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Torsional oscillations of a rigid disc bonded to multilayered poroelastic medium*

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Abstract: This paper deals mainly with the dynamic response of a rigid disc bonded to the surface of a layered poroelastic half-space. The disc is subjected to time-harmonic torsional moment loadings. The half space under consideration consists of a number of layers with different thickness and material properties. Hankel transform techniques and transferring matrix method are used to solve the governing equations. The continuity of the displacement and stress fields between different layers enabled derivation of closed-form solutions in the transform domain. On the assumption that the contact between the disc and the half space is perfectly bonded, this dynamic mixed boundary-value problem can be reduced to dual integral equations, which are further reduced to Fredholm integral equations of the second kind and solved by numerical procedures. Selected numerical results for the dynamic impedance and displacement amplitude of the disc resting on different saturated models are presented to show the influence of the material and geometrical properties of both the saturated soil-foundation system and the nature of the load acting on it. The conclusions obtained can serve as guidelines for practical engineering.

Key words: Layered saturated elastic medium, Rigid disc, Torsional oscillations, Transferring matrix method, Dynamic response
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INTRODUCTION

The dynamic response of foundations resting on the surface of a half space is of significant importance in soil-foundation interactions and machine foundation designs. Observation of earthquake damage indicated that the local soil properties, foundation geometries, etc., play a central role in the dynamic behavior of the soil-foundation system.

Ever-increasing research has been devoted to the problem of an elastic half space indented by a rigid footing by employing a variety of analytical and numerical techniques. Tsai (1989) studied the torsional vibrations of a circular disk on an infinite transversely isotropic medium. Luco and Westmann (1972) studied the vertical, horizontal and rocking motions of a rigid strip footing bonded to an elastic half-space using Green's functions. Gladwell (1968) considered the forced tangential and rotatory vibrations of a rigid circular disc on a semi-infinite solid.

Generally, geomaterials are two-phased materials consisting of a solid skeleton with voids filled with water. Such materials are commonly known as saturated ones and considered as a much more realistic representation of natural soils and rocks than ideal elastic ones. Since Biot (1956) established the first theory of wave propagation in a fluid-filled, poroelastic solid, many researchers used this theory and restudied the dynamic response of foundations on saturated media. Kassir and Xu (1988) considered the rigid strip bonded to a homogeneous poroelastic half space. Halpern and Christiano (1986) and Philippacopoulos (1989) studied the time-harmonic response of a rigid plate in smooth contact with saturated and partially saturated poroelastic half space,

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respectively. Dargush and Chopra (1996) and Japon *et al.*(1997) used boundary element method and examined the vibration of a rigid circular foundation and a rigid strip foundation on a poroelastic medium, respectively. Bo and Hua (1999) and Wang (2002) considered the vertical and torsional dynamic response of a foundation on a poroelastic saturated half-space.

During the sedimentation process, many naturally occurring soil profiles are created and tend to be horizontally layered, each layer comprising a different soil type with its own distinct properties. Using this stratified model, Gucunski and Peek (1993) investigated the vertical vibrations of circular flexible foundations on layered elastic half space by means of ring method. Rajapakse and Senjuntichai (1995) used Fourier transforms to study the dynamic response of a multi-layered poroelastic medium subjected to time-harmonic loads and fluid sources applied in the interior of the layered medium.

The main objective of this paper was to consider both stratification and coupling of the solid-fluid phase of the saturated half space for the first time. This layered saturated elastic model was used to study the torsional oscillations of a rigid disc resting on it by means of Hankel transforms and transferring matrix method. The analytical solutions of governing equations of a single saturated elastic layer were first obtained. Then, based on the boundary-value and continuity conditions of the displacement and stress fields between neighboring layers, dual integral equations were established, further reduced to Fredholm integral equations of the second kind and then solved by numerical procedures. Finally, as an illustration, selected examples of the model consisting of two saturated layers are given at the end of the paper. The numerical results indicated that the material and geometrical properties of both saturated layers and rigid disc and dynamic loading have important influence on the dynamic response. The results obtained can serve as guidelines for practical engineering.

FORMULATION OF THE PROBLEM

A study was made of the forced steady state torsional vibrations of a rigid circular footing of radius r_0 placed on the surface of a layered poroelastic medium consisting of $n-1$ layers with different depth and material properties resting on the half-space. Both the layers and the half-space were assumed to be homogeneous, isotropic and saturated elastic ones. The geometry of the model and coordinate system used are shown in Fig.1.

