived in Laplace transform domain. Furthermore, numerical results obtained by using Durbin (1974)'s inverse Laplace transform were used to analyze the influence of the partial permeable property of boundary and relative rigidity of shell and soil on the transient response of the spherical cavity.

SOLUTION OF SPHERICAL CAVITY WITHOUT SHELL

The thin elastic shell shown in Fig.1 is assumed to be bored in infinite viscoelastic saturated soil with outer radius r_2 and thickness h. The spherical coordinates are (r, θ, φ), where θ is the meridional angle and φ is the circumferential angle. For an axisymmetric non-torsional load, i.e. independent of the meridional angle θ and circumferential angle φ, acting on the shell surface, the non-vanishing components of the stress tensor are σ_r, σ_θ, σ_φ.

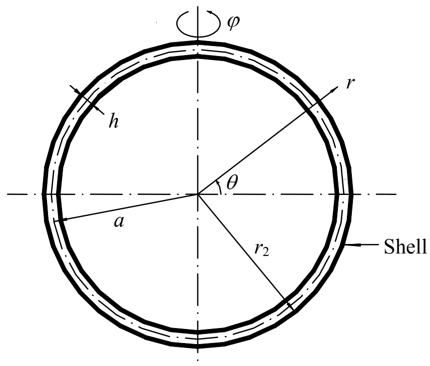

Fig.1 Geometry of the problem

The equilibrium equation for soil mass in spherical coordinate system can be written as:

$$\frac{\partial \sigma_r}{\partial r}+\frac{2\sigma_r-\sigma_\theta-\sigma_\varphi}{r}=\frac{\partial^2}{\partial t^2}(\rho u_r+\rho_{\mathrm{f}} w_r) \quad (1)$$

where, u_r and w_r are respectively radial displacement of soil skeleton and displacement of pore fluid with respect to the soil skeleton; $\rho=(1-n)\rho_{\mathrm{s}}+n\rho_{\mathrm{f}}$, the density of soil; ρ_{f} and ρ_{s} are densities of fluid and soil grains, respectively; n is porosity.

The pore fluid equilibrium equation is given by:

$$-\frac{\partial p}{\partial r}=\frac{\partial^2}{\partial t^2}\left(\rho_{\mathrm{f}} u_r+\frac{\rho_{\mathrm{f}}}{n} w_r\right)+\frac{\eta_0}{k_{\mathrm{d}}}\frac{\partial w_r}{\partial t} \quad (2)$$

where, p is excess pore pressure; η_0 is the fluid viscosity and k_{d} is the intrinsic permeability of soil.

Assuming that the viscoelastic property of soil can be simulated by Kelvin-Voigt model, the stress-strain relationship can be expressed by (Erigen, 1980):

$$\sigma_r=\lambda e+2G\frac{\partial u_r}{\partial r}+\lambda'\frac{\partial e}{\partial t}+2G'\frac{\partial}{\partial t}\left(\frac{\partial u_r}{\partial r}\right)-\alpha p \quad (3a)$$

$$\sigma_\theta=\sigma_\varphi=\lambda e+2G\frac{u_r}{r}+\lambda'\frac{\partial e}{\partial t}+2G'\frac{\partial}{\partial t}\left(\frac{u_r}{r}\right)-\alpha p \quad (3b)$$

$$p=M\xi-\alpha Me \quad (3c)$$

where, $e=\frac{\partial u_r}{\partial r}+\frac{2u_r}{r}$ and $\xi=-\left(\frac{\partial w_r}{\partial r}+\frac{2w_r}{r}\right)$, the dilatations of solid and fluid, respectively; λ and G are the Lame constants of the bulk material; λ' and G' are the dilatant and shear constant of the viscoelastic soil; α and M are the compressibility parameters of the two-phase medium, $0\le\alpha\le1$, $0\le M\le\infty$, and $M\to\infty$, $\alpha\to1$ for a material with incompressible constituents.

