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Transmission line realization of subwavelength resonator formed by a pair of conventional and LHM slabs*

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Abstract: In this paper, the authors present the transmission line (TL) realization of one-dimensional subwavelength resonator formed by a pair of conventional right-handed material (RHM) and left-handed material (LHM). In such resonator, a novel resonant mode with the resonant frequency depending on the length ratio of the RH/LH TL sections occurs as a consequence of the full phase compensation due to the backward wave in the LH TL section. The theoretical circuit-model analyses are supported by simulation and experimental evidence on resonators with different RH/LH length ratios.

Key words: Transmission line (TL), Subwavelength resonator, Left-handed material (LHM), Phase compensation
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INTRODUCTION

Left-handed materials (LHMs), first investigated theoretically by Veselago (1968), have received substantial attention in the scientific and engineering communities. The unique properties of these metamaterials are promising for a diversity of microwave applications, such as new types of beam steerers, modulators, filters, superlens (Pendry, 2000), couplers and antenna radoms (Smith *et al*., 2004). Recently, a transmission line (TL) approach has been successfully introduced to realize LHMs (Caloz and Itoh, 2002; Eleftheriades *et al*., 2002), which provides an efficient design tool for LHM applications with comparatively broad bandwidth and low loss.

As a potential application of the LHMs, Engheta (2002) proposed the idea of a one-dimensional (1D) resonator formed by a pair of conventional and LHM slabs between two perfect conducting walls, and demonstrated theoretically that in such a cavity a resonant mode can still exist even when the thicknesses of the slabs are electrically very thin. Such novel resonant mode is analyzed under the ideal condition that the LHM is assumed to be a homogenous non-dispersive medium. The stability and quality factor of the resonance is restricted (Shen *et al*., 2004; He *et al*., 2005). For practical LHM which is always dispersive and dissipative, the subwavelength resonant mode should be revisited.

In this paper, we employ the TL approach to verify the model of subwavelength resonator proposed by Engheta. We choose TL structures for the realization because such TL is broadband with low loss and easy to realize experimentally compared with other resonant-based structures (Smith *et al*., 2000). Theoretical analysis based on an effective homogeneous and lossless TLs network reveals that novel resonant mode with the resonant frequency depending on the length ratio of the RH/LH TL sections exists as a consequence of the full phase compensation. We have properly designed the TL resonators with a coplanar waveguide (CPW) structure and the microwave circuit simulations using the Agilent's Advanced Design System (ADS) have been compared

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with measurement results. In the simulation and measurement results, where TL loss has been taken into consideration, the novel resonant modes are clearly observed, which demonstrate the phenomenon of full phase compensation in such practical subwavelength resonators.

THEORETICAL ANALYSIS

The TL versions of RHMs and LHMs were thoroughly investigated by Caloz and Itoh (2002). From electromagnetic (EM) theory and TL theory, RHMs can be realized by the conventional TLs, which can be charactered using a periodic LC structure with the unit cell equivalent circuit shown in Fig.1a, where d is the length of unit cell. LHMs can be realized using the conventional TLs with loaded series capacitors and shunt inductors (named as LH TLs), with the unit cell equivalent circuit as shown in Fig.1b.

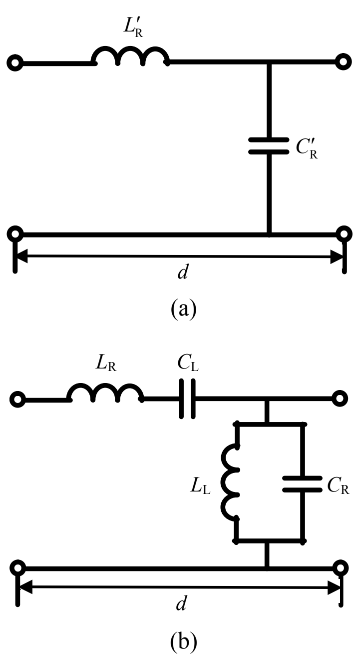


Fig.1 Equivalent circuit models of a unit cell. (a) RH TL; (b) LH TL

The 1D resonator based on TLs presented in Fig.2 consists of a segment of LH TL and a segment of RH TL short terminated at two ends similar to the two layers structure discussed in (Engheta, 2002). l_L and l_R are the lengths of the two sections.

