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A new fusion approach based on distance of evidences*

CHEN Liang-zhou (陈良洲)[†1], SHI Wen-kang (施文康)[1], DENG Yong (邓 勇)[1], ZHU Zhen-fu (朱振福)[2]

([1]School of Electronics & Information Technology, Shanghai Jiao Tong University, Shanghai 200030, China)

([2]National Defense Key Laboratory of Target and Environment Feature, Beijing 100854, China)

[†]E-mail: tiger-chen@sjtu.edu.cn

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Abstract: Based on the framework of evidence theory, data fusion aims at obtaining a single Basic Probability Assignment (BPA) function by combining several belief functions from distinct information sources. Dempster's rule of combination is the most popular rule of combinations, but it is a poor solution for the management of the conflict between various information sources at the normalization step. Even when it faces high conflict information, the classical Dempster-Shafer's (D-S) evidence theory can involve counter-intuitive results. This paper presents a modified averaging method to combine conflicting evidence based on the distance of evidences; and also gives the weighted average of the evidence in the system. Numerical examples showed that the proposed method can realize the modification ideas and also will provide reasonable results with good convergence efficiency.

Keywords: Data fusion, Evidence distance, Conflicting evidence, Evidence credibility, Combination rules
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INTRODUCTION

Interest in data fusion has markedly increased over the last decade, especially for military applications; and is also widely used in other fields, such as image processing and analysis and classification or target tracking (Goodman *et al*., 1997; Linas and Waltz, 1990; Hall and Linas, 2001). Based on the framework of evidence theory, data fusion relies on the use of combination rules allowing the belief functions for the different propositions to be combined. Dempster's (1967) combination rule plays an important role in evidence theory; and has several important mathematical properties such as commutation and association.

However, combining belief functions with this operator implies normalizing the results by scaling them proportionally to the conflicting mass in order to keep some basic properties. Zadeh (1986) underlined that this normalization involves counter-intuitive behaviors. In order to solve the problems of conflict management, Yager (1986), Smets (1990), Dubois (1998), and more recently Lefevre *et al*.(2002) proposed other combination rules which have more or less satisfactory behaviors. Particularly, Dubois' rule or Yager's rule of combination holds that the conflicting mass must be distributed over all subsets. Smets (1993) proposed that the conflicting mass results from the non-exhaustivity of the frame of discernment. Murphy (2000) suggested incorporating "average belief" into Dempster's combining rule, which handles highly conflicting evidence efficiently and has many attractive properties. However, simple averaging is not always adequate to get reasonable results, especially when the evidence has a high degree of conflict, because it does not consider the associative relationship among the evidences collected from multi-sources. In this paper, based on the distance of evidence, a modified average approach to combine conflicting evidence is developed.

This paper is organized as follows: The basic concepts of evidence theory are first briefly introduced including the problem of conflict in Demp-

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ster's rule of combination. Then, we introduce the distance measure between bodies of evidence and present a modified averaging approach to combining conflicting evidence. Following this, a numerical example is given to show the efficiency of the proposed method and the result of the modified averaging approach compared with that of other methods. Finally, a brief conclusion is drawn.

REVIEW OF THE THEORY OF EVIDENCE

In this section, we briefly review the basic concepts of evidence theory and introduce related functions and notations. The theory of evidence is initially based on Dempster's work concerning lower and upper probability distribution families. From these mathematical foundations, Shafer (1976) showed the ability of the belief functions to model uncertain knowledge. The functions defined in the theory of evidence allow one to quantify the confidence that a particular event could be the one observed. Then, while new information arrives, the identification system integrates it using conditioning rules to provide a representation of the obviousness of the situation. In the following, terminology of theory of evidence and the notation used in this paper are defined.