Substituting Eq.(3) into Eqs.(1) and (2), the governing equations of the transient response of a spherical cavity in viscoelastic saturated soil can be obtained as:

$$(\lambda+2G+\alpha^2 M)\frac{\partial e}{\partial r}+(\lambda'+2G')\frac{\partial}{\partial t}\left(\frac{\partial e}{\partial r}\right)-\alpha M\frac{\partial \xi}{\partial r}=\frac{\partial^2}{\partial t^2}(\rho u_r+\rho_{\mathrm{f}} w_r) \quad (4a)$$

$$\alpha M\frac{\partial e}{\partial r}-M\frac{\partial \xi}{\partial r}=\frac{\partial^2}{\partial t^2}\left(\rho_{\mathrm{f}} u_r+\frac{\rho_{\mathrm{f}}}{n} w_r\right)+\frac{\eta_0}{k_{\mathrm{d}}}\frac{\partial w_r}{\partial t} \quad (4b)$$

To solve Eq.(4), the following two scalar quantities $\Phi_1(r,t)$ and $\Phi_2(r,t)$ are introduced and defined as:

$$u_r=\frac{\partial}{\partial r}\left(\frac{\Phi_1(r,t)}{r}\right) \quad (5a)$$

$$w_r=\frac{\partial}{\partial r}\left(\frac{\Phi_2(r,t)}{r}\right) \quad (5b)$$

Defining the function of Laplace transform, $f(r,t)$ as:

$$s^n\overline{f}(r,s)=\int_0^{\infty}\frac{\partial^n f(r,t)}{\partial t^n}\mathrm{e}^{-st}\mathrm{d}t \quad (6a)$$

and the inversion by

$$f(r,t)=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\overline{f}(r,s)\mathrm{e}^{st}\mathrm{d}s \quad (6b)$$

in which, s is the Laplace transform variable; $i=\sqrt{-1}$; the parameter γ is so selected that the line Re(s)=γ is to the right of all singularities of $\overline{f}(r,s)$.

Substituting Eqs.(5a) and (5b) into Eqs.(4a) and (4b), and applying the Laplace transform yields:

$$\left[(\lambda+2G+\alpha^2M)\nabla^2+(\lambda'+2G')s\nabla^2-\rho s^2\right]\overline{\Phi}_1+(\alpha M\nabla^2-\rho_\mathrm{f}s^2)\overline{\Phi}_2=0 \quad (7a)$$

$$(\alpha M\nabla^2-\rho_\mathrm{f}s^2)\overline{\Phi}_1+\left(M\nabla^2-\frac{\rho_\mathrm{f}}{n}s^2-\frac{\eta_0}{k_\mathrm{d}}s\right)\overline{\Phi}_2=0 \quad (7b)$$

where, $\nabla^2=\dfrac{\partial^2}{\partial r^2}$.

Lou and Lin (1986) reported that the viscoelastic damp coefficient of rock and soft soil could be assumed as a constant in a great range of vibration frequencies. In this paper, the dimensionless damp coefficient η is assumed to be

$$\eta=\frac{\lambda'}{\lambda}=\frac{G'}{G} \quad (8)$$

In terms of all dimensionless variables in Eq.(7), and the use of Eq.(8) yields:

$$\left\{\left[(\lambda^*+2)(1+\eta s)+\alpha^2M^*\right]\nabla^2-s^2\right\}\overline{\Phi}_1+(\alpha M^*\nabla^2-\rho^*s^2)\overline{\Phi}_2=0 \quad (9a)$$

$$(\alpha M^*\nabla^2-\rho^*s^2)\overline{\Phi}_1+\left(M^*\nabla^2-\frac{\rho^*}{n}s^2-b^*s\right)\overline{\Phi}_2=0 \quad (9b)$$

where, $\lambda^*=\dfrac{\lambda}{G}$, $M^*=\dfrac{M}{G}$, $\rho^*=\dfrac{\rho_\mathrm{f}}{\rho}$, $b^*=\dfrac{\eta_0}{k_\mathrm{d}}\dfrac{a}{\sqrt{\rho G}}$, are the non-dimensional Lame constant, compressibility parameter, fluid density, and permeability coefficient of soil, respectively. $a=r_2-h/2$ (Fig.1).

