Journal of Zhejiang University SCIENCE
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Influences affecting the soil-water characteristic curve*

ZHOU Jian (周 建)[†], YU Jian-lin (俞建霖)

(*Institute of Geotechnical Engineering, Department of Civil Engineering, Zhejiang University, Hangzhou 310027, China*)

[†]E-mail: dzhoujian@yahoo.com

Received Mar. 10, 2004; revision accepted June 26, 2004

Abstract: The soil-water characteristic curve (SWCC) is the primary partially saturated soil information as its behavior and properties can be derived from it. Although there have been many studies of unsaturated soils and the SWCC, there is still no combined constitutive model that can simulate soil characteristics accurately. In cases when hydraulic hysteresis is dominant (e.g. under cyclic loading) it is particularly important to use the SWCC. In the past decades, several mathematical expressions have been proposed to model the curve. There are various influences on the SWCC as a source of information, so the curves obtained from conventional tests often cannot be directly applied; and the mathematical expressions from one scenario cannot be used to simulate another situation. The effects of void ratio, initial water content, stress state and high suction were studied in this work revealing that water content and stress state are more important than the other effects; but that the influences tend to decrease when suction increases. The van Genuchten model was modified to simulate better the changes in the degree of saturation at low values of suction. Predictions were compared with experimental results to determine the simulation capability of the model.

Key words: Soil-water characteristic curve (SWCC), Unsaturated soil, Mathematical expression
doi:10.1631/jzus.2005.A0797 **Document code:** A **CLC number:** TU411

INTRODUCTION

The soil-water characteristic curve (SWCC) gives the relationship between the amount of water in the soil (i.e. gravimetric or volumetric water content) and soil suction (i.e. matric suction at low suction and total suction at high suction). Many properties of a partially saturated soil such as the coefficient of permeability, shear strength and volume strain, pore size distribution, the amount of water contained in the pores at any suction, can be obtained from the SWCC.

This curve presents the basic characteristics of a partially saturated soil. Many experimental tests had been performed to obtain the SWCC for different types of soil and under different conditions. Due to the limitations of time and of accurately measuring suction, a wide range of suction tests have not been performed and the various factors influencing the SWCC have not received great attention. This paper focuses on various effects on the SWCC as reported in published data and draws some useful conclusions for engineering practice. An important phenomenon has been identified and a modified van Genuchten model is put forward at the end of the paper.

Fig.1 shows a typical curve during soil de-saturation usually consisting of three stages: capillary saturation, de-saturation and residual saturation. When the suction value exceeds the air-entry value (AEV), the degree of saturation decreases rapidly at relatively low suction values and then reduces more gradually when the suction becomes high.

Much attention has been paid recently to the constitutive modeling of partially saturated soils. Because of the complicated microstructure of partially saturated soil and its importance on unsaturated soil behavior, the SWCC should preferably be used to model the behavior accurately. But there are many influences on the SWCC, and without further studies on whether or not the SWCC from conventional tests can be used, the resulting large errors may lead to misunderstanding and wrong engineering solutions.

---

*Project (No. 22833012) supported by the China Scholarship Council

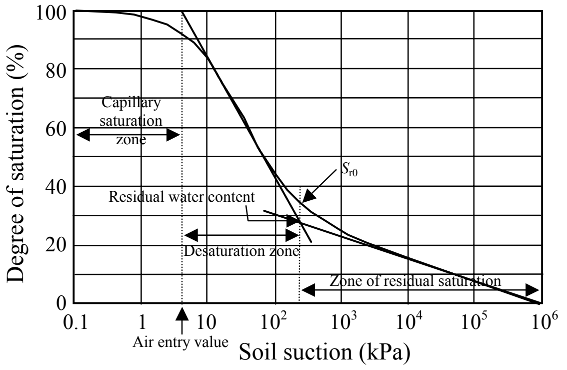


Fig.1 Soil-water characteristic curve showing the regions of de-saturation (Sillers *et al.*, 2001)

MAIN FACTORS INFLUENCING THE SWCC

The pressure plate test was commonly used to measure suction indirectly using the axis-translation technique, which is not capable of applying significant vertical stress, and so, cannot be used to study the effect of overload stress, i.e. the depositional history of specimens. Whether or not the SWCC curve generated from pressure plate tests can be used should be considered very carefully. Other factors such as soil structure (and aggregation), initial water content, void ratio, type of soil, mineralogy, and compaction method also have potentially significant effects on features of the SWCC. Among these factors, stress history and initial water content often have the greatest effect on soil structure, which in turn dominates the nature of the soil-water characteristic curve.