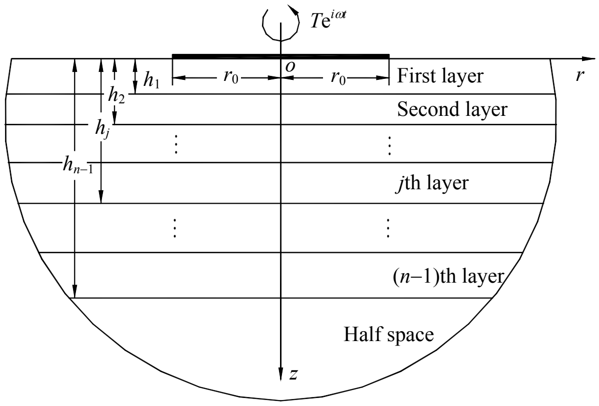


Fig.1 Description and coordinate system of saturated layered half-space

Following Biot (1956)'s theory of a two-phased material, the governing equations for a homogeneous poroelastic material can be expressed as

$$\mu\Delta\boldsymbol{U} + (\lambda_c + \mu)grad e - \alpha M grad\xi = \rho\ddot{\boldsymbol{U}} + \rho_f\ddot{\boldsymbol{W}} \quad (1)$$

$$\alpha M grad e - M grad\xi = \rho_f\ddot{\boldsymbol{U}} + m\ddot{\boldsymbol{W}} + b\dot{\boldsymbol{W}} \quad (2)$$

In the above equations, $\boldsymbol{U}$ and $\boldsymbol{W}$ are the displacement vectors of the solid and fluid relative to the solid, respectively; λ and μ are the Lame's constants; $\lambda_c=\lambda+\alpha^2M$, where α and M are the Biot's parameters accounting for compressibility in the two-phased material; $\rho=\beta\rho_f+(1-\beta)\rho_s$, where β is the porosity coefficient, ρ_f and ρ_s are the mass density of the fluid and the solid particle, respectively; e=div$\boldsymbol{U}$ and ξ=−div$\boldsymbol{W}$ are the dilatations of the solid and fluid relative to the solid; respectively; $m=\rho_f/\beta$ is a density-like parameter; $b=\eta/k$, where η is the pore fluid viscosity and k the permeability. Overdots denote the derivatives of field variables with respect to time t.

Due to the symmetry of the problem, the motion is independent of angle θ, and the only non-vanishing components of the displacement vectors are the solid tangential displacement $u_\theta(r,z)e^{i\omega t}$ and the tangential displacement of fluid relative to the solid $w_\theta(r,z)e^{i\omega t}$, where ω is the angular frequency and $i=\sqrt{-1}$. So Eqs.(1) and (2) can be further written as

$$\mu\left(\nabla^2-\frac{1}{r^2}\right)u_\theta=-(\rho\omega^2u_\theta+\rho_f\omega^2w_\theta) \tag{3}$$

$$\rho_f\omega u_\theta+m\omega w_\theta=ibw_\theta \tag{4}$$

The only non-vanishing bulk stress components of the soil are

$$\tau_{z\theta}=\mu\frac{\partial u_\theta}{\partial z} \tag{5}$$

$$\tau_{r\theta}=\mu\left(\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}\right) \tag{6}$$

It is convenient to introduce the dimensionless constants and variables: $\bar{r}=r/r_0$, $\bar{z}=z/r_0$, $\bar{u}_\theta=u_\theta/r_0$, $\bar{w}_\theta=w_\theta/r_0$, $\bar{\rho}_f=\rho_f/\rho$, $\bar{m}=m/\rho$, $a=\sqrt{\frac{\rho}{\mu}}r_0\omega$, $\bar{b}=\frac{r_0}{\sqrt{\rho\mu}}b$, $\bar{\tau}_{z\theta}=\tau_{z\theta}/\mu$, $\bar{\tau}_{r\theta}=\tau_{r\theta}/\mu$.