Uncoupling Eq.(9) yields:

$$\left(\nabla^2-\gamma_1^2\right)\left(\nabla^2-\gamma_2^2\right)\overline{\Phi}_{1,2}=0 \quad (10)$$

where, γ_1 and γ_2 are the complex wave numbers of two dilatational waves, i.e.

$$\gamma_1^2=\frac{\alpha_1-\sqrt{\alpha_1^2-4\alpha_2}}{2} \quad (11a)$$

$$\gamma_2^2=\frac{\alpha_1+\sqrt{\alpha_1^2-4\alpha_2}}{2} \quad (11b)$$

with

$$\alpha_1=\frac{[(\lambda^*+2)(1+\eta s)+\alpha^2M^*](\rho^*s^2/n+b^*s)+M^*s^2-2\alpha M^*\rho^*s^2}{(\lambda^*+2)(1+\eta s)M^*}$$

$$\alpha_2=\frac{s^2(\rho^*s^2/n+b^*s)-(\rho^*s^2)^2}{(\lambda^*+2)(1+\eta s)M^*}$$

Considering the limitation property of radial displacement when $r\to\infty$, the general solution of $\overline{\Phi}_1$ and $\overline{\Phi}_2$ in Eq.(10) can be written as:

$$\overline{\Phi}_1=A_1\mathrm{e}^{-\gamma_1 r}+A_2\mathrm{e}^{-\gamma_2 r} \quad (12a)$$

$$\overline{\Phi}_2=B_1\mathrm{e}^{-\gamma_1 r}+B_2\mathrm{e}^{-\gamma_2 r} \quad (12b)$$

The four constants A_1, A_2, B_1 and B_2 in Eq.(12) are linearly dependent, and can be related by using Eq.(10) to obtain:

$$B_i=\delta_iA_i \quad (i=1,2) \quad (13)$$

where, $\delta_i=\dfrac{-\alpha M^*\gamma_i^2+\rho^*s^2}{M^*\gamma_i^2-\rho^*s^2/n-b^*s}$; A_1 and A_2 are the variables that can be obtained from boundary conditions.

The Laplace transformed solutions of radial displacement $\overline{u}_r$, $\overline{w}_r$ can be obtained from Eqs.(5) and (12) as:

$$\overline{u}_r=-\frac{1}{r}\left(\gamma_1+\frac{1}{r}\right)\mathrm{e}^{-\gamma_1 r}A_1-\frac{1}{r}\left(\gamma_2+\frac{1}{r}\right)\mathrm{e}^{-\gamma_2 r}A_2 \quad (14a)$$

$$\overline{w}_r = -\frac{\delta_1}{r}\left(\gamma_1 + \frac{1}{r}\right)\mathrm{e}^{-\gamma_1 r}A_1 - \frac{\delta_2}{r}\left(\gamma_2 + \frac{1}{r}\right)\mathrm{e}^{-\gamma_2 r}A_2 \quad (14b)$$