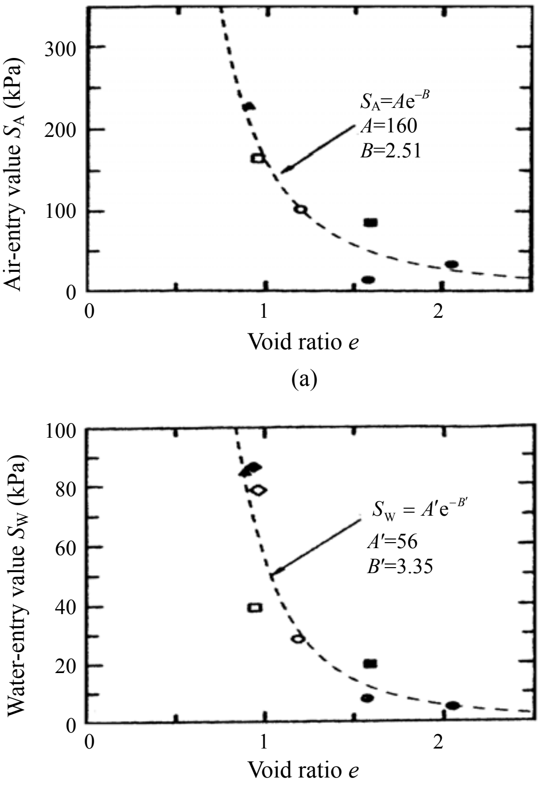
Effect of initial void ratio

Kawai *et al.*(2000) studied the effect of initial void ratio on the SWCC. The material properties of the testing silty clay soil are listed in Table 1. Each specimen was set in the oedometer apparatus modified for unsaturated soil, and suction was applied by means of the pressure plate method.

Table 1 Material properties of clay used

G_s	w_p	w_L	I_p
2.70	29.6%	43%	13.4

Porewater tends to migrate as suction increases and when the value reaches the air-entry value (AEV) bulk water begins to drain away. The AEV (denoted S_A in Fig.2a) reflects the magnitude of the capillary saturation zone in a soil (Fig.1). The larger the bulk pore sizes, the smaller the AEV. The AEV is thought to be in inverse proportion to the log of *e* of the soil.



(a)

(b)

Fig.2 Relationship between void ratio and air-entry value (a); water-entry value (b) (Kawai *et al.*, 2000)

Fig.2a is the plot of AEV value vs void ratio. Similar to the AEV when a soil is wetted up the degree of saturation increases markedly when suction reduces to a certain value called WEV (water-entry value). The relationship between WEV (denoted S_W in Fig.2b) and degree of saturation is given in Fig.2b.

The manner in which the residual degree of saturation (S_{r0}), the degree of saturation at the start of the zone of residual saturation changes with AEV, is shown in Fig.3. It can be seen from Figs.2 and 3 that the smaller the initial void ratio (i.e. the denser the soil), the higher the air-entry value, and the higher the residual degree of saturation as well. The air-entry value and the residual degree of saturation S_{r0} can be expressed together by void ratio *e* using empirical relationships.

The AEV is an important parameter for partially saturated soils since the degree of saturation starts to drop rapidly when the suction exceeds the AEV.

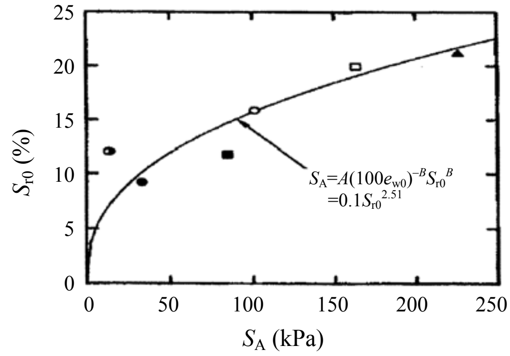


Fig.3 Relationship between AEV and residual degree of saturation (Kawai *et al.*, 2000)

There is a large range of AEVs corresponding to different void ratio values, as shown in Fig.2a. The denser the soil, the higher the AEV, which implies that for soils with low void ratio values, small changes in degree of saturation can be assumed at low suctions, i.e. the soil can be treated as fully saturated. This might be a helpful observation when soils from different depth are being dealt with.

