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Active suspension control of a one-wheel car model using single input rule modules fuzzy reasoning and a disturbance observer

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Abstract: This paper presents the construction of an active suspension control of a one-wheel car model using fuzzy reasoning and a disturbance observer. The one-wheel car model to be treated here can be approximately described as a nonlinear two degrees of freedom system subject to excitation from a road profile. The active control is designed as the fuzzy control inferred by using single input rule modules fuzzy reasoning, and the active control force is released by actuating a pneumatic actuator. The excitation from the road profile is estimated by using a disturbance observer, and the estimate is denoted as one of the variables in the precondition part of the fuzzy control rules. A compensator is inserted to counter the performance degradation due to the delay of the pneumatic actuator. The experimental result indicates that the proposed active suspension system improves much the vibration suppression of the car model.

Key words: One-wheel car model, Active suspension system, Single input rule modules fuzzy reasoning, Pneumatic actuator, Disturbance observer
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INTRODUCTION

Research and development of active suspension systems for car models are increasing much in recent years because the active suspension systems offer good riding comfort to passengers of high speed ground transportation. Investigation of linear active suspension systems has been mainly developed based on optimal control theory assuming that the car model is described by a linear or approximately linear system (Hác, 1986; Hrovat, 1993; Yoshimura and Sugimoto, 1990). However, as the car models are practically denoted as a complicated model with no negligibly strong non-linearity and uncertainty, nonlinear and intelligent active suspension systems, in place of the linear active suspension systems, have been proposed using fuzzy reasoning (Lin *et al*., 1993; Roukieh and Titli, 1993; Yeh and Tsao, 1994; Yoshimura *et al*., 2000; 2003; Yoshimura and Takagi, 2004), neural network (Moran and Nagai, 1994), sliding mode control theory (Kurimoto and Yoshimura, 1998; Yoshimura *et al*., 2001). Numerical and experimental results already reported showed that such active suspension systems give more satisfactory performance, but need more increasing inference load to achieve the active control, compared with the linear active suspension systems.

This paper proposes to establish a pneumatic active suspension system for a one-wheel car model approximately described as a nonlinear two degrees of freedom system subject to excitation from a road profile using fuzzy reasoning and a disturbance observer. The paper to be presented here is an extended version of the papers (Yoshimura *et al*., 2003; Yoshimura and Takagi, 2004) dealing with the active suspension systems of one-wheel car models. The active control for the suspension system is inferred by single input rule modules (SIRMs) fuzzy reasoning (Yoshimura *et al*., 2000; 2003). The excitation from the road profile is estimated by using a disturbance observer, and the estimate is denoted as one of the variables in the precondition part of the fuzzy control rules. The performance degradation in the vibration suppression of the car model due to the delay of the pneumatic actuator is improved much by inserting a compensator after the sensor to measure the acceleration of the car body.

ONE-WHEEL CAR MODEL

The experimental apparatus of a one-wheel car model vertically confined by two polls, due to the pneumatic active control, is shown in Fig.1 where it is slightly different from the experimental apparatus denoted in the paper (Yoshimura and Takagi, 2004). The masses of the car body and wheel parts are respectively denoted as m_1 and m_2 with the displacements respectively expressed as z_1 and z_2. The restoring force of the suspension part is practically assumed to be nonlinear and is denoted as $f(z_1-z_2)$. It consists of two coil springs with stiffness k_1 or four coil springs with stiffness k_1+k_1', depending on the suspension deflection z_1-z_2 as

$$f(z_1-z_2)=\begin{cases}(k_1+k_1')(z_1-z_2)-ak_1' & \text{for } z_1-z_2>a\\ k_1(z_1-z_2) & \text{for } |z_1-z_2|\le a\\ (k_1+k_1')(z_1-z_2)+ak_1' & \text{for } z_1-z_2<-a\end{cases} \quad (1)$$

where a is a positive constant. The gravity mainly due to the masses, m_1 and m_2, is supported by the mass m_3 with the displacement expressed as z_3, and the coil spring with stiffness K. The tyre part of the wheel consists of small tyre with stiffness k_2'. The excitation from the road profile is assumed to be the signal generated by the electric vibrator connected to the signal function generator. The damping force of the suspension part is the Coulomb damping caused by contact with the two polls and the viscous damping caused by the pneumatic cylinder is assumed to be linear with the damping coefficient c considered to be relatively small.