After introducing the dimensionless variables, Eqs.(3)–(6) can be written as

$$\left(\nabla^2-\frac{1}{\bar{r}^2}\right)\bar{u}_\theta=-(a^2\bar{u}_\theta+\bar{\rho}_fa^2\bar{w}_\theta) \tag{7}$$

$$\bar{\rho}_fa\bar{u}_\theta+\bar{m}a\bar{w}_\theta=i\bar{b}\bar{w}_\theta \tag{8}$$

$$\bar{\tau}_{z\theta}=\frac{\partial\bar{u}_\theta}{\partial\bar{z}} \tag{9}$$

$$\bar{\tau}_{r\theta}=\frac{\partial\bar{u}_\theta}{\partial\bar{r}}-\frac{\bar{u}_\theta}{r} \tag{10}$$

The Hankel integral transform of order υ of $\bar{f}(r,z)$ with respect to radial coordinate is defined by (Tranter, 1959)

$$\bar{\bar{f}}^\upsilon(p,z)=\int_0^\infty r\bar{f}(r,z)J_\upsilon(pr)\mathrm{d}r \tag{11}$$

and its inverse relationship is given by

$$\bar{f}(r,z)=\int_0^\infty p\bar{\bar{f}}^\upsilon(p,z)J_\upsilon(pr)\mathrm{d}p \tag{12}$$

where $J_\upsilon(pr)$ denotes the Bessel function of the first kind of order υ and p is the Hankel transform parameter.

Use of Hankel transform of the first order on Eqs.(7)–(9) yields

$$-p^2\bar{\bar{u}}_\theta^1+\frac{\mathrm{d}^2\bar{\bar{u}}_\theta^1}{\mathrm{d}\bar{z}^2}=-a^2\bar{\bar{u}}_\theta^1-\bar{\rho}_fa^2\bar{\bar{w}}_\theta^1 \tag{13}$$

$$\bar{\rho}_fa\bar{\bar{u}}_\theta^1+\bar{m}a\bar{\bar{w}}_\theta^1=i\bar{b}\bar{\bar{w}}_\theta^1 \tag{14}$$

$$\bar{\bar{\tau}}_{z\theta}^1=\frac{\mathrm{d}}{\mathrm{d}\bar{z}}\bar{\bar{u}}_\theta^1 \tag{15}$$

Considering that there exists both incoming and out-coming waves in the layered system, the solutions of Eqs.(13) and (14) can be obtained directly as

$$\bar{\bar{u}}_\theta^1=A_1\sinh j\bar{z}+A_2\cosh j\bar{z} \tag{16}$$

$$\bar{\bar{w}}_\theta^1=\frac{\bar{\rho}_fa}{i\bar{b}-\bar{m}a}(A_1\sinh j\bar{z}+A_2\cosh j\bar{z}) \tag{17}$$

in which $j^2=p^2-s^2$, $s^2=\frac{i\bar{b}a^2+\bar{\rho}_f^2a^3-\bar{m}a^3}{i\bar{b}-\bar{m}a}$, s is the dimensionless complex wave number associated with the rotational wave. A_1 and A_2 are arbitrary functions of p.

Substitution of Eq.(16) into Eq.(15) yields

$$\bar{\bar{\tau}}_{z\theta}^1=A_1j\cosh j\bar{z}+A_2j\sinh j\bar{z} \tag{18}$$

Letting $\bar{z}=\bar{z}_0$ in Eqs.(16) and (18) yields

$$\bar{\bar{u}}_\theta^1(p,\bar{z}_0)=A_1\sinh j\bar{z}_0+A_2\cosh j\bar{z}_0 \tag{19}$$

$$\bar{\bar{\tau}}_{z\theta}^1(p,\bar{z}_0)=A_2j\sinh j\bar{z}_0+A_1j\cosh j\bar{z}_0 \tag{20}$$

From which A_1 and A_2 can be obtained easily as

$$A_1=\frac{1}{j}\cosh j\bar{z}_0\bar{\bar{\tau}}_{z\theta}^1(p,\bar{z}_0)-\sinh j\bar{z}_0\bar{\bar{u}}_\theta^1(p,\bar{z}_0) \tag{21}$$

$$A_2=\cosh j\bar{z}_0\bar{\bar{u}}_\theta^1(p,\bar{z}_0)-\frac{1}{j}\sinh j\bar{z}_0\bar{\bar{\tau}}_{z\theta}^1(p,\bar{z}_0) \tag{22}$$

Substituting Eqs.(21) and (22) back into Eqs.(16) and (18) and rearranging it in matrix form, we can get

$$\bar{\boldsymbol{B}}(p,\bar{z})=\boldsymbol{\phi}(p,\bar{z},\bar{z}_0)\bar{\boldsymbol{B}}(p,\bar{z}_0) \tag{23}$$

where $\overline{\boldsymbol{B}}(p,\bar{z})=[\bar{\bar{u}}_\theta^1(p,\bar{z}),\bar{\bar{\tau}}_{z\theta}^1(p,\bar{z})]^{\mathrm{T}}$, $\boldsymbol{\phi}(p,\bar{z},\bar{z}_0)$ is a 2×2 matrix with elements $\phi_{11}=\cosh j(\bar{z}-\bar{z}_0)$, $\phi_{12}=\frac{1}{j}\sinh j(\bar{z}-\bar{z}_0)$, $\phi_{21}=j\sinh j(\bar{z}-\bar{z}_0)$, $\phi_{22}=\phi_{11}$.