Effect of initial water content

Fig.4 illustrates the effect of initial water content on the SWCC obtained from Vanapalli *et al.*(1999). The samples were of sandy clay till obtained from Indian Head, Saskatchewan, Canada. This soil is classified as a clay with low liquid limit. The liquid limit, plastic limit and grading properties are given in Table 2. The clay fraction is predominantly calcium montmorillonite. The AASHTO standard compacted maximum density is 1.80 g/cm^3 at optimum water content of 16.3%. The specific density of the soil solids is 2.73.

Table 2 Summary of the soil properties (from Vanapalli *et al.*, 1999)

Soil type	San (%)	Si (%)	Clay (%)	w_L (%)	w_p (%)
Indian head till	28	42	30	35.5	16.8

All samples were compacted and prepared with the required initial water content and density, and then sandwiched between filter paper and porous stones in consolidation rings and were loaded to 3.5 kPa in a conventional oedometer. These samples were submerged in distilled water for 36 h at degrees of saturation of greater than 99%; removed from the odeometers and placed in pressure plate apparatus. They are referred to as specimens with 0 kPa equivalent pressure in the following part.

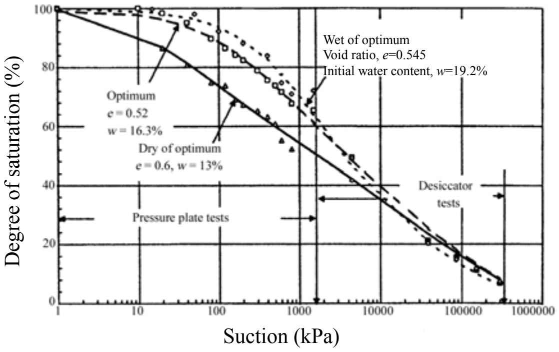


Fig.4 Soil-water characteristic curves for specimens compacted at different initial water contents (Vanapalli *et al.*, 1999)

The initial water content has considerable influence on the shape of SWCC curves. The higher the initial water content, the steeper the curve. The air-entry value also increases with initial water content. The resistance to de-saturation is relatively low in the dry of optimum specimens in comparison to optimum and wet of optimum specimens. So for soils of high initial water content the effect of de-saturation is more obvious, especially at low suction values. Curves with different initial water contents tend to converge at high suction values.

Effect of stress state

In the field, due to its depositional history, soil normally experiences a certain stress, which is recognized to have some influence on SWCC (Fredlund and Rahardjo, 1993). Vanapalli *et al.*(1996; 1998; 1999) studied the influence of total stress state on the SWCC of a compacted fine-grained soil indirectly. As a conventional pressure plate apparatus does not allow any external loading, an equivalent pressure is applied to study its effect on the SWCC.

Equivalent pressure can be explained using the following example, as seen in Fig.5. A specimen was placed in an oedometer, saturated under constant volume conditions and loaded to 200 kPa (point *A* in Fig.5). Then it was allowed to swell under a nominal pressure (3.5 kPa) (point *B*). When the specimen had experienced maximum pre-stress pressure (200 kPa), it had a void ratio corresponding to 100 kPa on the initial compression branch after swelling under the applied pressure of 3.5 kPa (point *C*). The equivalent pressure for this specimen is equal to 100 kPa.

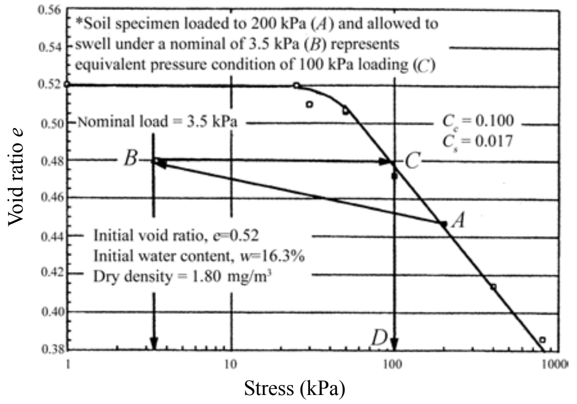


Fig.5 Void ratio versus the applied stress for an initial void ratio of 0.52 (Vanapalli *et al.*, 1999)

The SWCCs developed for the specimens compacted dry of optimum and with equivalent pressures of 25, 35, 80 and 200 kPa are shown in Fig.6 showing obviously that the air-entry value of specimens increases with increasing equivalent pressure. In general, beyond the air-entry value of suction, specimens subjected to higher equivalent pressures have higher degrees of saturation at any given suction.

