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Inverse method for the determination of elastic properties of coating layers by the surface ultrasonic waves*

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Abstract: As the coated materials are widely applied in engineering, estimation of the elastic properties of coating layers is of great practical importance. This paper presents an inversion algorithm for determining the elastic properties of coating layers from the given velocity dispersion of surface ultrasonic waves. Based on the dispersive equation of surface waves in layered half space, an objective function dependent on coating material parameters is introduced. The density and wave velocities, which make the object function minimum, are taken as the inversion results. Inverse analyses of two parameters (longitudinal and transverse velocities) and three parameters (the density, longitudinal and transverse velocities) of the coating layer were made.

Key words: Coating layers, Surface waves, Dispersive equation, Inverse algorithm
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INTRODUCTION

The elastic properties of coating layers in coating materials are very important in the design and evaluation for engineering purpose. They can serve as ground knowledge in the selection of surface coatings for particular applications, and can be used to monitor the quality of physical processes (Every, 2002). Therefore, estimation of the elastic properties of coating layers is of great practical value. Much effort has been put into measuring the material properties of coating layers by ultrasonic non-destructive testing technique, which has been theoretically and experimentally proved to be effective (Every, 2002; Wu, 1999; Mallah *et al*., 1999; Lakestani *et al*., 1995; Windels *et al*., 2001). It is well known that the dispersion characteristics of surface waves in a coated medium are strongly dependent on the elastic properties of the coating layer. By introducing a suitable inverse algorithm, the elastic properties can be recovered from the measured phase velocity dispersion curves. To determine the thickness and elastic properties of a bonding layer, Wu and Liu (1999) introduced an error function defining the difference between the guessed and measured phase velocities and proposed a method to minimize it. A similar inversion method was adopted by Makkonen *et al.*(2004) and Pastorelli *et al.*(2000) in estimating the material parameters of thin and ultra-thin films. Schneider *et al.*(2000) deduced Young's modulus by fitting the measured dispersion curves. Inverting the fundamental and higher mode data simultaneously, Xia *et al.*(2003) made the inversion process stable and improved the measurement. The aforementioned work contributed a lot to the inversion algorithm. However, there exist some pending problems: (1) the accuracy of theoretical dispersion curves can only be judged numerically; (2) the uniqueness characteristic of the inversion for inverting fundamental mode data is unknown, therefore, it is rather arbitrary to select the modes.

This paper is aimed at presenting a new inversion algorithm for determining the elastic properties

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based on the dispersive equation of surface waves propagating in layered half space. An objective function depending on phase velocities, wave frequencies and three material parameters of coating layers is introduced, and an inversion scheme is proposed to seek a group of material parameters by minimizing it. In the numerical analyses section, the inversion process of the density, the longitudinal and transverse wave velocities is illustrated in detail.

DISPERSIVE EQUATION AND INVERSE SCHEME

Consider a semi-infinite medium composed of a layer and a substrate, which are isotropic, homogenous, and linear materials as shown in Fig.1. Let the incident wave be a time harmonic plane wave from the free surface, then the displacements and stresses in the layer and substrate are only dependent on the coordinates of x_1 and x_2.

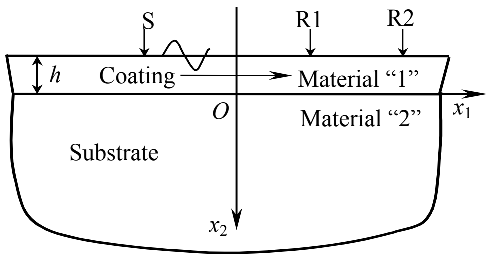

Fig.1 Coordinates and geometry of the layered half space

With the time factor $e^{-i\omega t}$ omitted, the time-reduced form of the displacement equations of motion for the layer and the substrate can be expressed as