When the medium is a homogeneous isotropic saturated half space, there exist only out-coming waves. Hence the solutions of Eqs.(16) and (18) are

$$\overline{\boldsymbol{B}}(p,\bar{z})=\boldsymbol{\psi}(p,\bar{z})[C_1] \tag{24}$$

in which $C_1=(A_1-A_2)/2$, $\boldsymbol{\psi}(p,\bar{z})$ is a 2×1 matrix with elements $\psi_{11}=-\mathrm{e}^{-j\bar{z}}$, $\psi_{21}=j\mathrm{e}^{-j\bar{z}}$.

Applying Eq.(23) to the saturated layers and Eq.(24) to the half space yields the following recurrence relations:

$$\begin{aligned}&\overline{\boldsymbol{B}}(p,0^+)=\boldsymbol{\phi}(p,0,\bar{h}_1)\overline{\boldsymbol{B}}(p,\bar{h}_1^-)\\&\overline{\boldsymbol{B}}(p,\bar{h}_1^+)=\boldsymbol{\phi}(p,\bar{h}_1,\bar{h}_2)\overline{\boldsymbol{B}}(p,\bar{h}_2^-)\\&\qquad\cdots\qquad\cdots\\&\overline{\boldsymbol{B}}(p,\bar{h}_{n-2}^+)=\boldsymbol{\phi}(p,\bar{h}_{n-2},\bar{h}_{n-1})\overline{\boldsymbol{B}}(p,\bar{h}_{n-1}^-)\\&\overline{\boldsymbol{B}}(p,\bar{h}_{n-1}^+)=\boldsymbol{\psi}(p,\bar{h}_{n-1})\overline{\boldsymbol{B}}(p,\bar{h}_{n-1}^-)\end{aligned} \tag{25}$$

in which $\bar{h}_j=h_j/r_0$, h_j is the depth from the free surface of the model to the lower surface of the jth layer; $\overline{\boldsymbol{B}}(p,\bar{h}_j^-)$ denotes the field variables at the lower face of the jth layer, while $\overline{\boldsymbol{B}}(p,\bar{h}_j^+)$ denotes the field variables at the upper face of the (j+1)th layer.

Considering the continuity of stresses and displacements between neighboring layers, we have

$$\overline{\boldsymbol{B}}(p,\bar{h}_j^-)=\boldsymbol{H}_j\overline{\boldsymbol{B}}(p,\bar{h}_j^+)\quad(j=1,2,\cdots,n-1) \tag{26}$$

where $\boldsymbol{H}_j=\operatorname{diag}\left\{1,\frac{\mu_{j+1}}{\mu_j}\right\}$.

Combining Eqs.(25) and (26) yields

$$\overline{\boldsymbol{B}}(p,0)=\left[f_{ij}\right]_{2\times1}[C_1] \tag{27}$$

where $\left[f_{ij}\right]_{2\times1}=\boldsymbol{\phi}(p,0,\bar{h}_1)\boldsymbol{H}_1\boldsymbol{\phi}(p,\bar{h}_1,\bar{h}_2)\boldsymbol{H}_2\cdots\boldsymbol{\phi}(p,\bar{h}_{n-2},\bar{h}_{n-1})\boldsymbol{H}_{n-1}\boldsymbol{\psi}(p,\bar{h}_{n-1})$.

From which the tangential displacement and the shear stress in the integral transform domain can be expressed as

$$\bar{\bar{u}}_\theta^1(p,0)=f_{11}C_1 \tag{28}$$

$$\bar{\bar{\tau}}_{z\theta}^1(p,0)=f_{21}C_1 \tag{29}$$

THE MIXED BOUNDARY VALUE PROBLEMS OF LAYERED SATURATED MEDIUM

The torsional vibrations of a rigid disc resting on the layered saturated half space belong to the class of mixed boundary-value problems. Here we assume that the contact between the foundation and the underlying layer is perfectly bonded, i.e., there is no relative displacement between the interface of the soil-foundation system and that the stress outside of the interface is zero. Hence, the dimensionless boundary-value conditions of the layered problem can be stated as follows:

$$\bar{u}_\theta(\bar{r},0)=\bar{r}\vartheta\quad(0\le\bar{r}\le1) \tag{30}$$

$$\bar{\tau}_{z\theta}(\bar{r},0)=0\quad(\bar{r}>1) \tag{31}$$

here ϑ is the amplitude of the angular displacement of the rigid disc.