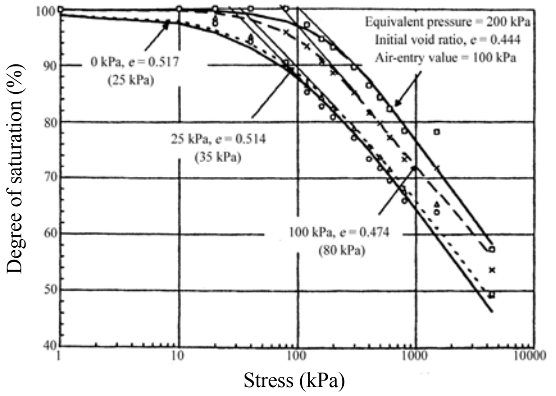


Fig.6 Soil-water characteristics for specimens compacted dry of optimum water content (Vanapalli *et al.*, 1999)

To explain this phenomenon, Vanapalli *et al.*(1999) suggested that when investigating the structure of partially saturated soils, there are two levels of structure to be considered: macrostructure and microstructure. Soil microstructure is described as the elementary particle associations within soil, whereas the arrangement of soil aggregates is referred to as the macrostructure (Mitchell, 1976). Macrostructure governs soil-water characteristic behavior for specimens compacted with initial water contents dry of optimum, particularly at low suction values. The air-entry value and the residual state of saturation increase with the equivalent pressure for specimens with dry of optimum initial water content conditions. Microstructure seems to govern the soil-water characteristic behavior of specimens compacted wet of optimum and resists the de-saturation (drying). This interpretation has to be confirmed by the inspection of soil structure at different water contents.

Ng and Pang (2000) investigated the influence of stress state on the SWCC of an "undisturbed" or natural, completely decomposed volcanic soil. A conventional volumetric pressure plate extractor and a modified one were used together. Three undisturbed or natural specimens were directly cut from the block into oedometer rings. The net normal stress levels considered in the modified volumetric pressure plate extractor were 40 and 80 kPa, which were appropriate for many relatively shallow slope failures in Hong Kong. Samples were first loaded to 40 and 80 kPa applied net normal stress, respectively, in oedometers with free drainage for 24 h for pre-consolidation purpose. Then they were removed and placed in the modified volumetric pressure plate extractor to subject the SWCCs to a predetermined stress. The required stress applied to each specimen was maintained throughout the tests. The SWCCs from their research are shown in Fig.7.

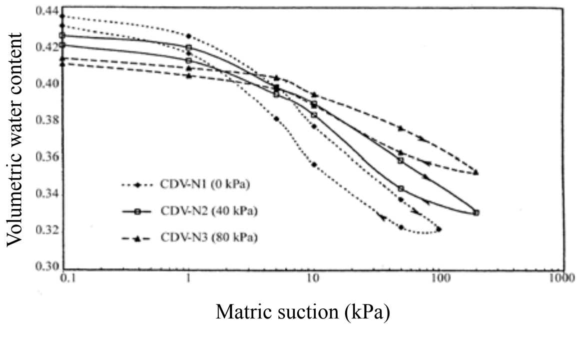


Fig.7 Effects of stress state on SWCC (Ng and Pang, 2000)

The results indicated that as matric suction increases, the volumetric water content of all specimens decreases but at different rates. The higher the applied load, the lower the rate of reduction in volumetric water content. The point where the volumetric water content starts to decrease significantly indicates the air-entry value. Fig.7 shows a general tendency that soil specimens subjected to higher stresses possess higher air-entry values, which is related to the presence of a smaller average pore sizes distribution in soil specimens under higher applied load.

Stress history or applied stress seems not to affect significantly the shape of SWCC, although the AEV increases and the rate of change of the degree of saturation decreases with the increasing net total stress.