Terminology

In the theory of evidence, if Θ denotes the set of θ_N ($\theta_N \in \Theta$) corresponding to N identifiable objects, then Θ is called the frame of discernment, defined as follows:

$$\Theta=\{\theta_1, \theta_2, \ldots, \theta_N\} \quad (1)$$

It is composed of N mutually exhaustive and exclusive hypotheses. The power set of Θ is the set containing the all the 2^N possible subsets of Θ, represented by $P(\Theta)$:

$$P(\Theta)=\{\varnothing,\{\theta_1\},\{\theta_2\},\ldots,\{\theta_N\},\{\theta_1,\theta_2\},\{\theta_1,\theta_3\}\ldots,\Theta\} \quad (2)$$

where $\varnothing$ denotes the empty set. The $\{\theta_N\}$ subsets containing only one element are called singletons. A key point of evidence theory is the Basic Probability Assignment (BPA). A BPA is a function from $P(\Theta)$ to [0, 1] defined by:

$$m{:}P(\Theta)\rightarrow[0,1], \quad A|\rightarrow m(A) \quad (3)$$

and which satisfies the following conditions:

$$\sum_{A\in P(\Theta)} m(A)=1, \quad m(\varnothing)=0 \quad (4)$$

where A is the element of $P(\Theta)$. If the condition $m(\varnothing)=0$ is specified, it corresponds to the "closed-world assumption". If it is not, it corresponds to an "open-world assumption" (Smets, 1990). The elements of $P(\Theta)$ that have non-zero mass are called focal elements. A body of evidence (BOE) is the set of all focal elements. And the union of all the focal elements is called the core of the m-function. Given a BPA m, a belief function Bel is defined as:

$$Bel: P(\Theta)\rightarrow[0,1], A\mapsto Bel(A)=\sum_{B\subseteq A} m(B) \quad (5)$$

and a plausibility function Pl is defined as:

$$Pl: P(\Theta)\rightarrow[0,1], A\mapsto Pl(A)=\sum_{A\cap B\neq\varnothing} m(B) \quad (6)$$

$Bel(A)$ measures the total belief that the hypotheses is true. While $Pl(A)$ measures the total belief that can move into A. In particular, we have $Bel(\varnothing)=0$, $Bel(\Theta)=1$ and $Pl(\varnothing)=0$, $Pl(\Theta)=1$. Because the functions m, Bel and Pl have a one-to-one correspondence, it is equivalent to talking about one of them, or about the corresponding body of evidence.

Combination rule of evidence

In the case of imperfect data, fusion is an interesting solution to obtain more relevant information. Evidence theory offers appropriate aggregation tools. Two BPAs m_1 and m_2 can be combined to yield a new BPA m by a combination rule. Dempster's rule of combination (also called the orthogonal sum), noted by $m=m_1\oplus m_2$, is the first classical one within the framework of evidence theory as defined by Dempster (1967):

$$m(A)=\frac{\sum_{B\cap C=A} m_1(B)m_2(C)}{1-K} \quad (7)$$

where $K = \sum_{B \cap C = \varnothing} m_1(B) m_2(C)$ (8)

K is the mass that the combination assigned to the null subset and represents contradictory evidence, called conflict because it measures the degree f of the conflict between m_1 and m_2, K=0 corresponds to the absence of conflict between m_1 and m_2, whereas K=1 implies complete contradiction between m_1 and m_2. The belief function resulting from the combination of J information sources S_J is defined as:

$$m = m_1 \oplus m_2 \oplus \cdots \oplus m_j \oplus \cdots \oplus m_J \quad (9)$$

In fact, the Dempster's rule of combination is not the only rule to combine two BPAs. Smets (1993) proposed the conjunctive and disjunctive rules, which are also un-normalized rules that allow the empty set to have a non-null mass.

Conflict in combining evidence

In fact, when conflicting evidence is present, Dempster's rule for combining beliefs often produces counter-intuitive results. A typical example given by Zadeh (1986) is as follows:

Example 1 Consider a situation in which we have two belief structures m_1 and m_2 as follows:

$m_1(a)$=0.9, $m_1(b)$=0.1, $m_2(b)$=0.1, $m_2(c)$=0.9.

Application of the Dempster's rule yields:

$m(a)$=$m(c)$=0, $m(b)$=1.