Combining Eq.(12) and Eqs.(28)–(31) yields the following dual integral equation

$$\int_0^\infty p^{-1}(1+Q(p))R(p)J_1(p\bar{r})\mathrm{d}p=\bar{r}\vartheta\quad(0\le\bar{r}\le1) \tag{32}$$

$$\int_0^\infty R(p)J_1(p\bar{r})\mathrm{d}p=0\quad(\bar{r}>1) \tag{33}$$

where $Q(p)=\frac{pf_{11}}{f_{21}}-1$, $R(p)=pf_{21}C_1$.

Noble (1963) considered the solutions of the above dual integral equations in great detail. Based on the methods advanced by him, let

$$R(p)=\frac{4}{\pi}p\vartheta\int_0^1\theta(t)\sin(pt)\mathrm{d}t \tag{34}$$

Substitution of Eq.(34) into Eqs.(32) and (33), Eq.(33) is automatically satisfied and Eq.(32) can be reduced to the following Fredholm integral equation of the second kind

$$\theta(t)+\frac{2}{\pi}\int_0^1\theta(\tau)K(t,\tau)\mathrm{d}\tau=t \tag{35}$$

where the kernel

$$K(t,\tau)=\int_0^\infty Q(p)\sin(pt)\sin(p\tau)\mathrm{d}p \tag{36}$$

Suppose a torque $Te^{i\omega t}$ is applied on the disc. According to the dynamic equilibrium of the disc, it has

$$\overline{T}=-2\pi\int_0^1\overline{r}^2\overline{\tau}_{z\theta}(\overline{r},0)\mathrm{d}\overline{r} \tag{37}$$

where $\overline{T}=\dfrac{T}{\mu_I r_0^3}$, μ_1 is the shear modulus of the first soil layer.

After taking Eq.(29)'s inverse Hankel transform of the first order and considering Eq.(34), Eq.(37) can be further written as

$$\overline{T}=\frac{16\phi}{3}\cdot 3\int_0^1 t\theta(t)\mathrm{d}t \tag{38}$$

So the dimensionless torsional dynamic compliance coefficient of the rigid disc on the layered saturated media is

$$C_T=\frac{1}{3\int_0^1 t\theta(t)\mathrm{d}t} \tag{39}$$

and the equivalent dimensional torsional stiffness and damping of the disc can be expressed as

$$k_T=\mu_1 r_0^3\frac{f_1}{f_1^2+f_2^2} \tag{40}$$

$$\eta_T=-\sqrt{\rho_1\mu_1}r_0^4\frac{f_2}{f_1^2+f_2^2} \tag{41}$$

where $f_1=\dfrac{3}{16}\mathrm{Re}[C_T]$, $f_2=\dfrac{3}{16}\mathrm{Im}[C_T]$; ρ_1 and μ_1 are the mass density and shear modulus of the first layer.

So the tangential displacement amplitude of the rigid disc resting on layered saturated half-space can be written as

$$A_T=\overline{T}\sqrt{\frac{f_1^2+f_2^2}{(1-\overline{I}_\theta a^2 f_1)^2+(\overline{I}_\theta a^2 f_2)^2}} \tag{42}$$

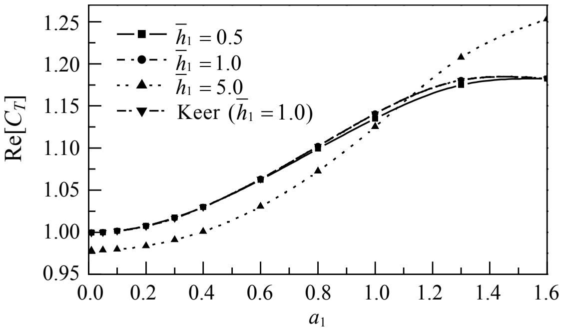
where $\overline{I}_\theta=\dfrac{I_\theta}{\rho_1 r_0^5}$, I_θ is the mass inertia moment of the disc.