Effect of high suction values

It has been shown that different initial values of void ratio, water content and stress state will influence the SWCC, especially at low suctions. In this section the effects at high suction values are investigated.

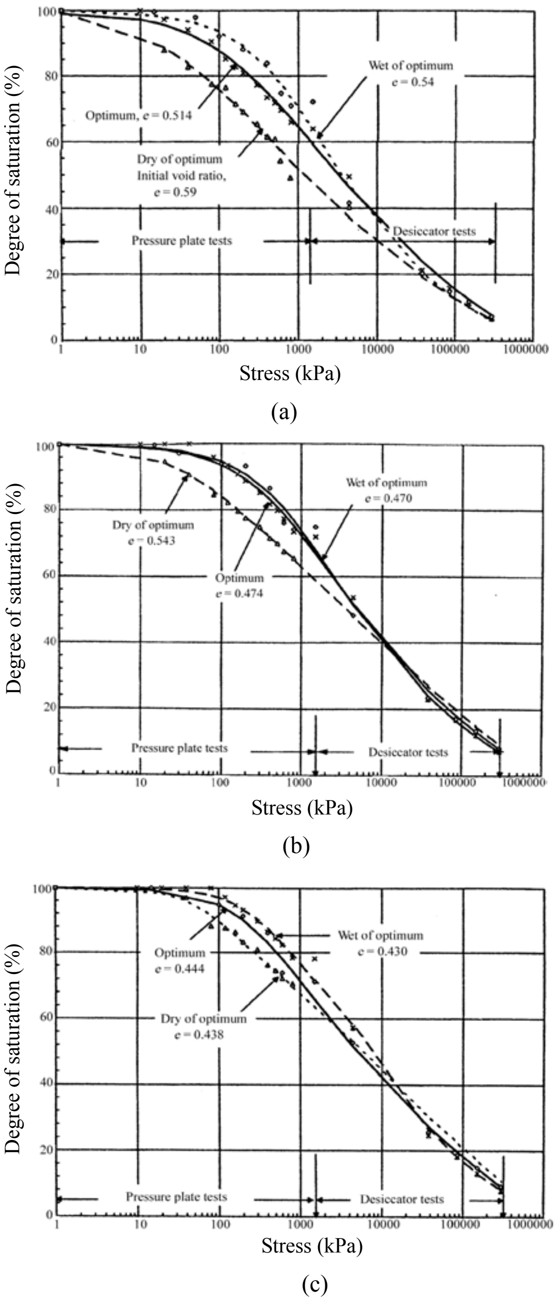
Regardless of the initial conditions of water content (i.e. dry of optimum, optimum and wet of optimum) and stress history, the soil-water characteristic behavior appears to be similar at higher suctions (i.e. 20000–300000 kPa), as shown in Fig.8. In other words, as Vanapalli *et al.*(1999) explained, the inner forces between soil aggregates are very strong in resisting de-saturation behavior at the higher suction values. Presumably, water films at these suctions are so thin that all the water is within the range of influence of osmotic and adsorptive fields. So soil structure (and aggregation) seems to have negligible influence on the soil-water characteristic behavior in this high suction range. From this, it can be concluded that when suction is very high, the effect of initial water content and stress history can be ignored.

MATHEMATICAL EXPRESSIONS FOR THE SWCC

Brief review of mathematical expression for the SWCC

Due to the different SWCC shape for various soil types, it is essential to find a general mathematical expression for practical use. Many researchers have tried to find a good expression.

Sillers *et al.*(2001) summarized numerous mathematical models for the SWCC. The mathematical models presented in their paper can be categorized in a number of ways to illustrate the characteristic equations, as well as their advantages and disadvantages. Models such as that of Gardner (1956), Brooks and Corey (1964), Brutsaert (1966), van Genuchten (1980), McKee and Bumb (1987), Burdine (1953), Mualem (1976), Kosugi (1994), Fredlund and Xing (1994) were analyzed; the main expressions are given in Table 3.