Thus it can be seen that while m_1 and m_2 offer little support to b, the results offer complete support to b. This appears somewhat counter-intuitive. Because of the illogical aspects mentioned above, conflict management in belief functions is a very important problem and had been studied in the past. In general, there are three types of problems concerning the classical Demspter's combination rule: (1) D-S combination can assign 100% certainty to a minority opinion, which is shown in Example 1 (Zadeh, 1986). (2) The "ignorance" interval disappears forever whenever a single piece of evidence imparts all its weight to a proposition and its negation, gives the false impression that precise probabilistic information underlies the belief (Pearl, 1990). (3) Elements of sets with larger cardinality can gain a disproportionate share of belief (Voorbraak, 1991).

To solve these problems, several alternatives to the normalization process were proposed. Murphy (2000) carefully analyzed the proposed approaches, comprehensively compared their results, and argued that, of the presented alternatives, averaging best solves the normalization problems.

Murphy's averaging approach

Murphy's average rule of combination is very simple: just average all the BPAs to get a new BPA. Using the average approach, the results in Example 1 are:

$m(a)$=0.45, $m(b)$=0.1, $m(c)$=0.45.

The results are more reasonable than that of Dempster's. On the one hand, since m_1 and m_2 offer little support to b, $m(b)$ is small as expected. On the other hand, the support of a and c are equal and both of them do not draw a clear conclusion in the step. The final decision should be made depending on the collection of additional evidence.

This averaging approach has many attractive properties. However, it does not offer convergence toward certainty. This drawback can be shown in the following example introduced by Voorbraak (1991).

Example 2 Consider a situation in which we have two belief structures m_1 and m_2 as follows:

$m_1\{a\}$= $m_1\{b$ or $c\}$=0.5, $m_2\{c$ or $b\}$=$m_2\{c\}$=0.5.

Results after combining the two bodies of evidence:

$m\{a\}$= $m\{b\}$= $m\{c\}$=1/3

The equal distribution of belief is counter-intuitive because both $\{a\}$ and $\{c\}$ had individually assigned mass, as well as a share with $\{b\}$; but $\{b\}$ had only two shared masses. Because the problem occurs in the intersection operation, omitting the normalization step does not change the relative assignments. Thus, the proposed alternatives cannot handle the problem in this case. However, averaging the masses yields:

$m\{a\}$=$m\{c\}$=$m\{a$ or $b\}$=$m\{b$ or $c\}$=0.25.

The results showed that, on the one hand, "averaging" solves the problem presented by Voorbraak (1991). On the other hand, it does not offer convergence toward certainty. To improve the performance of convergence, Murphy incorporated average belief into the combining rule. Thus, averaging followed by the D-S combination gives:

$m\{a\}=m\{c\}=0.3$, $m\{b\}=0.2$,
$m\{a \text{ or } b\}=m\{b \text{ or } c\}=0.1$.

This result assigns a higher mass to $\{a\}$ and $\{c\}$ than to $\{b\}$. Though the averaging approach efficiently solves the problem of combining conflicting evidence, simple average is not always reasonable, especially in the case when the number of the evidences is not enough to make a decision. The reason is that simple averaging does not consider the associative relationship among the evidences collected from multi-sources. A modified averaging approach to combine conflicting evidence is developed below.

THE MODIFIED AVERAGING APPROACH

In this section, based on the Definition 1−5 of distance measure between bodies of evidence presented by Jousselme *et al.*(2001), a credibility degree coefficient is introduced to show the relative weight of each piece of evidence, then a modified (or weighted) averaging approach is developed.

Distance of evidence (Jousselme *et al.*, 2001)

Unlike other's viewpoints, Jousselme *et al.* give a geometrical interpretation for BPAs. The main idea is detailed as follows: Let us call $\mathbb{R}_{P(\Theta)}$ the space generated by all subsets of Θ. $\mathbb{R}_{P(\Theta)}$ is a vector space of any linear combination of the object of $\mathbb{R}_{P(\Theta)}$.

Definition 1 Let Θ be a frame of discernment containing N mutually exclusive and exhaustive hypotheses, and let $\mathbb{R}_{P(\Theta)}$ be the space generated by all subsets of Θ. A basic probability assignments (BPA) is a vector $\boldsymbol{m}$ of $\mathbb{R}_{P(\Theta)}$ with coordinates $m(A_i)\geq 0$ such that

$$\sum_{i=1}^{2^N} m(A_i)=1,\quad A_i\in P(\Theta),\quad i=1,\ldots,2^N \tag{10}$$

In this definition, we do not need to impose $m(\varnothing)=0$.