$$(\lambda^{(n)}+\mu^{(n)})\nabla\nabla\cdot\boldsymbol{u}^{(n)}(x_1,x_2)+\mu^{(n)}\nabla^2\boldsymbol{u}^{(n)}(x_1,x_2)+\rho^{(n)}(\omega^{(n)})^2\boldsymbol{u}^{(n)}(x_1,x_2)=0 \quad (1)$$

where n=1,2 denotes the layer and the substrate, respectively, and $\boldsymbol{u}^{(n)}(x_1,x_2)$ is the complex valued displacement vector with time-reduced. $\lambda^{(n)}$ and $\mu^{(n)}$ are the Láme constants, $\rho^{(n)}$ is the density and $\omega^{(n)}$ is the circular frequency. For the case of plane strain, the displacement vector can be expressed in terms of two scalar potentials $\phi^{(n)}$ and $\psi^{(n)}$.

$$\boldsymbol{u}^{(n)}(x_1,x_2)=\nabla\phi^{(n)}(x_1,x_2)+\nabla\times[\psi^{(n)}(x_1,x_2)\cdot\boldsymbol{e}_3] \quad (2)$$

where $\boldsymbol{e}_3$ is coordinate basis vector, and $\phi^{(n)}$ and $\psi^{(n)}$ must satisfy the equations of motion

$$\left.\begin{aligned}&[\nabla^2+(\omega^{(n)}/c_L^{(n)})^2]\phi^{(n)}=0\\&[\nabla^2+(\omega^{(n)}/c_T^{(n)})^2]\psi^{(n)}=0\end{aligned}\right\} \quad (3)$$

$$c_L^{(n)}=\sqrt{(\lambda^{(n)}+2\mu^{(n)})/\rho^{(n)}},\ c_T^{(n)}=\sqrt{\mu^{(n)}/\rho^{(n)}} \quad (4)$$

here $c_L^{(n)}$ and $c_T^{(n)}$ are the velocities of longitudinal waves and transverse waves, respectively.

The boundary conditions at the free surface and the continuity conditions at the interface between the layer and the substrate can be written as

$$\left.\begin{aligned}&\sigma_{2\beta}^{(1)}=0 && (x_2=-h)\\&\sigma_{2\beta}^{(1)}=\sigma_{2\beta}^{(2)},\ u_\beta^{(1)}=u_\beta^{(2)} && (x_2=0)\end{aligned}\right\}(\beta=1,2) \quad (5)$$

where the stress components $\sigma_{\alpha\beta}^{(n)}$ can be expressed by introducing two-dimension array symbols

$$e_{11}=e_{22}=0,\quad e_{12}=-e_{21}=1 \quad (6)$$

$$\begin{aligned}\sigma_{\alpha\beta}^{(n)}=&\lambda^{(n)}\nabla^2\phi^{(n)}\delta_{\alpha\beta}+2\mu^{(n)}\phi_{,\alpha\beta}^{(n)}\\&+\mu^{(n)}(e_{\alpha\gamma}\psi_{,\beta\gamma}^{(n)}+e_{\beta\gamma}\psi_{,\alpha\gamma}^{(n)})\end{aligned} \quad (7)$$

where $(^*)_{,\alpha\beta}=\dfrac{\partial^2(^*)}{\partial x_\alpha\partial x_\beta}$, α,β,γ=1,2.

Consider a surface wave propagating in the direction x_1, Eq.(3) admits the plane-wave solution of the form

$$\left.\begin{aligned}\phi^{(n)}&=\Phi^{(n)}(x_2)e^{ik^{(n)}x_1}\\\psi^{(n)}&=\Psi^{(n)}(x_2)e^{ik^{(n)}x_1}\end{aligned}\right\} \quad (8)$$

where $k^{(n)}=\omega^{(n)}/c^{(n)}$ is the wave number, and $c^{(n)}$ is the propagation velocity.