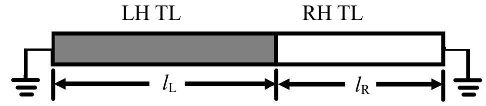


Fig.2 Ideal one-dimensional (1D) resonator based on TLs

The lengths of the unit cells of both the RH TL and LH TL structures are much less than the wavelength, so the TLs can be justifiably considered as effective media with nearly homogeneous wave propagation. By applying periodic boundary conditions (PBCs) related to the Bloch-Floquet theorem on TL unit cells, the propagation constant of RH TL can be given by ($\beta_R d << 1$) (Eleftheriades *et al*., 2002)

$$\beta_R = (\omega/d)\sqrt{L'_R C'_R}\ , \tag{1}$$

and the propagation constant of LH TL can be given by ($\beta_L d << 1$)

$$\beta_L = (\omega/d)\sqrt{(L_R - 1/(\omega^2 C_L))(C_R - 1/(\omega^2 L_L))}. \tag{2}$$

β_L becomes negative and the LH TL presents LHM properties when the operating frequency is lower than $\min(1/\sqrt{L_L C_R}, 1/\sqrt{L_R C_L})$. This should be considered in the design procedure.

The resonant condition of the resonator can be written as

$$\beta_L l_L + \beta_R l_R = n\pi, \tag{3}$$

here n can be positive, negative or zero, since the propagating constants in the two sections have opposite signs. However, if both sections are made of two RH TLs, n in Eq.(3) will remain positive. From Eqs.(1) and (2), we obtain

$$(\omega l_L/d)\sqrt{(L_R - 1/(\omega^2 C_L))(C_R - 1/(\omega^2 L_L))} + (\omega l_R/d)\sqrt{L'_R C'_R} = n\pi, \tag{4}$$

If n=0, it reduces to

$$[L_R C_R - L'_R C'_R d^2 (l_R/l_L)^2]\omega^4 - [L_R/L_L + C_R/C_L]\omega^2 + 1/(L_L C_L) = 0, \tag{5}$$

which means the resonant frequency only depends on the length ratio of the two segments. The total length of the TLs will not influence this particular resonant frequency, therefore the resonator could be made very small with the subwavelength dimension. This novel resonant mode has never been observed in conventional resonators. Due to the backward wave propagation in the LH TL section (Grbic and Eleftheriades, 2002), the wave traversing along this structure achieves full phase compensation in the RH and LH TL sections. Therefore, the resonant frequency does not depend on the total length of the resonator; rather it depends on the length ratio of the RH and LH TL sections. In principle, we can reduce the total length of the resonator to far less than the conventional $\lambda/2$ limitation.

PHYSICAL IMPLEMENTATION

The previous section presented the concept to create 1D subwavelength resonator by TL approach. However, the unit cells must be constructed with physical components that can realize the required RH/LH TL. Currently, the conventional TL can be implemented via microstrip, stripline, coplanar waveguide, or other structures. Surface-mount technology (SMT) chip components or distributed components (for example, the interdigital capacitor or spiral inductor) can be used to realize the loaded capacitors and inductors. The choice of SMT chip elements or distributed components depends on several factors. In terms of analysis and design, SMT component-based LH TL structures are generally easier to implement. SMT chip components are readily available and do not need to be designed and fabricated, unlike their distributed counterparts. However, SMT components are only available in discrete values and are limited to low frequency operation. As a result, specific phase responses and operation frequency ranges are limited for SMT-based LH TL structures. In addition, the choice between SMT chip elements and distributed components depends on the intended application. For example, SMT chip elements are not suitable to circuits for radiation-type applications.

Here we demonstrate a subwavelength resonator of coplanar waveguide loaded with SMT components. The topology of coplanar waveguide is suitable for mounting the SMT components. The structure of such a resonator is shown in Fig.3. Series capacitors are mounted on the gaps of the center conductor and stub inductors are symmetrically mounted on the gaps between the center conductor and the ground to realize LH TL. Two ends of the resonator are connected to the SMT coaxial connectors through the transition segments by coupling capacitors (C_c). This setup is prepared for measurement of the S parameters in a coaxial testing system.