(a)

(b)

(c)

Fig.8 SWCC under different equivalent pressures (a) 25 kPa; (b) 100 kPa; (c) 200 kPa (Vanapalli *et al.*, 1999)

Table 3 Model for the soil-water characteristic curve and corresponding pore size distribution

Model name	Model	Pore size distribution $f(\psi)$
Gardner (1956)	$\theta=B/\psi-D$	Constant pore size
Linear function representing water content (Fredlund and Xing, 1994)	$\theta=B-D\psi$	A/ψ^2
Brooks and Corey (1964)	$S=B-D\psi^m$	$A/\psi^{(m+1)}$
Brutsaert (1966)	$S=\frac{1}{1+(\psi/a)^n}$	$\frac{n(\psi/a)^{n-1}/a}{[1+(\psi/a)^n]^2}$
Normal distribution	$S=\frac{1}{2}erfc\left(\frac{\psi-\mu}{\sqrt{2}s}\right)$	Normal distribution
Van Genuchten (1980)	$S=\frac{1}{(1+(a\psi)^n)^m}$	$\frac{mna(a\psi)^{n-1}}{(1+(a\psi)^n)^{m+1}}$
McKee and Bumb (1987) (Boltzman)	$S=\exp(-\psi/B)$	Exponential distribution
Fredlund and Xing (1994)	$S=\frac{1}{(\ln(e+(\psi/a)^n))^m}$	$\frac{mn(\psi/a)^{n-1}}{a(e+(\psi/a)^n)(\ln(e+(\psi/a)^n))^{m+1}}$
Kosugi (1994)	$S=\frac{1}{2}erfc\left(\frac{\ln\left(\frac{\psi_{\text{aev}}-\psi}{\psi_{\text{aev}}-\psi_{\text{mode}}}\right)-s^2}{\sqrt{2\pi}s}\right)$	$\frac{\exp\left(\frac{\ln\left(\frac{\psi_{\text{aev}}-\psi}{\psi_{\text{aev}}-\psi_{\text{mode}}}\right)-s^2}{\sqrt{2\pi}s}\right)}{\sqrt{2\pi}s(\psi_{\text{aev}}-\psi)}$

The variables in Table 3 are defined as follows: θ is the normalized water content, which can be calculated by using residual and saturated water content as described in the Gardner (1956) model; ψ is suction, and S is the degree of saturation. ψ_{aev} is the air-entry value related to the largest pore radius within the soil, designated by the capillary pressure equation and ψ_{mode} is the mode of the pore size distribution. Details for determining ψ_{aev} and ψ_{mode} the reader can refer to the Kosugi model (Kosugi, 1994) and *erfc* is the complementary error function. Other undefined symbols listed in the table are experimentally determined variables.

Modification of the van Genuchten expression

A good mathematical expression should have only a few parameters with clear physical meaning and be easy to use. Among all the expressions mentioned above, the van Genuchten expression is widely used due to its flexibility and simplicity and is given as follows:

$$S=\frac{1}{(1+(a\psi)^n)^m} \tag{1}$$

where a, n and m are fitting parameters. In this equation when the parameter m equals 1.0 it is equivalent to the Brutsaert (1966) model with a parameter inverted. m is related to the asymmetry of the curve; a smaller m corresponds to a moderate slope in low suction range and a steeper slope in high suction range. Parameter n is related to the pore size distribution index. The more uniform pore sizes in soil have larger n values whilst the larger the value of n, the steeper the curve in the de-saturation zone will be. Parameter a does not affect the curve shape, but shifts the curve towards the higher or lower suction regions of the plot.

Many test data indicated that for many unsaturated soils when suction is reduced to zero during wetting up, the degree of saturation could remain significantly below unity. Researchers sometimes ignore this behavior since it implies that there might be a big gap between saturated soil and partially saturated soil in terms of the degree of saturation. Coping with it mathematically is still a challenge. When unsaturated soils are subjected to cyclic loading, the degree of saturation will be far below unity at zero suction value. This hysteresis behavior is very important for the stability of slopes, embankments or dams. The existing mathematical expressions do not describe this characteristic behavior fully.

In order to simulate this situation, the van Genuchten expression needs some modification. It can be easily modified if the unity in the denominator is replaced by a parameter B_0, as shown in Eq.(2). This independent parameter is related to the value of degree of saturation corresponding to zero suction. Here fitting parameters a, n and m should change to be consistent with Eq.(1), but they still follow the same changing laws as described in the van Genuchten model. The disadvantage is the additional complexity involved in introducing another parameter.

$$S=\frac{1}{(B_0+(a\psi)^n)^m} \tag{2}$$

Fig.9 is from Kawai *et al.*(2000). The properties of the testing silty clay soil are given in Table 1 and different initial conditions are listed in Table 4. Compared with the total suction change often measured in generating the SWCC (i.e. 0 to 1000000 kPa), the range of suction in this case is not very wide, hence an arithmetic scale was used, as seen in Fig.9. However, it is common to plot soil water characteristic behavior in semi-logarithmic coordinates.