According to Eq.(8), it is obvious that $k^{(1)}$ equals to $k^{(2)}$ and $c^{(1)}$ equals to $c^{(2)}$. Let $k=k^{(1)}=k^{(2)}$, $c=c^{(1)}=c^{(2)}$, c and k are defined as the phase velocity and wave number for the surface wave. Solving Eq.(3) we have

$$\left.\begin{aligned}\phi^{(n)}&=\left(A_1^{(n)}e^{kq_L^{(n)}x_2}+A_2^{(n)}e^{-kq_L^{(n)}x_2}\right)e^{ikx_1}\\\psi^{(n)}&=\left(B_1^{(n)}e^{kq_T^{(n)}x_2}+B_2^{(n)}e^{-kq_T^{(n)}x_2}\right)e^{ikx_1}\end{aligned}\right\} \quad (9)$$

where $q_L^{(n)}=\sqrt{1-c^2/(c_L^{(n)})^2}$, $q_T^{(n)}=\sqrt{1-c^2/(c_T^{(n)})^2}$. $A_1^{(n)}, A_2^{(n)}, B_1^{(n)}, B_2^{(n)}$ are unknown constants.

In the substrate, displacement components approach zero when $x_2\to\infty$. Combining this condition and the boundary conditions, we get $A_1^{(2)}=B_1^{(2)}=0$ and the equation

$$\boldsymbol{M}\boldsymbol{X}^{\mathrm{T}}=\boldsymbol{0} \tag{10}$$

where $\boldsymbol{X}=\left\{A_1^{(1)}, A_2^{(1)}, B_1^{(1)}, B_2^{(1)}, A_2^{(2)}, B_2^{(2)}\right\}$,

$$\boldsymbol{M}=\begin{bmatrix} 2\mathrm{i}q_L^{(1)}\mathrm{e}^{-2\pi f q_L^{(1)}h/c} & -2\mathrm{i}q_L^{(1)}\mathrm{e}^{2\pi f q_L^{(1)}h/c} & \zeta_T^{(1)}\mathrm{e}^{-2\pi f q_T^{(1)}h/c} & \zeta_T^{(1)}\mathrm{e}^{2\pi f q_T^{(1)}h/c} & 0 & 0 \\ \zeta_T^{(1)}\mathrm{e}^{-2\pi f q_L^{(1)}h/c} & \zeta_T^{(1)}\mathrm{e}^{2\pi f q_L^{(1)}h/c} & -2\mathrm{i}q_T^{(1)}\mathrm{e}^{-2\pi f q_T^{(1)}h/c} & 2\mathrm{i}q_T^{(1)}\mathrm{e}^{2\pi f q_T^{(1)}h/c} & 0 & 0 \\ \mathrm{i} & \mathrm{i} & q_T^{(1)} & -q_T^{(1)} & -\mathrm{i} & q_T^{(2)} \\ q_L^{(1)} & -q_L^{(1)} & -\mathrm{i} & -\mathrm{i} & q_L^{(2)} & \mathrm{i} \\ 2\mathrm{i}q_L^{(1)} & -2\mathrm{i}q_L^{(1)} & \zeta_T^{(1)} & \zeta_T^{(1)} & 2\mathrm{i}rq_L^{(2)} & -r\zeta_T^{(2)} \\ \zeta_T^{(1)} & \zeta_T^{(1)} & -2\mathrm{i}q_T^{(1)} & 2\mathrm{i}q_T^{(1)} & -r\zeta_T^{(2)} & -2\mathrm{i}rq_T^{(2)} \end{bmatrix}$$

here, $\zeta_T^{(n)}=1+(q_T^{(n)})^2$, $r=\mu^{(2)}/\mu^{(1)}$, $f=kc/(2\pi)$.