Applying Laplace transform to Eq.(3) and using Eq.(14) yields the general solutions of pore pressure and stress expressed as:

$$\frac{\overline{p}}{G} = -\sum_{i=1}^{2}\frac{1}{r}\left(\alpha + \delta_i\right)M^*\gamma_i^2\mathrm{e}^{-\gamma_i r}A_i \quad (15a)$$

$$\frac{\overline{\sigma}_r}{G} = \sum_{i=1}^{2}\mathrm{e}^{-\gamma_i r}A_i\left[\frac{1}{r}(\lambda^* + 2)(1+\eta s)\gamma_i^2 + \frac{4}{r^2}(1+\eta s)\left(\gamma_i + \frac{1}{r}\right) + \frac{\alpha}{r}\gamma_i^2(\alpha + \delta_i)M^*\right] \quad (15b)$$

$$\frac{\overline{\sigma}_\theta}{G} = \sum_{i=1}^{2}\left[\frac{1}{r}\lambda^*(1+\eta s)\gamma_i^2 - \frac{2}{r^2}(1+\eta s)\left(\gamma_i + \frac{1}{r}\right) + \frac{\alpha}{r}\gamma_i^2(\alpha + \delta_i)M^*\right]\mathrm{e}^{-\gamma_i r}A_i \quad (15c)$$

SOLUTION OF SHELL EMBEDMENT

The general solution of the transient response of a spherical cavity in viscoelastic saturated soil has been obtained. In the following we will consider the case of a thin, elastic shell embedded in infinite viscoelastic saturated soil subjected to axisymmetric surface load and fluid pressure. The motion equation of the prescribed shell under the condition of non-torsional axisymmetric loading can be derived from Akkas and Zakout (1997):

$$2(1+\mu_l)u_r^l + \gamma_0^2\frac{\partial^2 u_r^l}{\partial t^2} = q_0(t)\frac{a^2(1-\mu_l^2)}{E_l h} \quad (16)$$

where $\gamma_0^2 = \frac{c_1^2}{c_p^2}$; $c_1 = \sqrt{\frac{\lambda + 2G}{\rho}}$ and $c_p = \sqrt{\frac{E_l}{\rho_l(1-\mu_l^2)}}$ are the dilatational wave velocity and the plate velocity, respectively; E_l, μ_l are the modulus and Possion's rate of shell, respectively; $q_0(t)$ is the net outward radial pressure.

The dynamic loads applied on the surface of shell considered herein are an axially symmetric surface load and gradually applied fluid pressure which in the Laplace transform domain can be expressed as:

$$\overline{q}(s) = \frac{q_0}{T^*}\frac{1-\mathrm{e}^{-T^* s}}{s^2}, \quad r = r_2 - h \quad (17)$$

where T^* is the non-dimensional gradually applied load time ($T^* = T\sqrt{G/\rho}/a$), T is actual time; q_0 is the maximum of the gradually applied load.

Solution corresponding to axisymmetric loading

When the shell surface is subjected to axially symmetric surface load, due to the perfect bonding, the displacement and stress components must be continuous at the kinematic interface between the spherical shell and infinite medium. For a thin shell, the thick $h/2$ can be omitted without significant error (Akkas and Zakout, 1997). So, the interface between shell and soil can be defined as $r=a$. The stress and displacement condition at the interface can be expressed as:

$$q_0(t) = q(t) - \sigma_r, \qquad r = a \quad (18a)$$

$$u_r = -u_r^l, \qquad r = a \quad (18b)$$

where, $q(t)$ is the radial stress applied at the inner surface of the shell; σ_r is the stress exerted by the soil on the shell and can be given by Eq.(15b).

Previous workers (Xu and Wu, 1998; Xu and Cai, 2001) considered two extreme flow boundary conditions: permeable and impermeable. In practical situation, the condition is frequently found in two extreme cases. By applying the concept of Li (1999), the partial permeable flow boundary condition can be written as:

$$\frac{\partial p}{\partial r} = \frac{kp}{a} \qquad \text{at } r = a \quad (18c)$$

where, $k = \frac{k_l}{k_\mathrm{d}}\frac{1}{\ln(r_2) - \ln(r_2 - h)}$ is a dimensionless permeability parameter that defines the flow capacity of the shell. This parameter depends on the relative permeability of the shell and soil as well as the geometry of the shell: when the spherical shell is impermeable (i.e. k_l=0), k approaches to zero; and when the shell is permeable (i.e. k_l=∞), k approaches to an infinite value.