Definition 2 Let m_1 and m_2 be two BPAs on the same frame of discernment Θ, containing N mutually exclusive and exhaustive hypotheses. The distance between m_1 and m_2 is:

$$d_{\text{BPA}}(m_1,m_2)=\sqrt{\frac{1}{2}(\boldsymbol{m}_1-\boldsymbol{m}_2)\underline{\underline{\boldsymbol{D}}}(\boldsymbol{m}_1-\boldsymbol{m}_2)^{\mathrm{T}}} \tag{11}$$

where m_1 and m_2 are the BPAs according to Definition 1 and $\underline{\underline{\boldsymbol{D}}}$ is a $2^N\times 2^N$ matrix whose elements are

$$D(A,B)=\frac{|A\cap B|}{|A\cup B|},\ A,B\in P(\Theta) \tag{12}$$

Here, the factor 1/2 is needed in Eq.(11) to normalize d_{BPA} and to guarantee that $0\leq d_{\text{BPA}}(m_1, m_2)\leq 1$. From Definition 2, another way to write d_{BPA} is:

$$d_{\text{BPA}}(m_1,m_2)=\sqrt{\frac{1}{2}(\|\boldsymbol{m}_1\|^2+\|\boldsymbol{m}_2\|^2-2\langle\boldsymbol{m}_1,\boldsymbol{m}_2\rangle)} \tag{13}$$

where $\langle\boldsymbol{m}_1, \boldsymbol{m}_2\rangle$ is the scalar product defined by

$$\langle\boldsymbol{m}_1,\boldsymbol{m}_2\rangle=\sum_{i=1}^{2^N}\sum_{j=1}^{2^N} m_1(A_i)m_2(B_j)\frac{|A_i\cap B_j|}{|A_i\cup B_j|} \tag{14}$$

while $A_i, A_j\in P(\Theta)$ for $i,j=1,2,\ldots,2^N$. $\|\boldsymbol{m}\|^2=\langle\boldsymbol{m},\boldsymbol{m}\rangle$ is then the square norm of $\boldsymbol{m}$.

Credibility of evidence

To do the fusion, we want to know if the evidence we will use is credible. Maybe one piece of evidence is more important than the other, and each piece of evidence has its own weight in the sum of evidence we had. For example, in a long time measurement, some sensors may be more reliable than others because of their high stability, so the relative importance of each sensor may not be equal. Therefore, how to assign the weight of evidence is very important, especially when the evidence that is collected by multi-sensor array has a high degree of conflict. In this case, the system should determine, among the evidences, which one is more reliable and which one should we pay less attention to. A reasonable way to handle this problem is as follows: If a piece of evidence is supported by other evidences in the system, then the evidence should have a higher weight than that of a piece of evidence which has high conflicting degree with other evidences. Based on this rule, we give some definitions to describe the "credibility degree" of evidence, which actually is a weight and showing the relative importance of each piece of evidence in the system.

Definition 3 The similarity measure $Sim_{i,j}$ between the two bodies of evidence m_i and m_j is defined as:

$$Sim_{i,j}=\frac{1}{2}[\cos\pi d_{\text{BPA}}(m_i,m_j)+1] \quad i,j=1,\ldots,N \tag{15}$$

Suppose the number of bodies of evidence is N. We can construct a Similarity Measure Matrix (***SMM***) of all the N bodies of evidence, which gives us insight into the agreement between every two bodies of evidence Eq.(16).

$$\boldsymbol{SMM}=\begin{bmatrix} 1 & Sim_{1,2} & \cdots Sim_{1,N-1} & Sim_{1,N} \\ Sim_{2,1} & 1 & \cdots Sim_{2,N-1} & Sim_{2,N} \\ \vdots & \vdots & \ddots \quad \vdots & \vdots \\ Sim_{N-1,1} & Sim_{N-1,2} & \cdots \quad 1 & Sim_{N-1,N} \\ Sim_{N,1} & Sim_{N,2} & \cdots Sim_{N,N-1} & 1 \end{bmatrix} \tag{16}$$