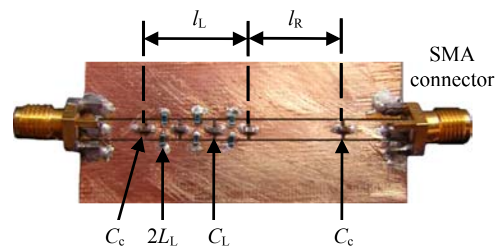
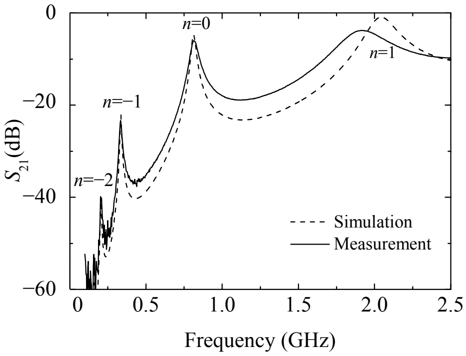


Fig.3 A coplanar waveguide resonator consisting of SMT capacitors and inductors

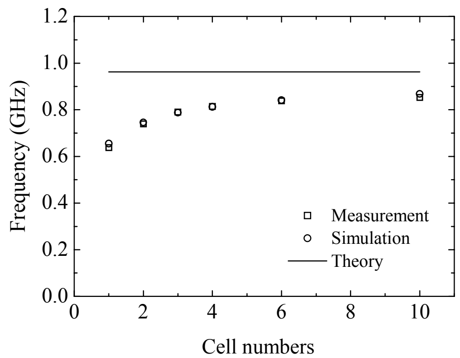
SIMULATION AND MEASUREMENT

Several resonators have been fabricated with different length ratios and different cell numbers. The entire circuit was implemented on an FR4 PCB board with dielectric constant ε_r=4.5 and thickness h=1.5 mm. We choose C_L=10 pF, $2L_L$=56 nH, C_c=0.5 pF. The length of the unit cell is d=5 mm. The spacing between the center conductor and the ground plane is 0.35 mm. The extracted LC parameters are C_R= C'_R=10 pF, L_R= L'_R=10 nH. Theoretical analysis of resonant frequency for the full phase compensation mode can be calculated from Eq.(5) if the length ratio of the two sections is determined. The equivalent circuit model is also simulated using the Agilent's ADS. The experimental results are compared with the theoretical results and circuit simulation results in Figs.4 and 5.

If the LH and RH sections of the resonator are composed of 4 unit cells, the resonant frequency for n=0 is obtained from Eq.(5) to be 962 MHz. The S_{21} parameter obtained for both the circuit simulation and the measurement are compared in Fig.4a. Several resonant peaks are observed for different values of n, and the two curves agree with each other quite well.



(a)



(b)

Fig.4 Results comparison when $l_R=l_L$. (a) S_{21} when each section has 4 cells; (b) Measured resonant frequency compared with simulation and theoretical results

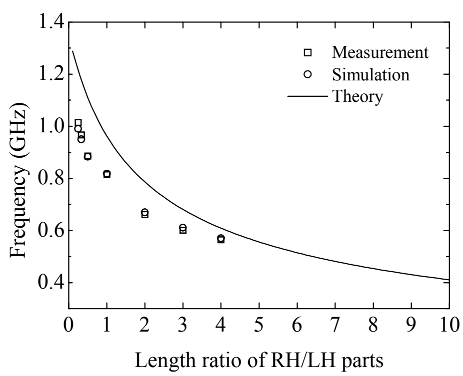


Fig.5 Relations between zeroth resonant frequency and the length ratio of the two sections

The resonance for n=0 occurs at about 812 MHz, which is a little bit lower (about 16% discrepancy) than the theoretical analysis. The main reason for the disagreement is that the propagation constants in Eqs.(1) and (2) were obtained under periodic boundary conditions, which are not satisfied strictly in the practical realization with finite unit cells. Fig.4b shows the relation between the cell numbers of each section and the zeroth (n=0) resonant frequency. When increasing the cell number, the practical resonance approaches to the ideal periodic boundary condition. This is clearly demonstrated in Fig.4b showing that both the simulated and the measured zeroth resonance frequency approach to the theoretical result when the cell number increased.