Substitution of Eqs.(14a), (15b), (18a) and (18b) into Eq.(16), and Eq.(15a) into Eq.(18c) yields:

$$m_1 A_1 + m_2 A_2 = \frac{a^2(1-\mu_l^2)}{E_l^* h}\frac{\overline{q}(s)}{G} \quad (19a)$$

$$n_1 A_1 + n_2 A_2 = 0 \quad (19b)$$

where,

$$m_i = \left[2(1+\mu_l)+\gamma_0^2 s^2\right]\frac{1}{a}\left(\gamma_i+\frac{1}{a}\right)\mathrm{e}^{-\gamma_i a} + \left[\frac{1}{a}(\lambda^*+2)(1+\eta s)\gamma_i^2+\frac{4}{a^2}(1+\eta s) \times\left(\gamma_i+\frac{1}{a}\right)+\frac{\alpha}{a}\gamma_i^2(\alpha+\delta_i)M^*\right]\frac{a^2(1-\mu_l^2)}{E_l^* h}\mathrm{e}^{-\gamma_i a},$$

$$n_i = \frac{1}{a}\left(\gamma_i+\frac{k+1}{a}\right)(\delta_i+\alpha)M^*\gamma_i^2\mathrm{e}^{-\gamma_i a}, (i=1,2);$$

$$E_l^* = \frac{E_l}{G}.$$

The expressions for variables A_1 and A_2 can be obtained from Eq.(19):

$$A_1 = -\frac{n_2}{n_1}A_2 \quad (20a)$$

$$A_2 = -\frac{n_1}{n_1 m_2 - n_2 m_1}\frac{a^2(1-\mu_l^2)}{E_l^* h}\frac{\overline{q}(s)}{G} \quad (20b)$$

Solution of fluid pressure

When the shell surface is subjected to fluid pressure, the displacement and stress components must be continuous at the kinematic interface between the spherical shell and soil, and the flow boundary condition can be expressed as:

$$q_0(t) = -\sigma_r \quad \text{at } r=a \quad (21a)$$

$$u_r = -u_r^l \quad \text{at } r=a \quad (21b)$$

$$\frac{\partial p}{\partial r} = \frac{k}{a}(p+q(t)) \quad \text{at } r=a \quad (21c)$$

Substitution of Eqs.(14a), (15b), (21a) and (21b) into Eq.(16), and Eq.(15a) into Eq.(21c), yields:

$$m_1 A_1 + m_2 A_2 = 0 \quad (22a)$$

$$n_1 A_1 + n_2 A_2 = \frac{k}{a}\frac{\overline{q}(s)}{G} \quad (22b)$$

where, the expression of m_i and n_i (i=1,2) are the same as in Eq.(9).

From Eq.(22), we can obtain the expression for variables A_1 and A_2 as:

$$A_1 = -\frac{m_2}{m_1}A_2 \quad (23a)$$

$$A_2 = \frac{k}{a}\frac{m_1}{m_1 n_2 - m_2 n_1}\cdot\frac{\overline{q}(s)}{G} \quad (23b)$$

Once the variables A_1 and A_2 are determined, the specific solutions are obtained. The final solutions in time domain can be obtained by using inverse Laplace transform and numerical computation.

NUMERICAL RESULTS AND DISCUSSION

The numerical results are presented for the material and geometric parameters which are listed in Table 1. In this paper, we will discuss the influence of partial permeable property of boundary and relative rigidity of shell and soil (defined as $RR=E_l/E_s$) on the transient response of the spherical cavity. Numerical results in time domain were obtained by Durbin's numerical inversion Laplace transform. The formula suggested by Durbin (1974) is given by:

$$f(t) = \frac{2\mathrm{e}^{ct}}{T_0}\left[-\frac{1}{2}\mathrm{Re}\{F(c)\}+\sum_{n=0}^{N}\left(\mathrm{Re}\left\{F\left(c+in\frac{2\pi}{T_0}\right)\right\}\right.\right.$$