Therefore, the equations of motion for the car model are given by

$$m_1\ddot{z}_1+c(\dot{z}_1-\dot{z}_2)+f(z_1-z_2)=u \quad (2)$$

$$m_2\ddot{z}_2-c(\dot{z}_1-\dot{z}_2)-f(z_1-z_2)+k_2'(z_2-z_3)=-u \quad (3)$$

$$m_3\ddot{z}_3-k_2'(z_2-z_3)+Kz_3=f_e \quad (4)$$

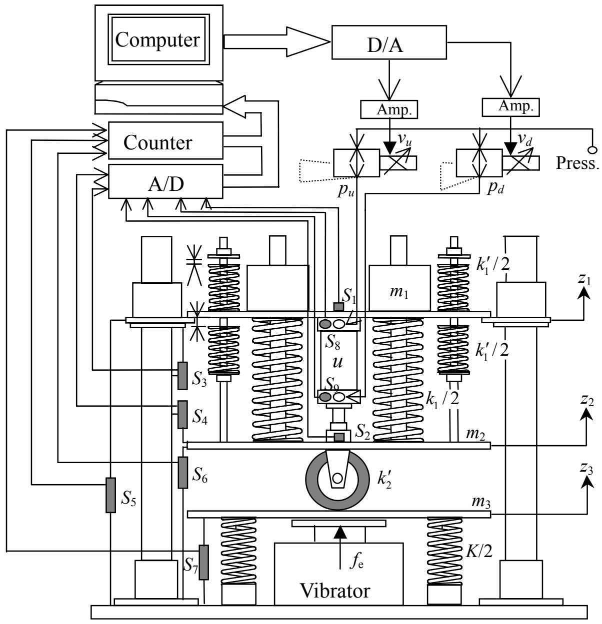

Fig.1 Experimental apparatus of one-wheel car model

where u is the active control achieved by actuating the pneumatic actuator, and f_e is the exciting force generated by the electric vibrator. After dividing both sides of Eq.(4) by K, neglecting the resultant first term on the left-hand side of Eq.(4) as its effect is considered relatively small compared with the effect of the other terms, defining that

$$k_2=\frac{k_2'K}{k_2'+K},\quad w=\frac{f_e}{K},\quad z_2-w=\left(\frac{k_2'}{k_2}\right)(z_2-z_3),$$

and substituting the above definitions into Eq.(3), the resultant equation becomes

$$m_2\ddot{z}_2-c(\dot{z}_1-\dot{z}_2)-f(z_1-z_2)+k_2(z_2-w)=-u \quad (5)$$

where w and z_2-w in Eq.(5) correspond respectively to the excitation from the road profile and the tyre deflection.

The state variables for the car model are measured by the sensors, the control signal is calculated by using the personal computer based measured data of the state variables, the control valve of the pneumatic actuator is regulated by the control signal, and finally the active control force u is u=77.0v, with v being the voltage of the control valve.

PNEUMATIC ACTIVE SUSPENSION SYSTEM

The pneumatic active suspension system is constructed as follows. Firstly, the compensator is constructed to deal with the performance degradation due to the delay of the pneumatic actuator. Secondly, the fuzzy control as the active control is inferred by using SIRMs fuzzy reasoning (Yoshimura *et al*., 2000; 2003), and the excitation from the road profile is estimated by using a disturbance observer and the estimate is denoted as one of the variables in the precondition part of the fuzzy control rules.

Compensator

The performance degradation in the vibration suppression of the car model due to the delay of the pneumatic actuator is countered by inserting the compensator after the sensor to measure the acceleration of the car body. The transfer function for the pneumatic actuator is experimentally identified as (Yoshimura and Takagi, 2004)

$$G_a(s)=\frac{P_u(s)}{K_aV_u(s)}=\frac{900}{s^2+30s+900}\mathrm{e}^{-0.035s} \tag{6}$$

where K_a=90 kPa/V. Assuming that the transfer function for the compensator is given as

$$G_c(s)=\frac{s^2+2\varsigma_b\omega_b s+\omega_b^2}{s^2+2\varsigma_a\omega_a s+\omega_a^2} \tag{7}$$

then the parameters characterizing $G_c(s)$ are determined by considering the frequency response function $G_a(j2\pi f)G_c(j2\pi f)$.