Definition 4 The support degree Sup of each piece of evidence m_i is defined as Eq.(17).

$$Sup(m_i)=\sum_{\substack{j=1\\ j\neq i}}^{N} Sim_{i,j}, \quad i,j=1,2,\cdots,N \tag{17}$$

Definition 5 The credibility degree Crd_i of each piece of evidence is defined as:

$$Crd_i=\frac{Sup(m_i)}{\sum_{i=1}^{N} Sup(m_i)}, \quad i=1,2,\cdots,N \tag{18}$$

It can be easily seen that $\sum_{i}^{N} Crd_i=1$. According to Definition 4, for a given piece of evidence, the shorter the distance to other pieces of evidence, the more the support degree. If a piece of evidence is greatly supported by others, its credibility degree is high and this evidence has more effect on the final combinatorial results. Contrarily, if a piece of evidence is always in conflict with other evidences to a high degree, its credibility degree is low and this evidence should have less effect on the final combinatorial results. Thus the credibility degree is actually a weight, which shows the relative importance of the collected evidence. The modified (or weight) average mass m_M of the evidence is given as:

$$m_M=\sum_{i=1}^{N}(Crd_i\times m_i), \quad i=1,2,\cdots,N \tag{19}$$

If there are N pieces of evidence, the classical Dempster's rule is used to combine the weighted average of the masses N−1 times, which is the same as Murphy's (2000) approach. We have proposed a weighted averaging approach to combining evidence. This presented approach will be summarized in five steps:

Step 1: Within N pieces of evidence, calculate the distance between each piece of evidence with others.

Step 2: Calculate $Sim_{i,j}$ and Sup_i of each evidence.

Step 3: Calculate the weighted average masses of the evidence.

Step 4: Use the classical Dempster's rule to combine the weighted average masses N−1 times.

Step 5: When get the (N+1)th evidence, go to Step 1 and repeat.

NUMERICAL EXAMPLES

In this section, two numerical examples are illustrated. The first one shows how to combine the conflicting evidence and the second one shows the efficiency of the proposed method by comparing the results of different combining rules.

Example 3 Three pieces of evidence m_1, m_2, m_3 and their masses are as follows:

m_1: $m_1(A)$=0.6, $m_1(B)$=0.1, $m_1(C)$=0.3

m_2: $m_2(A)$=0.2, $m_2(B)$=0, $m_2(C)$=0.8

m_3: $m_3(A)$=0.7, $m_3(B)$=0.1, $m_3(C)$=0.2

According to Eqs.(11)–(18), the credibility degree Crd_i of each piece of evidence can be calculated as:

Crd_1=0.3947, Crd_2=0.2501, Crd_3=0.3552.

And the weighted masses of the evidences m_1, m_2, m_3 are respectively as:

$m_M(A)$=0.3947×0.6+0.2501×0.2+0.3552×0.7=0.5355
$m_M(B)$=0.3947×0.1+0.2501×0+0.3552×0.1=0.0750
$m_M(C)$=0.3947×0.3+0.2501×0.8+0.3552×0.2=0.3895

As can be seen from the results above, evidence m_1 and m_3 are similar to each other. So the credibility degrees of m_1 and m_3 are higher than m_2. Contrarily, the evidence m_2 is highly conflicting with m_1 and m_3, which leads to the rather low degree of credibility Crd_2, compared with the other two pieces of evidence in the system. After the weighted mass of the three pieces of evidence is obtained, classical Dempster's rule is used to combine the weighted masses twice. So, the final combination results by the modified average approach are given as follows:

$m(A)$=0.7207, $m(B)$=0.0020, $m(C)$=0.2773

The following example illustrates the efficiency of the proposed combination rule. Since Murphy (2000) has compared the simple averaging approach with other alternatives, in this paper, we just compared the proposed method with the classical Dempster's combination rule and the averaging method presented by Murphy.