Table 1 Parameters used in computation

Quantity	Notation	Value
Elastic modulus of soil (MPa)	E_s	8
Poisson rate of soil	μ	0.4
Poisson rate of shell	μ_l	0.1
Dimensionless shell density	ρ_l^*	1.5
Dimensionless shell thickness	h/a	0.05
Dimensionless fluid density	ρ^*	0.5
Material parameter	α	0.98
Compressibility parameter	M^*	20
Permeability coefficient	b^*	10
Viscoelastic damp coefficient	η	0.1
Porosity	n	0.4
Gradually applied step load time	T^*	1

$$\times\cos\left(nt\frac{2\pi}{T_0}\right)-\mathrm{Im}\left\{F\left(c+in\frac{2\pi}{T_0}\right)\right\}\sin\left(nt\frac{2\pi}{T_0}\right)\Bigg)\Bigg] \quad (24)$$

where, cT_0=5 to 10 gives good results for N ranging from 50 to 5000. The computation value of all parameters in Eq.(24) were then taken as T_0=20, c=0.25 and N=1000 in this paper. The numerical results are shown in Fig.2 to Fig.9.

Solutions corresponding to radial load

The histories of dimensionless radial displacement at the interface of shell and soil induced by axially symmetric radial surface load are shown in Fig.2 when the parameter k=0.1. With the increase of the dimensionless time ($t^* = t\sqrt{G/\rho}/a$), radial displacement increases to maximum value, then decreases and, is noted once again. Eventually, it tends to an asymptotic value. The influence of the relative rigidity (Fig.2) of shell and soil on radial displacement is remarkable. When relative rigidity RR=0, the shell is complete flexible, there is the maximum radial displacement at the interface of shell and soil. The value of radial displacement decreases with the increase of relative rigidity. The influence of permeability parameter k on radial displacement is indicated in Fig.3. It can be seen that the influence of parameter k on radial displacement induced by axisymmetric radial surface load is not remarkable.

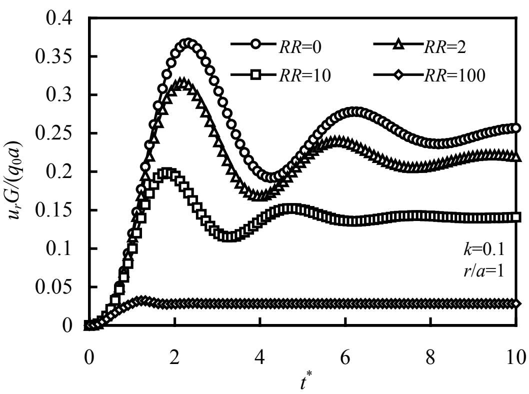

Fig.2 Curve of radial displacement vs relative rigidity

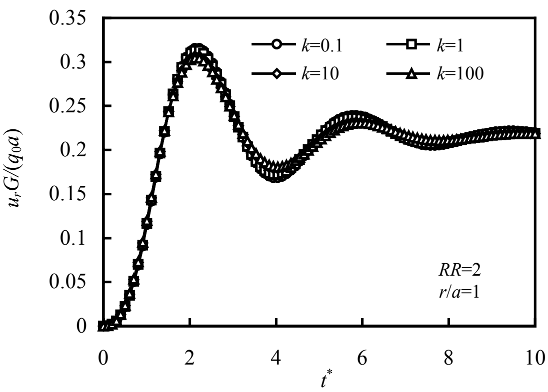

Fig.3 Curve of radial displacement vs parameter *k*

The histories of dimensionless pore pressure are shown in Fig.4 at the interface of shell and soil for the parameter k=0.1. Pore pressure is zero at t^*=0 and increases rapidly with time in the interval $0<t^*\le T^*$ and reaches to its peak value nearly at $t^*=T^*$. Thereafter, it decreases with time and reaches to its maximum suction values. With increasing time the values of suction decreases and pore pressure is noted once again. Pore pressure dissipates at t^*=10 approximately. With the increase of the values of relative rigidity, pore pressure (Fig.4) decreases at the interface. On the other hand, the pore pressure decreases with the increase of parameter k (Fig.5). As a result, both the relative rigidity and parameter k have great influence on the pore pressure under the condition of axisymmetric radial surface load.