NUMERICAL EXAMPLES

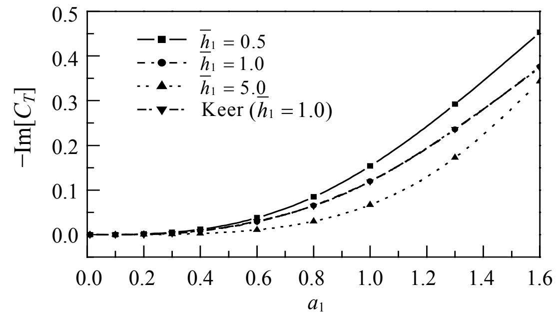
The Fredholm integral equation of the second kind derived above can be solved numerically by discretizing it to a system of algebraic equations in the interval [0,1] using trapezoidal rule of integration with N subintervals. The key for solving it is to calculate the kernel $K(t,\tau)$. Due to the presence of internal friction between solid and fluid (i.e. $b\neq0$), the branch points and poles of the integrand in the kernel appearing in Eq.(36) are complex-valued quantities with negative imaginary parts. Therefore, the real p axis is free from any singularities and the kernel can be evaluated by direct numerical integration along the real p axis. For poroelastic materials with $b\to0$ and dry elastic materials, 1% attenuation (i.e. complex shear modulus μ) is incorporated in the analysis to ensure that the real p axis is free from any singularities. In this paper, the numerical evaluation of the kernel is performed using Filon's methods (Watson, 1958).

After getting the solution of Eq.(35), we can use Eqs.(39) and (42) to get the compliance functions and tangential displacement amplitude of the disc resting on the layered saturated elastic half-space.

The above problem can be reduced to the torsional vibrations of a rigid disc resting on layered, elastic half-space if we let ρ_f=0. The degenerated results for layered, elastic half-space are compared with those of Keer *et al*.(1974). Fig.2 shows the comparison between the real and imaginary parts of the torsional compliance coefficients vs the dimensionless vibration frequency a_1 of the first layer for μ_1/μ_2=2.0 and h_1/r_0=0.5, 1.0 and 5.0, respectively. From the comparison, it can be seen that the two results agree very well at all points.



(a)



(b)

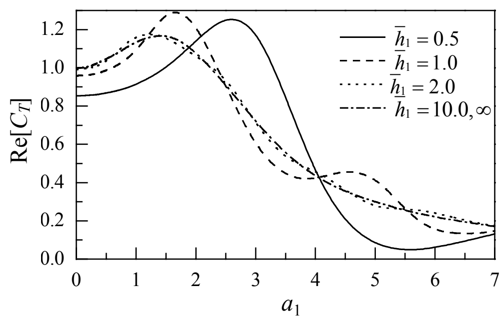
Fig.2 Comparison of compliance coefficients of rigid disc on layered elastic half space
(a) Re[C_T] vs a_1; (b) Im[C_T] vs a_1

In the remainder of the paper, selected numerical examples of models consisting of a saturated layer resting on saturated half-space are given to examine the effect of geometrical and physical properties of the soil-foundation system and dynamic loading on the response of the disc. The following parameters are used throughout: $\bar{h}_1=1.0$, μ_1/μ_2=0.2, ρ_1/ρ_2=0.8, $\bar{\rho}_{f1}/\bar{\rho}_{f2}=1.25$, $\bar{b}_1=10.0$, $\bar{I}_\theta=2.0$ and 10.0 respectively, unless otherwise stated.

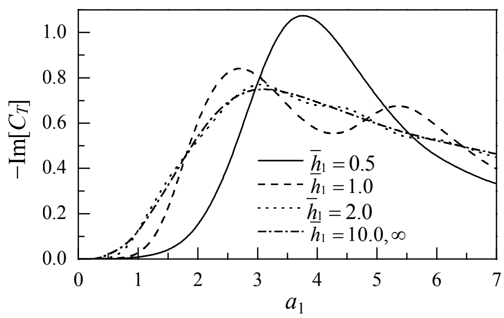
Effect of layer depth

In order to explore the influence of layer depth on the impedance functions and on the angular displacement, five values of $\bar{h}_1$ were considered, namely, $\bar{h}_1$=0.5, 1.0, 2.0, 10.0 and ∞. $\bar{h}_1$=∞ indicates that the model tends to be isotropic saturated half space. The numerical results are shown in Figs.3 and 4.

From Figs.3a and 3b, it can be seen that the depth of the soil layer significantly affects the response of the disc. When $\bar{h}_1\le 2.0$, with the increase of the vibration frequency, the curves of the real and imaginary part of C_T oscillate heavily. But when the depth of the upper layer increases, the curves tend to be stable, especially when $\bar{h}_1\ge 10.0$, the dynamic compliance coefficient is very close to that of isotropic, homogeneous saturated half space (Wang, 2002).