To ensure $\boldsymbol{X}$ has nonzero solutions, the determinant of the matrix $\boldsymbol{M}$ must be zero. Thereby, the dispersive equation of the surface wave propagating in a layered half space can be deduced as

$$\Omega=\det(\boldsymbol{M})=0 \tag{11}$$

Calculating the determinant of the matrix $\boldsymbol{M}$, $\Omega(c,f)$ can be expressed in the following form by adopting hyperbolic sine functions and hyperbolic cosine functions.

$$\begin{aligned}\Omega=&\,\Gamma_1+\Gamma_2\cosh(2\pi f h q_T^{(1)}/c)\sinh(2\pi f h q_L^{(1)}/c)\\&+\Gamma_3\sinh(2\pi f h q_T^{(1)}/c)\sinh(2\pi f h q_L^{(1)}/c)\\&+\Gamma_4\cosh(2\pi f h q_T^{(1)}/c)\cosh(2\pi f h q_L^{(1)}/c)\\&+\Gamma_5\sinh(2\pi f h q_T^{(1)}/c)\cosh(2\pi f h q_L^{(1)}/c)\end{aligned} \tag{12}$$

where

$$\Gamma_1=16q_L^{(1)}q_T^{(1)}\zeta_T^{(1)}[2\zeta_T^{(1)}(q_L^{(2)}q_T^{(2)}-1)+r(2+\zeta_T^{(1)})\eta+r^2\chi],$$

$$\Gamma_2=4rq_T^{(1)}(2-\zeta_T^{(1)})(\zeta_T^{(2)}-2)\times[(\zeta_T^{(1)})^2q_L^{(2)}-4q_T^{(2)}(q_L^{(1)})^2],$$

$$\begin{aligned}\Gamma_3=&\,4[(\zeta_T^{(1)})^4+16(q_L^{(1)}q_T^{(1)})^2](1-q_L^{(2)}q_T^{(2)})+8r[(\zeta_T^{(1)})^3\\&+8(q_L^{(1)}q_T^{(1)})^2]\eta+4r^2[(\zeta_T^{(1)})^2+4(q_L^{(1)}q_T^{(1)})^2]\chi,\end{aligned}$$

$$\begin{aligned}\Gamma_4=&\,4q_L^{(1)}q_T^{(1)}[8(\zeta_T^{(1)})^2(1-q_L^{(2)}q_T^{(2)})-4\zeta_T^{(1)}(2+\zeta_T^{(1)})\\&\times r\eta-(3+2\zeta_T^{(1)}+(q_T^{(1)})^4)r^2\chi],\end{aligned}$$

$$\Gamma_5=4rq_L^{(1)}(2-\zeta_T^{(1)})(\zeta_T^{(2)}-2)\times[(\zeta_T^{(1)})^2q_T^{(2)}-4q_L^{(2)}(q_T^{(1)})^2],$$

$$\chi=4q_L^{(2)}q_T^{(2)}-(\zeta_T^{(2)})^2,\quad \eta=\zeta_T^{(2)}-2q_L^{(2)}q_T^{(2)}.$$

According to Eq.(11), it is clear that the phase velocity is dependent on the wave frequency, the thickness, and the elastic properties (the density, the longitudinal and transverse velocities) of the coating layer.

The laser-acoustic method is described in detail in Every (2002). The surface acoustic signals are received by two transducers R1 and R2, as shown in Fig.1. The distance between the receivers is Δx. Fourier transforming the waveforms received yields their phase difference $\Delta\varphi$. Putting them into the relation $c_j=2\pi f_j\Delta x/\Delta\varphi$, a phase velocity c_j can be measured responding to a certain frequency f_j.

To determine the elastic properties of the coating layer from the measured data, an objective function is introduced and defined as

$$\Pi_e=\sqrt{\sum_{j=1}^{N}\left|\Omega(\tilde{c}_L^{(1)},\tilde{c}_T^{(1)},\tilde{\rho}^{(1)},c_j,f_j)\right|} \tag{13}$$

where N is the number of data points utilized in the inversion process, and $(\tilde{c}_L^{(1)},\tilde{c}_T^{(1)},\tilde{\rho}^{(1)})$ is a group of guessed values of the longitudinal wave velocity, the transverse wave velocity and the density of the coating layer. The inversed wave velocities and density $\tilde{c}_L^{(1)},\tilde{c}_T^{(1)},\tilde{\rho}^{(1)}$ are those leading the objective function Π_e to the minimum.