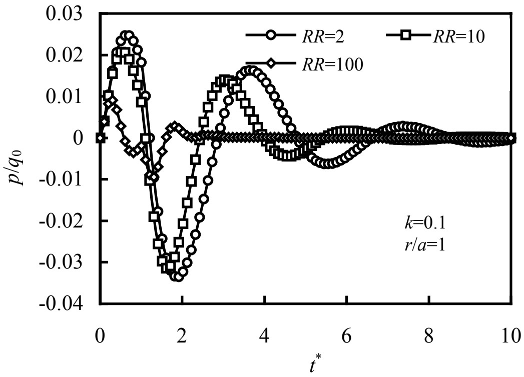

Fig.4 Curve of pore pressure vs relative rigidity

Solutions corresponding to fluid pressure

The histories of dimensionless radial displacement under fluid pressure are shown in Fig.6 when parameters k=0.1. It is noted that at a certain time instant as shown in Fig.6, there exists maximum displacement at the interface of shell and soil. With the increase of time, radial displacement decreased vibrationally and finally to an asymptotic value of zero.

Radial displacement decreased obviously with increasing relative rigidity, and increased with increasing of parameter k (Fig.7). It is worth mentioning that there were great differences among the radial displacements for different parameters k. No radial displacement occurred when the shell was completely sealed ($k\to0$), and maximum radial displacement occurred when the tunnel boundary was permeable ($k\to\infty$). Comparison of Fig.6 or Fig.7 with Fig.2 or Fig.3 shows obvious dimensionless radial displacement induced by axisymmetric radial load.

The excess pore pressures induced by fluid pressure are shown in Figs.8 and 9. The histories of pore pressure shown in Fig.8 for k=10 indicate that the influence of relative rigidity can be ignored. However, the influence of parameter k (Fig.9) is significant. When the shell boundary became almost impermeable ($k\to0$), almost no excess pore pressure existed, whereas with the increasing of time, the excess pore pressure at the interface equaled the fluid pressure ($P=-q_0$) when the shell boundary became almost permeable ($k\to\infty$).

Therefore, it is important to consider realistic values of k in order to accurately predict the radial displacement and pore pressure in the transient response of spherical cavity with a shell embedded in viscoelastic saturated soil subjected to fluid pressure.

CONCLUSION

By introducing the partial sealing boundary condition, analytical solutions were derived in Laplace transform domain for the transient response of a sph-