Fuzzy reasoning and disturbance observer

The number of fuzzy control rules increases more exponentially if the number of variables in the precondition parts increases more. Decreasing the number of fuzzy control rules, the SIRMs fuzzy reasoning (Yoshimura *et al*., 2000; 2003) is proposed. The fuzzy control rules are expressed as

$$SIRM-1:\{\text{if}\ \alpha_1\ \text{is}\ A_{1j}\ \text{then}\ \beta_1\ \text{is}\ B_{1j}\}_{j=1}^{n_1}$$
$$\cdots$$
$$SIRM-i:\{\text{if}\ \alpha_i\ \text{is}\ A_{ij}\ \text{then}\ \beta_i\ \text{is}\ B_{ij}\}_{j=1}^{n_i}$$
$$\cdots$$
$$SIRM-n:\{\text{if}\ \alpha_n\ \text{is}\ A_{nj}\ \text{then}\ \beta_n\ \text{is}\ B_{nj}\}_{j=1}^{n_n} \tag{8}$$

where α_i and β_i are respectively the variables in the precondition and the conclusion parts, A_{ij} and B_{ij} are respectively the fuzzy sets whose membership functions are respectively denoted as $\mu_{A_{ij}}(\alpha_i)$ and $\mu_{B_{ij}}(\beta_i)$, with n_i denoting the number of fuzzy control rules. Reducing the inference load to obtain the defuzzificated value of the variable in the conclusion part, the product-sum-gravity method (Yoshimura *et al*., 2000; 2003) is proposed. Measuring α_i as α_i^0, the degree of fitness ω_{ij} is given by

$$\omega_{ij}=\mu_{A_{ij}}(\alpha_i^0) \tag{9}$$

and the variable in the conclusion part is inferred as

$$\beta_{ij}=\mu_{B_{ij}}^{-1}(\omega_{ij}) \tag{10}$$

Then, the defuzzificated value β_i^0 of β_i in the conclusion part becomes

$$\beta_i^0=\sum_{j=1}^{n_i}\omega_{ij}\beta_{ij}/\sum_{j=1}^{n_i}\omega_{ij} \tag{11}$$

Summing up all the defuzzificated values, the fuzzy control u becomes

$$u=\sum_{i=1}^{n}g_i\beta_i^0 \tag{12}$$

where g_i is the control gain, and u is achieved by actuating the pneumatic actuator.

The disturbance observer is constructed to estimate the excitation from the road profile w. Assuming that the state variables $(\dot{z}_1, z_1, \dot{z}_2, z_2)$ can be measured and w cannot be measured, w is estimated by using the minimum-order observer as the proposed disturbance observer (Yoshimura and Takagi, 2004). Approximating $f(z_1-z_2)$ from $k_1(z_1-z_2)$, and defining the augmented state vector $\boldsymbol{x}$ and the measurement vector $\boldsymbol{y}$ as

$$\boldsymbol{x}=[\dot{z}_1\ \ z_1\ \ \dot{z}_2\ \ z_2 \mid w\ ]^{\mathrm{T}}\underline{\underline{\Delta}}[\ \boldsymbol{y}\mid w\ ]^{\mathrm{T}}$$

the estimate $\hat{w}$ for w is obtained by using the measurement vector $\boldsymbol{y}$ (Yoshimura and Takagi, 2004), and is denoted as one of the variables in the precondition part of the fuzzy control rules.

EXPERIMENTAL RESULT

The parameters characterizing the frequency response function of the compensator $G_c(j2\pi f)$ given as Eq.(7) are determined to improve the performance degradation in the vibration suppression of the car model. The $G_c(j2\pi f)$ is experimentally determined by evaluating the frequency response function $G_a(j2\pi f)G_c(j2\pi f)$, and finally the parameters characterizing $G_c(j2\pi f)$ are given as

$$\varsigma_a=0.3,\ \varsigma_b=0.5,\ \omega_a=60\ \text{rad/s},\ \omega_b=60\ \text{rad/s}.$$

It is seen from the frequency response functions shown in Fig.2 that $G_a(j2\pi f)G_c(j2\pi f)$ (with compensation) raises the gain more and more leads the phase shift by more than $G_a(j2\pi f)$ (without compensation).