Fig.10a to Fig.10e are curves generated using the modified van Genuchten expression to simulate the

Table 4 Initial conditions of samples in Fig.9 (Kawai *et al.*, 2000)

Samples	w (%)	e	S_r (%)
a	23.2	2.11	29.7
b	30.2	1.62	50.5
c	17.5	1.59	29.7
d	23.1	1.20	52.0
e	30.4	0.99	82.9

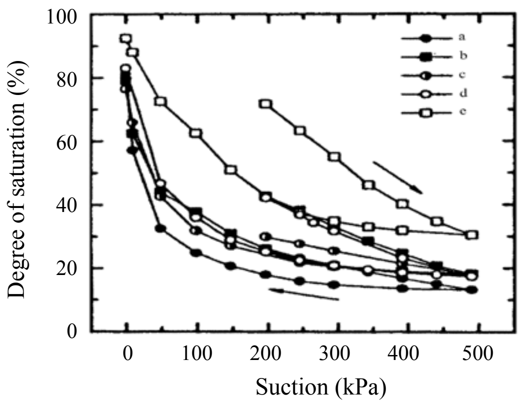


Fig.9 Different initial conditions on SWCC (Kawai *et al.*, 2000)

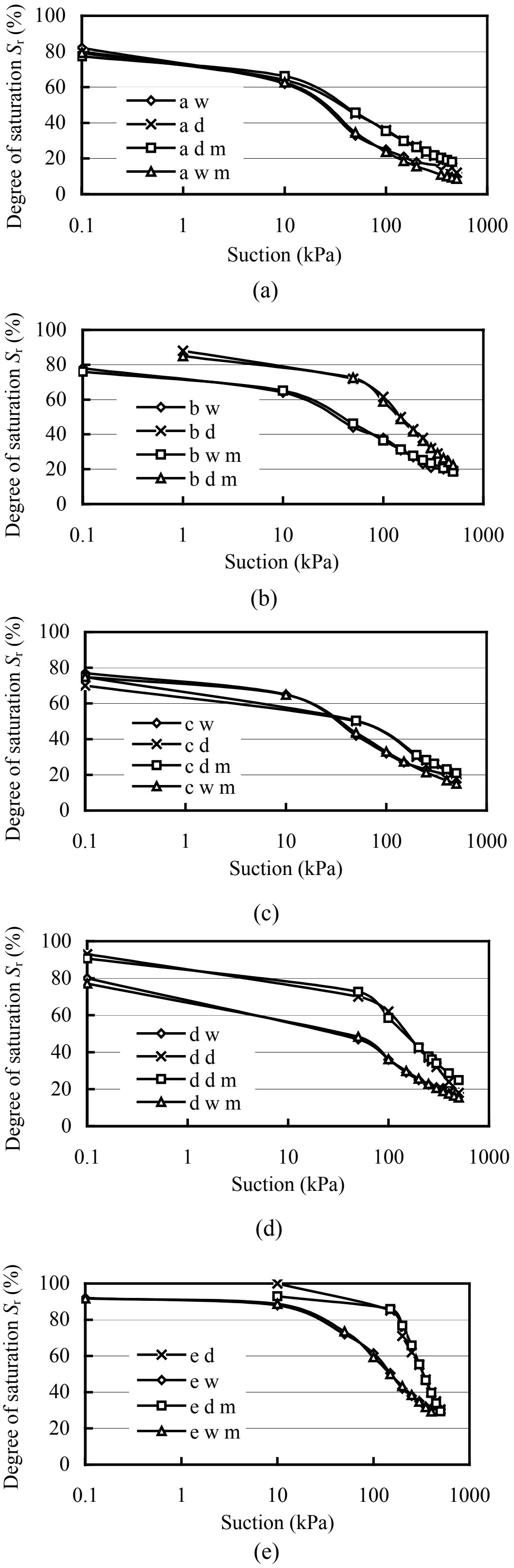


Fig.10 Measured and best-fit curve of samples in Fig.9 (a) Sample a; (b) Sample b; (c) Sample c; (d) Sample d; (e) Sample e

data shown in Fig.9 from Kawai *et al.*(2000), in which "w" refers to wetting path, "d" to drying path and "m" to the modeled curve. It can be seen from these figures that the modified expression is suitable for both wetting and drying paths.