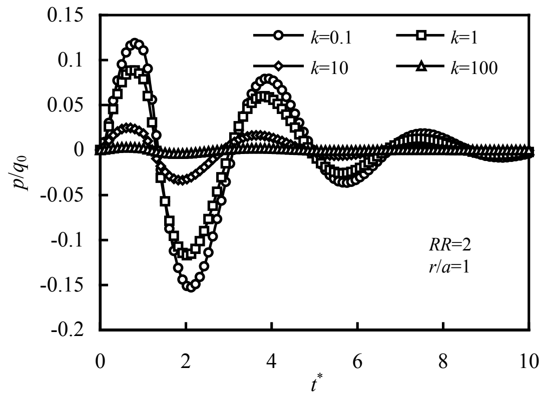

Fig.5 Curve of pore pressure vs parameter k

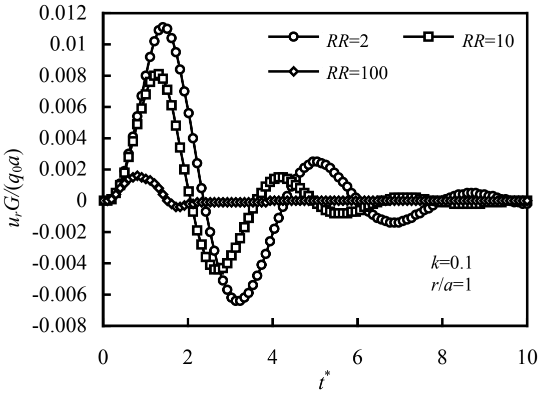

Fig.6 Curve of radial displacement vs relative rigidity (fluid pressure)

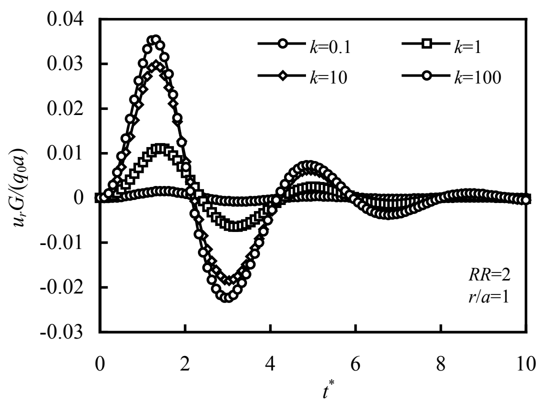

Fig.7 Curve of radial displacement vs parameter k (fluid pressure)

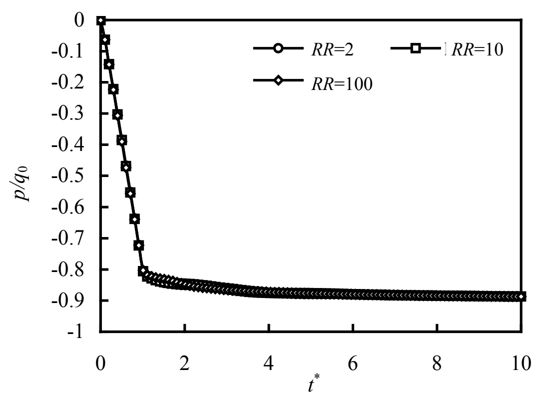

Fig.8 Curve of pore pressure vs relative rigidity (fluid pressure)

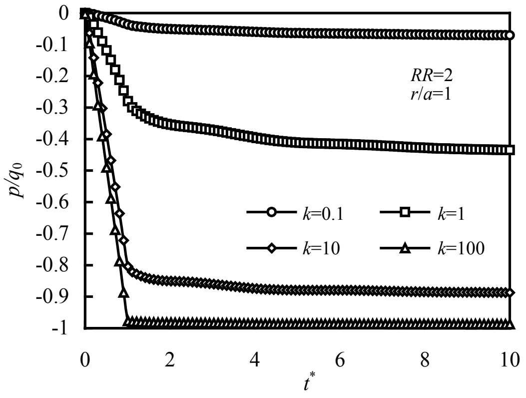

Fig.9 Curve of pore pressure vs parameter *k* (fluid pressure)

erical cavity with a shell embedded in viscoelastic saturated soil subjected to axially symmetric radial load and fluid pressure. Numerical results in time domain were obtained by Durbin's inverse Laplace transform. An extensive parameters study conducted to investigate the influence of the relative rigidity of shell and soil and permeability parameter *k*, showed that it is dependent on the relative permeability of the liner and soil as well as the geometry of the liner. It was found that both relative rigidity and parameter *k* have great influence on the transient response of spherical cavity with a shell embedded in viscoelastic saturated soil. The available solutions under permeable and impermeable boundary conditions are only two extreme cases of the solutions developed in this paper. Therefore, it is of significance to consider the partially sealed boundary condition and the relative rigidity of shell and soil in the designing and computation of spherical shell in viscoelastic saturated medium.

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