The parameters characterizing the experimental apparatus of the one-wheel car model are given by

$$m_1=46.16\ \text{kg},\ m_2=18.1\ \text{kg},\ m_3=1.32\ \text{kg}$$
$$k_1=6.8\ \text{kN/m},\ k_1'=20\ \text{kN/m},\ k_2'=100\ \text{kN/m}$$
$$K=100\ \text{kN/m},\ c=400\ \text{Ns/m},\ a=1.4\ \text{mm}$$

and the nonlinear restoring force of the suspension part is constructed (Yoshimura and Takagi, 2004). The excitation force generated by the electric vibrator is assumed to be random with 5 Hz bandwidth. The sampling interval of the measurement and the active control is assumed to be 10 ms, and the root mean squares (*RMS*) values of the time responses of the state variables and the active control are computed by using the 10 s measurement data.

The $(\ddot{z}_1,\dot{z}_1,z_1,\ddot{z}_2,\dot{z}_2,z_2,\hat{w})$ are assumed as the variables in the precondition part of the fuzzy control rules and are respectively normalized as

$$\alpha_1=[\ddot{z}_1/c_{1\max}],\ \alpha_2=[\dot{z}_1/c_{2\max}],\ \alpha_3=[z_1/c_{3\max}],$$
$$\alpha_4=[\ddot{z}_2/c_{4\max}],\ \alpha_5=[\dot{z}_2/c_{5\max}],\ \alpha_6=[z_2/c_{6\max}],$$
$$\alpha_7=[\hat{w}/c_{7\max}]$$

where $c_{1\max}$~$c_{7\max}$ are the scaling factors. Two kinds of fuzzy sets, *P* and *N*, are assumed to reduce the inference load, and their membership functions are given by Fig.3. The fuzzy control rules are proposed as

If α_i is *P* (or *N*) then β_i is *N* (or *P*), i=1,2,…,7 (13)

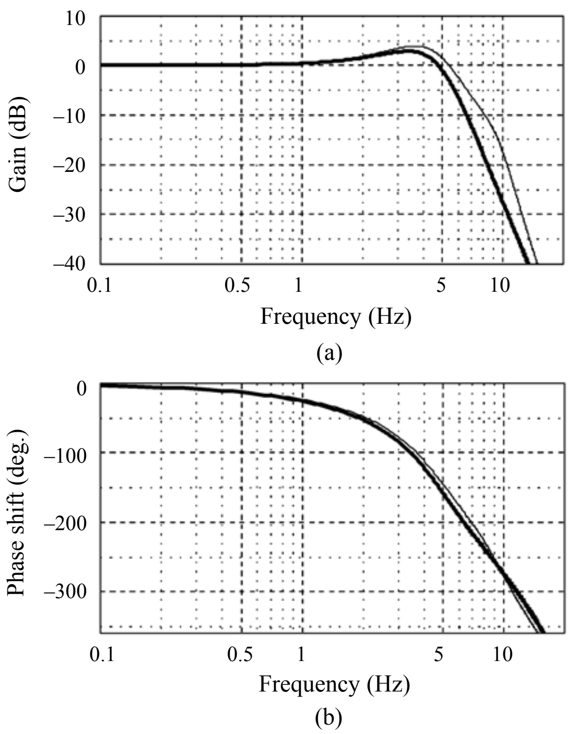

Fig.2 Frequency response functions
(a) Gain; (b) Phase shift
(—— Without; —— With)