CONCLUSION

The SWCC is very important in research of partially saturated soils. When combined with constitutive models, different factors corresponding to field situations should be considered. In this paper some of the main influencing factors are discussed in the context of how they affect the SWCC. Among these factors stress state and initial water content have the greatest influence. However, at high suction values the effect of these factors tends to diminish.

More tests are needed to grasp the general features of the SWCC, especially to provide information about soil structure, microstructure and macrostructure. It is not practicable and unnecessary to test samples under every condition. For this reason a basic series of tests for each type of soil should be performed to establish the main effects and the influence they have on the SWCC of the soil.

A good expression for the SWCC is essential to combine with constitutive modelling. A modified equation is proposed based on van Genuchten expression. Comparisons between measured and modelled SWCCs proved the model's capability and accuracy. Further studies should focus on how to deal with the value of the degree of saturation at zero suction.

References

Brooks, R., Corey, A., 1964. Hydraulic Properties of Porous Media, Hydrology Paper No. 3. Colorado State University, Fort Collins, CO.

Brutsaert, W., 1966. Probability laws for pore size distribution. *Soil Science*, **101**:85-92.

Burdine, N.T., 1953. Relative permeability calculations from pore size distribution data. *Journal of Petroleum Technology*, **5**:71-78.

Fredlund, D.G., Rahardjo, H., 1993. Soil Mechanics for Unsaturated Soils. Wiley, New York.

Fredlund, D.G., Xing, A., 1994. Equations for the soil-water characteristic curve. *Canadian Geotechnical Journal*, **31**:521-532.

Gardner, W., 1956. Mathematics of Isothermal Water Condition in Unsaturated Soils. Highway Research Board Special Report 40 International Symposium on Physico-Chemical Phenomenon in Soils. Washington D.C., p.78-87.

Kawai, K., Karube, D., Kato, S., 2000. The Model of Water Retention Curve Considering Effects of Void Ratio. *In*: Rahardjo, H., Toll, D.G., Leong, E.C.(Eds.), Unsaturated Soils for Asia. Balkema, Rotterdam, p.329-334.

Kosugi, K., 1994. The parameter lognormal distribution model for soil water retention. *Water Resource Research*, **30**:891-901.

McKee, C., Bumb, A., 1987. Flow-testing coalbed methane production wells in the presence of water and gas. *SPE Formation Evaluation*, (10):599-608.

Mitchell, J., 1976. Fundamentals of Soil Behaviour. John Wiley and Sons Inc., New York.

Mualem, Y., 1976. A new model for predicting the hydraulic conductivity of unsaturated porous media. *Water Resources Research*, **12**:593-622.

Ng, C.W.W., Pang, Y.W., 2000. Influence of stress state on soil-water characteristics and slope stability. *Journal of Geotechnical and Geoenvironmental Engineering*, **126**(2):157-166.

Sillers, W.S., Fredlund, D.G., Zakerzadeh, N., 2001, Mathematical attributes of some soil-water characteristic curve models. *Geotechnical and Geological Engineering*, **19**:243-283.

Vanapalli, S.K., Fredlund, D.G., Pufahl, D.E., Clifton, A.W., 1996. Model for the prediction of shear strength with respect to soil suction. *Can. Geotech. J., Ottawa*, **33**:379-392.

Vanapalli, S.K., Pufahl, D.E., Fredlund, D.G., 1998. The Meaning and Relevance of Residual Water Content to Unsaturated Soils. Proceedings of 51st Canadian Geotechnical Conference, Edmonton, AB, p.101-108.

Vanapalli, S.K., Pufahl, D.E., Fredlund, D.G., 1999. The influence of soil structure and stress history on the soil-water characteristic of a compacted till. *Geotechnique*, **49**(2):143-159.

van Genuchten, M.T., 1980. A closed form equation predicting the hydraulic conductivity of unsaturated soils. *Soil Science Society of America Journal*, **44**:892-898.