NUMERICAL ANALYSIS

In this section, numerical examples are given to illustrate the inverse method outlined above. The copper-steel configuration is considered, and the corresponding values of the material parameters are given as (Windels *et al.*, 2001):

The copper coating:

$c_L^{(1)}$=4660 m/s, $c_T^{(1)}$=2260 m/s, $\rho^{(1)}$=8900 kg/m^3,

The steel substrate:

$c_L^{(2)}$=5790 m/s, $c_T^{(2)}$=3100 m/s, $\rho^{(2)}$=7900 kg/m^3.

The material parameters of the copper coating are taken as the theoretical reference values. The thickness of the copper coating h is 100 μm. A group of phase velocities with frequency of 5 MHz to 25 MHz as shown in Fig.2 were assumed as the measured data for determining the parameters of the coating layer.

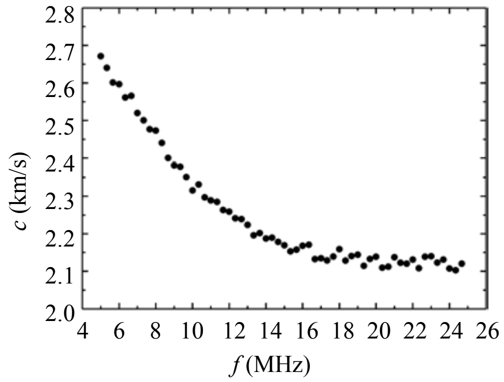

Fig.2 The given phase velocity dispersion

Inversion of the wave velocities of the layer when the density is known

The longitudinal and transverse wave velocities are related to the elastic modulus of a solid, and therefore, determining the elastic wave velocities of a coating layer is useful in accessing the coating quality. When the density of the coating layer is known, the inversion of the wave velocities becomes simple.

Based on the inversion process introduced above, Fig.3 shows the dependence of the objective function on the longitudinal and transverse velocities of the coating layer when the density is fixed. In the contour plot, the global minimum appears when $\left(c_L^{(1)}, c_T^{(1)}\right)$ reaches the triangle mark in Fig.3. Therefore, calculating by the inversion program, we obtained the inversed results: $\hat{c}_L^{(1)}$=4673.76 m/s and $\hat{c}_T^{(1)}$=2258.57 m/s.

Inversion of the density and the wave velocities of the layer

To determine the elastic constants from the wave

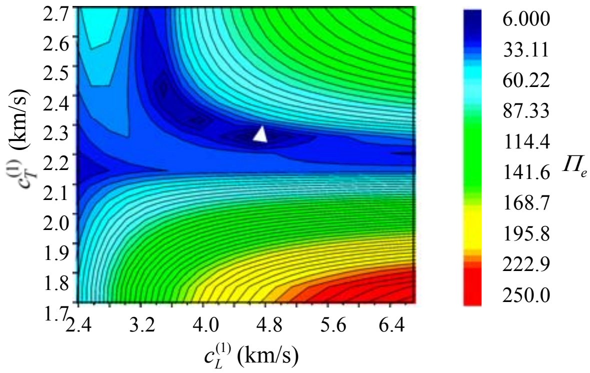

Fig.3 The dependence of the objective function on the longitudinal and transverse wave velocities of the coating layer with the density fixed at $\rho^{(1)}$=8900 kg/m^3

velocities, the density of the material should be estimated first. Making use of the proposed inversion process, the density of the coating layer can be determined simultaneously, though the minimum objective function should be judged by three parameters.