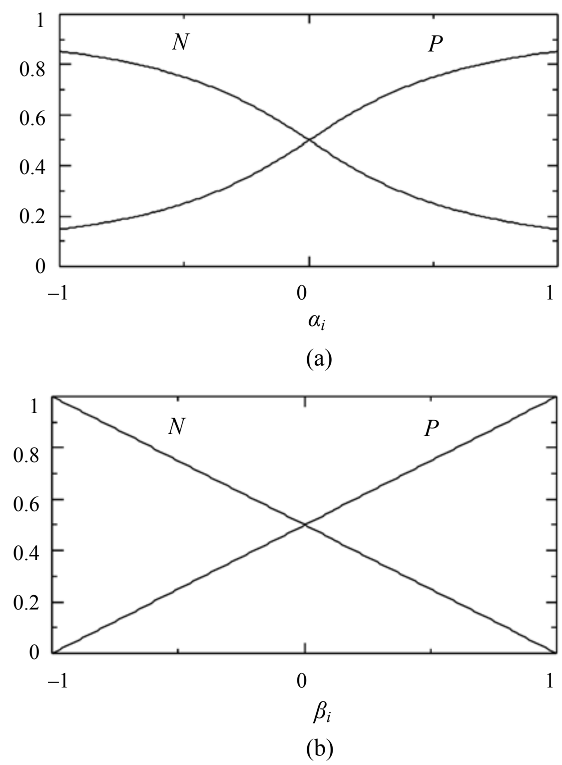

Fig.3 Membership functions
(a) Precondition part; (b) Conclusion part

The number of fuzzy control rules obtained by the proposed method is calculated as 2×7=14, while that obtained by the conventional method with seven variables in the precondition part is calculated as 2^7=128. Therefore, it is seen from the number of fuzzy control rules that the proposed method compared to the conventional method, reduces much more the inference load.

The performance index determining the parameters characterizing the active control is assumed to be

$$J = E[\lambda_1 \ddot{z}_1^2 + \lambda_2 (z_1 - z_2)^2 + \lambda_3 (z_2 - w)^2 + \rho u^2] \quad (14)$$

and the weighting factors are given as

$$\lambda_1 = 1,\ \lambda_2 = 0.1,\ \lambda_3 = 0.1,\ \rho = 3\times10^{-4}$$

The scaling factors of the variables in the precondition parts are respectively given as

$$c_{1\max} = 4 \text{ m/s}^2,\ c_{2\max} = 0.13 \text{ m/s},\ c_{3\max} = 4\times10^{-3} \text{ m}$$
$$c_{4\max} = 8 \text{ m/s}^2,\ c_{5\max} = 6.4\times10^{-2} \text{ m/s},$$
$$c_{6\max} = 3.2\times10^{-3} \text{ m},\ c_{7\max} = 3.2\times10^{-3} \text{ m}$$

Then, the control gains characterizing the active control are respectively obtained as

$$g_1 = 35,\ g_2 = 25,\ g_3 = -25,\ g_4 = 25$$
$$g_5 = 20,\ g_6 = -15,\ g_7 = -30$$

Yoshimura *et al*.(2000) showed that the active control based on SIRMs fuzzy reasoning improved more the performance in the vibration suppression of the car model than the skyhook damper control. Therefore, the performance in the vibration suppression obtained by the disturbance observer of the proposed active suspension system is discussed in this paper.

The following three kinds of suspension systems are presented to compare their performance: Method A: Passive suspension system; Method B: Active suspension system without disturbance observer; Method C: Active suspension system with disturbance observer.

Table 1 shows that the *RMS* values of the time responses of the one-wheel car model obtained from the three kinds of methods. It is seen from the table that Method C improves more the performance in the vibration suppression of the car model, especially in the acceleration of the car body $\ddot{z}_1$, than the other two methods. The spectral density calculated from the time response of $\ddot{z}_1$ is shown in Fig.4. It is seen from the figure that Method C reduces more the peak of the spectral density in the neighborhood of 4 Hz compared with the other two methods. The time response of the active control u is shown in Fig.5, and the range of the amplitude almost lies in (–50*N*, 50*N*). The estimation of w obtained from the proposed disturbance observer is shown in Fig.6, indicating that the proposed disturbance observer is very effective as the exact and estimated values of the excitation from the road profile have good agreement.

Therefore, it is concluded from the experimental results that the proposed active suspension system has effective performance in vibration suppression of the car model.