Example 4 In a multi-sensor based automatic target recognition system: suppose the real target is A and the system has collected 5 pieces of evidence m_1, m_2, m_3, m_4, m_5 as follows:

m_1: $m_1(A)$=0.6, $m_1(B)$=0.2, $m_1(C)$=0.3
m_2: $m_2(A)$=0, $m_2(B)$=0.9, $m_2(C)$=0.1
m_3: $m_3(A)$=0.55, $m_3(B)$=0.1, $m_3(C)$=0.35
m_4: $m_4(A)$=0.55, $m_4(B)$=0.1, $m_4(C)$=0.35
m_5: $m_5(A)$=0.55, $m_5(B)$=0.1, $m_5(C)$=0.35

The results by using different combination rules are shown in Table 1.

As can be seen from Table 1, when conflicting evidence is present, the classical Dempster's rule for combining beliefs produces results that do not reflect the actual distribution of beliefs. In this case, for the collection of the "bad" evidence m_2, which may have been caused by many factors such as foul weather, spurious external or internal signal noise, drifting circuit board component values, or even an enemy's jamming activity or the flaws of the sensor array itself, Dempster's combination results show that, though more pieces of evidence, collected later, support target A, it is impossible that the target is A, which is contrary to the truth. With incremental evidence, both the simple averaging and weight averaging methodes provide reasonable results. However, when the number of evidences is not adequate to make a decision, the proposed method is superior to the simple averaging of the sums. For example, when the system collects only

Table 1 Results of using different combination rules of evidence

	m_1, m_2	m_1, m_2, m_3	m_1, m_2, m_3, m_4	m_1, m_2, m_3, m_4, m_5
D-S combination rule	$m(A)$=0 $m(B)$=0.8571 $m(C)$=0.1429	$m(A)$=0 $m(B)$=0.6316 $m(C)$=0.3648	$m(A)$=0 $m(B)$=0.3288 $m(C)$=0.6712	$m(A)$=0 $m(B)$=0.1228 $m(C)$=0.8772
Yager's combination rule	$m(A)$=0 $m(B)$=0.18 $m(C)$=0.03 $m(\Theta)$=0.79	$m(A)$=0 $m(B)$=0.018 $m(C)$=0.0105 $m(\Theta)$=0.9715	$m(A)$=0 $m(B)$=0.0018 $m(C)$=0.0037 $m(\Theta)$=0.9945	$m(A)$=0 $m(B)$=0.0002 $m(C)$=0.0013 $m(\Theta)$=0.9985
Murphy's average combination rule	$m(A)$=0.1543 $m(B)$=0.7469 $m(C)$=0.0988	$m(A)$=0.3504 $m(B)$=0.5231 $m(C)$=0.1265	$m(A)$=0.6027 $m(B)$=0.2627 $m(C)$=0.1346	$m(A)$=0.7958 $m(B)$=0.0932 $m(C)$=0.1110
Modified average combination rule	$m(A)$=0.1543 $m(B)$=0.7469 $m(C)$=0.0988	$m(A)$=0.4626 $m(B)$=0.3845 $m(C)$=0.1529	$m(A)$=0.7419 $m(B)$=0.1120 $m(C)$=0.1461	$m(A)$=0.8827 $m(B)$=0.0142 $m(C)$=0.1031

three pieces of evidence m_1, m_2, m_3, the presented approach draws a correct conclusion that the target is *A*, while the simple average still supports that the target is *B*. What is more, as can be seen from Table 1, the convergence performance of the modified method is better than that of the simple average. The main reason for these finding mentioned above is that, by making use of the distance of the evidences, the modified average approach decreases the weight of the "bad" evidence, so the "bad" evidence has less effect on the final combination of results.

CONCLUSION

Dempster's combination operator is a poor solution for the management of the conflict between the various information sources at the normalization step. Of the alternative methods that address the problems, averaging best solves the normalization problems and has much more attractive features. The modified average approach based on the distance between the evidence preserves all of the desirable properties of the simple average. In addition, compared with simple averaging, the proposed method reflects the associative relationship of the evidences and can efficiently handle conflicting evidences with better convergence performance.

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