The global minimum of the objective function was researched in the range of 2400< $c_L^{(1)}$ < 6700 m/s, 1700< $c_L^{(1)}$ < 2700 m/s and 6700<$\rho^{(1)}$<11000 kg/m^3. Calculation by the inversion program yields the inversed results $\hat{c}_L^{(1)}$ =4588.68 m/s, $\hat{c}_T^{(1)}$ =2263.51 m/s, $\hat{\rho}^{(1)}$ =8856.41 kg/m^3. Fig.4 shows the dependence of the objective function on the density and the longitudinal wave velocity when the transverse wave velocity is fixed. Fig.5 is the dependence of the objective function on the density and the transverse wave velocity with the longitudinal wave fixed.

Fig.6 compares the assumed phase velocity dis-

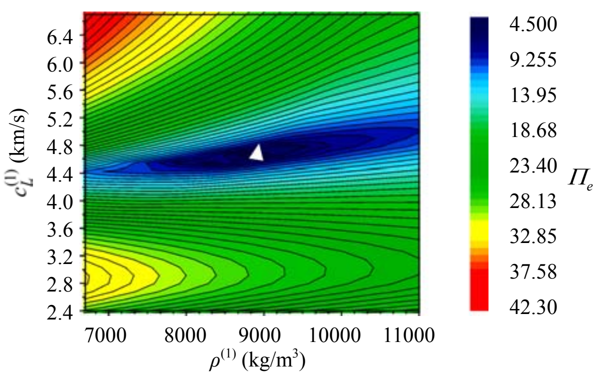

Fig.4 The dependence of the object function on the longitudinal wave velocity and the density of the coating layer with the transverse velocity fixed at $c_T^{(1)} = \hat{c}_T^{(1)}$

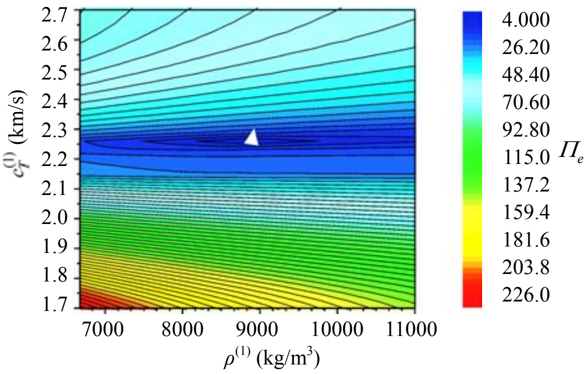

Fig.5 The dependence of the object function on the transverse wave velocity and the density of the coating layer with the longitudinal velocity fixed at $c_L^{(1)} = \hat{c}_L^{(1)}$

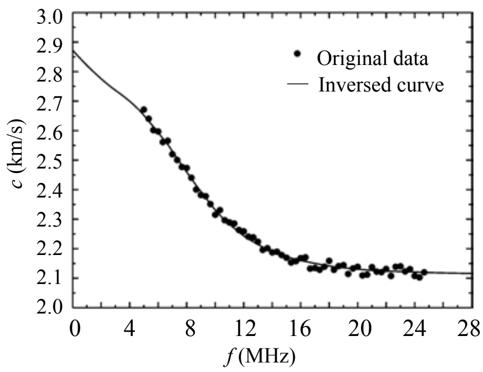

Fig.6 Comparison of the given and calculated dispersion

persion curve used in the inversion with the one calculated from the inversed results. It can be found that they agree well.

CONCLUSION

An analytical model was developed for surface waves propagating in a coated medium, and the dispersive equation was derived from the wave motion equations and the boundary conditions. It was found that the phase velocity dispersion is dependent on the elastic properties of the coating layer. An inversion algorithm is proposed for determining coating material parameters. It is worth noting that the objective function in the inversions scheme is established based on the dispersive equation. The inversed solutions of the material parameters, which make the objective function minimum, are considered to satisfy the dispersive equation. Therefore, the inversion algorithm is applicable to invert not only fundamental mode data, but also high mode data. In the numerical section, inversion results demonstrate that elastic properties of the coating layer can be successfully determined.

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