(a)



(b)

Fig.3 Effect of layer depth on the compliance of the disc
(a) Re[C_T] vs a_1; (b) Im[C_T] vs a_1

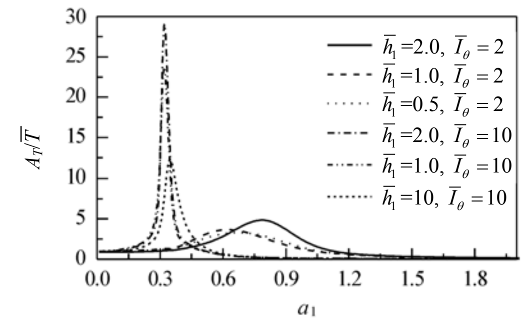


Fig.4 Effect of layer depth on the tangential displacement amplitude

The results shown in Fig.3 indicate that the curves of the dynamic compliance of the foundation resting on the layered saturated medium oscillate more heavily than those of the isotropic, homogeneous one. This is due to the shear waves being reflected back and forth many times between the half-space and the free layer, modifying the response of the disc. When $\bar{h}_1$ increases gradually, the energy of the waves in the upper layer is absorbed gradually. So the energy that reflects and refracts at the interface is smaller and the curves tend to those of the saturated half space.

Fig.4 of the tangential displacement amplitude of the disc vs the vibrating frequency shows that the depth of the upper layer significantly affects the tangential displacement amplitude of the disc. When $\bar{h}_1$ is smaller, the peak value of the curves is smaller. This phenomenon is more pronounced, especially when $\bar{I}_\theta$ is larger. It indicates that the existence of the upper layer can weaken the vibration of the foundation, especially when the depth is relatively smaller and $\bar{I}_\theta$ is larger.

Effect of μ_1/μ_2

The layering characteristics of soil also have important influence on the torsional compliance and displacement. In this section, four values of μ_1/μ_2(=50.0, 10.0, 1.0 and 0.1) are used to investigate the effect of different shear modulus on the dynamic response of a foundation resting on layered saturated medium and the results are presented in Figs.5 and 6.

As expected, the effect of layering is noticeable as μ_1/μ_2 changes. Fig.5 shows that the dynamic response of the disc is different for different values of μ_1/μ_2 and somewhat oscillatory; because with the decreasing of the relative stiffness between the top and bottom layer (decreasing in μ_1/μ_2), less and less waves are reflected from the interface and so the curves tend to be stable; especially when μ_1/μ_2=1.0, the curves are almost very smooth and agree very well with those in isotropic, saturated half-space.

From Fig.5a and 5b, it is hard to determine whether this tendency strengthens or weakens the total response of the foundation. But Fig.6 indicates that the peak values of the tangential displacement amplitude increase with the decrease of μ_1/μ_2. This phenomenon is more pronounced when $\bar{I}_\theta$ is larger.

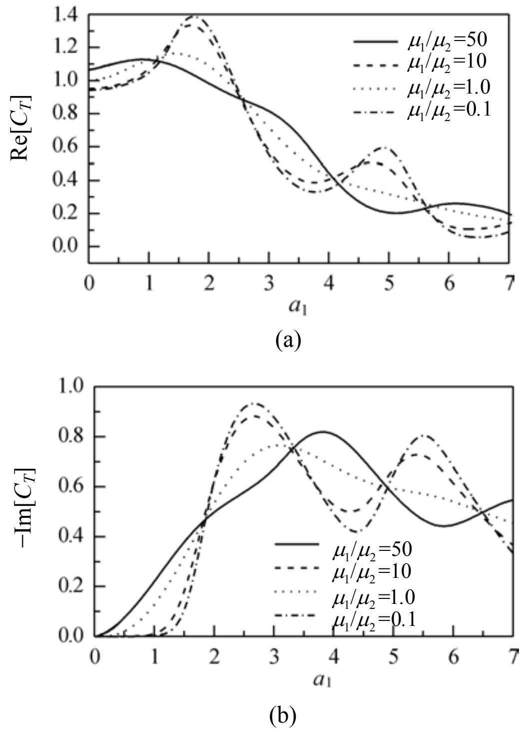


Fig.5 Effect of μ_1/μ_2 on the compliance coefficient
(a) Re[C_T] vs a_1; (b) Im[C_T] vs a_1