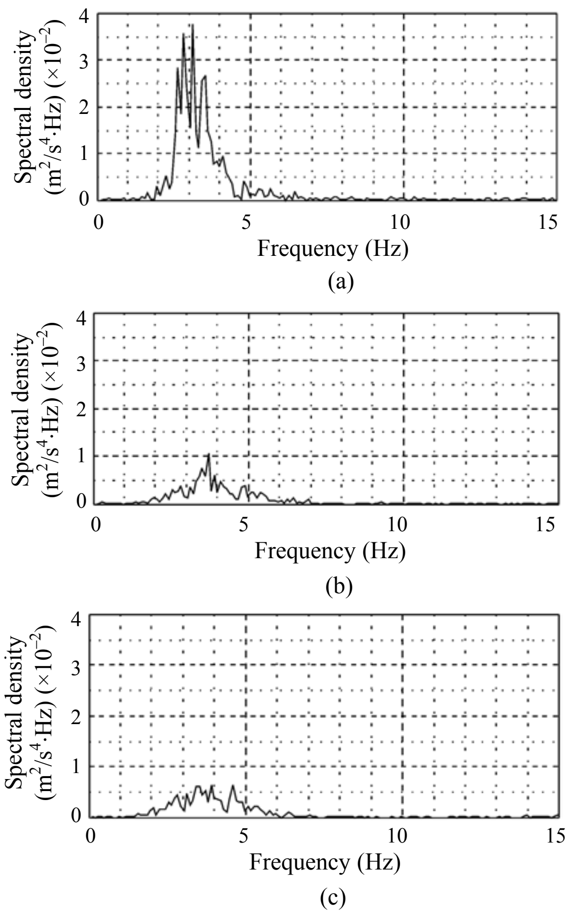

Fig.4 Spectral density for $\ddot{z}_1$

(a) Method A; (b) Method B; (c) Method C

Table 1 *RMS* values of the time responses of the variables

	Method A	Method B	Method C	Unit
$\ddot{z}_1$ ($\times10^{-1}$)	9.714	5.616	5.346	m/s^2
$\dot{z}_1$ ($\times10^{-2}$)	4.198	2.140	2.129	m/s
z_1 ($\times10^{-3}$)	2.539	1.488	1.615	m
$\ddot{z}_2$	1.046	0.999	0.931	m/s^2
$\dot{z}_2$ ($\times10^{-2}$)	2.175	2.125	2.094	m/s
z_2 ($\times10^{-3}$)	1.639	1.238	1.199	m
z_1–z_2 ($\times10^{-3}$)	2.170	1.303	1.294	m
z_2–w ($\times10^{-4}$)	9.881	4.151	4.003	m
u ($\times10^{-1}$)	0	2.148	2.232	N
J ($\times10^{-1}$)	9.436	4.538	4.353	–

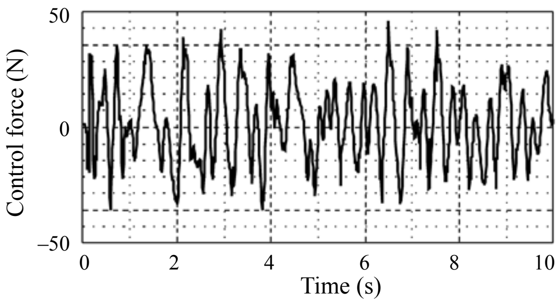

Fig.5 Time response of the proposed active control

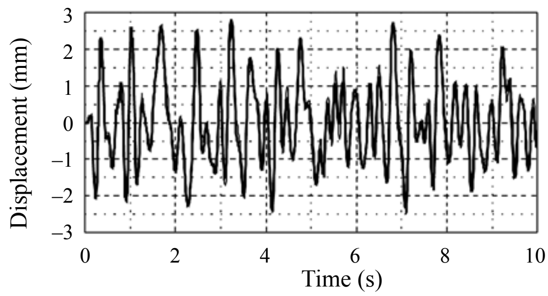

Fig.6 Estimation of the excitation from the road profile
(—— Exact; —— Estimated)

CONCLUSION

This paper proposes an active suspension system for a one-wheel car model using fuzzy reasoning and a disturbance observer. The fuzzy control as the proposed active control was obtained by single input rule modules fuzzy reasoning where excitation from a road profile was estimated by using the disturbance observer. The active control force was released by actuating a pneumatic actuator, and the performance degradation due to the delay of the pneumatic actuator was improved by inserting a compensator after the sensor to measure the acceleration of the car body. The experimental results indicated that the proposed active suspension system improved much the performance in the vibration suppression of the car model.

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