Fig.5 shows the zeroth resonant frequency for different length ratios of the RH/LH sections. The resonant frequency is determined by the length ratio and tends to reduce for large length ratio. Excellent agreement can be observed between circuit simulations and experimental results. Small discrepancies from the theoretical result were detected and had been explained previously.

Both the theoretical analysis and the experiment revealed that the novel resonant mode is determined by the length ratio of the RH/LH sections, and has little relation with the total length of the resonator. Therefore, such resonator could be made more compact with subwavelength dimension. For example, the resonator described in Fig.4 with 4 unit cells in both sections has a total subwavelength dimension of about 0.12λ.

CONCLUSION

A TL realization of 1D subwavelength resonator formed by LHM/RHM pair has been demonstrated. The novel resonant mode, which is based on the phase compensation theory, has been analyzed by circuit models, and verified by circuit simulation and experimental results. We have shown that the novel resonant mode for such a resonator only depends on the ratio of length of the RH/LH TL sections, not the sum of them. In other words, the resonator can be made small as long as the length ratio is fixed in the special dispersion relation leading to 1D subwavelength resonator. The TL approach enables simple design and implementation of such resonator. We believe the subwavelength resonator we discussed could lead to the development of novel types of compact and broadband microwave devices.

References

Caloz, C., Itoh, T., 2002. Application of the transmission line theory of left-handed (LH) materials to the realization of a microstrip LH transmission line. *IEEE-APS Int. Symp. Dig. Proc.*, **2**:412-415.

Eleftheriades, G.V., Iyer, A.K., Kremer, P.C., 2002. Planar negative refractive index media using periodically L-C loaded transmission lines. *IEEE Trans. Microwave Theory Tech.*, **50**:2702-2712. [doi:10.1109/TMTT.2002.805197]

Engheta, N., 2002. An idea for thin subwavelength cavity resonators using metamaterials with negative permittivity and permeability. *IEEE Antennas and Wireless Propagat. Lett.*, **1**:10-13. [doi:10.1109/LAWP.2002.802576]

Grbic, A., Eleftheriades, G.V., 2002. Experimental verification of backward-wave radiation from a negative refractive index metamaterial. *J. Appl. Phys.*, **92**:5930-5935. [doi:10.1063/1.1513194]

He, S.L., Jin, Y., Ruan, Z.C., Kuang, J.G., 2005. On subwavelength and open resonators involving metamaterials of negative refraction index. *New Journal of Physics*, **7**:210. [doi:10.1088/1367-2630/7/1/210]

Pendry, J.B., 2000. Negative refraction makes a perfect lens. *Phys. Rev. Lett.*, **85**:3966-3969. [doi:10.1103/PhysRevLett.85.3966]

Shen, L.F., He, S.L., Xiao, S.S., 2004. Stability and quality factor of a one-dimensional subwavelength cavity resonator containing a left-handed material. *Phys. Rev. B*, **69**:115111. [doi:10.1103/PhysRevB.69.115111]

Smith, D.R., Padilla, W.J., Vier, D.C., Nemat-Nasser, S.C., Schultz, S., 2000. Composite medium with simultaneously negative permeability and permittivity. *Phys. Rev. Lett.*, **84**:4184-4187. [doi:10.1103/PhysRevLett.84.4184]

Smith, D.R., Pendry, J.B., Wiltshire, M.C.K., 2004. Metamaterials and negative refractive index. *Science*, **305**:788-792. [doi:10.1126/science.1096796]

Veselago, V.G., 1968. The electrodynamics of substances with simultaneously negative values of ε and μ. *Soviet Physics Uspekhi*, **10**(4):509-514. [doi:10.1070/PU1968v010n04ABEH003699]