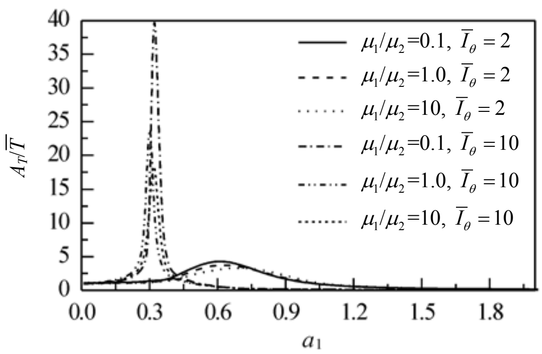


Fig.6 Effect of μ_1/μ_2 on the tangential displacement amplitude

Effect of pore fluid

Finally, in order to demonstrate the effect of pore fluid on the dynamic response of the foundation, we calculate the dynamic response of the soil-foundation system using $\bar{h}_1=1.0$, $\mu_1/\mu_2=1.0$, $\bar{b}_1/\bar{b}_2$=0.001 and 1.0, respectively. The bigger the value of $\bar{b}_1/\bar{b}_2$ is, the smaller the relative permeability and the larger the relative viscosity between the upper layer and the underlying half space are. If there is no fluid in the model, the half space will become a dry one, which had been considered by Luco and Westmann (1971). The results are presented in Figs.7 and 8.

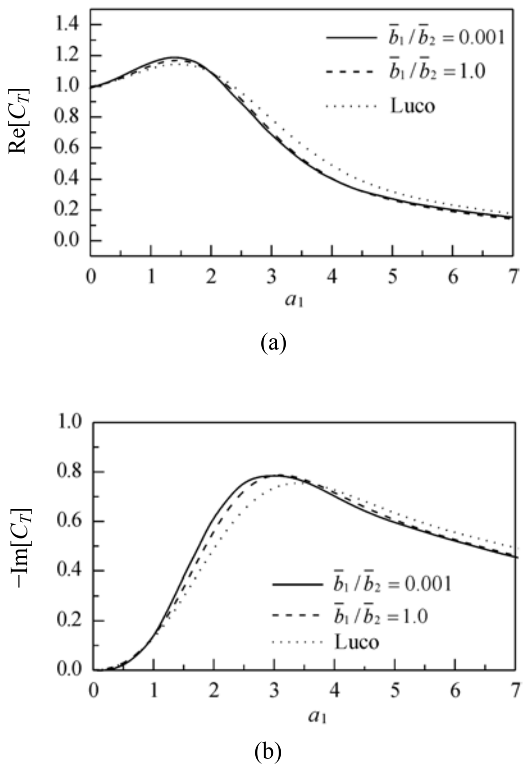


Fig.7 Effect of pore fluid on the compliance coefficient
(a) Re[C_T] vs a_1; (b) Im[C_T] vs a_1

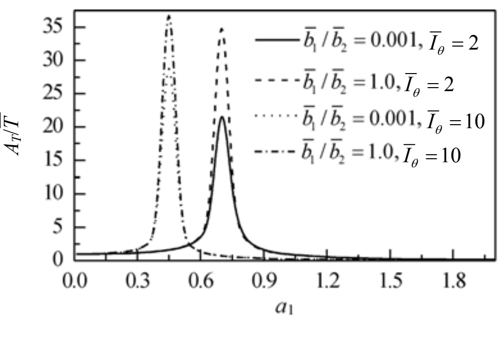


Fig.8 Effect of pore fluid on the tangential displacement amplitude

Fig.7 shows that $\bar{b}_1/\bar{b}_2$ has significant influence on the response of the disc and tends to reduce the real part and imaginary one of C_T (compared to the dry case). Furthermore, these effects are more pronounced at higher dimensionless frequencies a_1. The total influence can be seen clearly from Fig.8 that the tangential displacement amplitude of the disc increases for larger value of $\bar{b}_1/\bar{b}_2$. Such differences can be attributed to the inter-granular energy losses in the solid phase and the viscous resistance to the flow of the pore fluid.

SUMMARY AND CONCLUSION

In this work, the torsional oscillations of a rigid disc resting on the surface of a saturated elastic layered half-space were studied analytically using the transferring matrix and integral transform approaches. This complicated dynamic contact problem is solved by using Fredholm integral equation of the second kind which is easy to calculate numerically. The selected numerical examples presented indicate that the response of the rigid disc depends on several parameters, such as depth of the layers, the geometrical properties of the soil-foundation system, the load distribution and soil stratification, and in most cases, significantly differs from that of the rigid disc bearing on isotropic, homogeneous saturated half-space. The results obtained can provide some guidelines for engineering practices.

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