

# **Bragg reflectors based on alternate RHTL-LHTL structures**<sup>\*</sup>

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**Abstract:** New types of Bragg reflectors, multilayered periodic structures, based on alternating left-handed transmission line (LHTL) and right-handed transmission line (RHTL) are proposed. These new structures based on ideal microstrip TLs and L-C lumped elements, are designed and analyzed. We report on unusual narrow transmission bands in such kind of structures. In such multilayered structures both Bragg reflectance and the Fabry-Perot resonance exist and the phenomenon of unusual transmission is a result of competition between these two transmission effects, in which the Fabry-Perot resonance is dominant. According to our simulation results we find that this unusual transmission property exits no matter if the electrical length of the LHTL layer cancels the electrical length of the RHTL layer or not.

Key words: Bragg reflector, Left-handed transmission line (LHTL), Right-handed transmission line (RHTL), L-C lumped elements, Narrow bands

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#### INTRODUCTION

Left-handed materials (LHMs) with negative permittivity and negative permeability, first theoretically investigated by Veselago (1968), have attracted much interest in the past few years (Smith et al., 2000; Pendry, 2000). Such materials were first implemented using a structure which combined periodic arrays of metallic lines and split ring resonators (SRRs) (Shelby et al., 2001). Since then much work has been dedicated to fabricating such materials. These new artificial electromagnetic materials (metamaterials) exhibit interesting properties which can be used for e.g. subwavelength imaging (Engheta, 2002; Simovski et al., 2003). Left-handed (LH) structures formed by L-C lumped elements or L-C loaded microstrip transmission lines (TLs) have also been studied at microwave frequentcies both theoretically and experimentally (Eleftheriades et al., 2002; 2003; Lai et al., 2004). Such microwave im-

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plementations of LHMs offer many attractive features, such as large bandwidths of LH operation and very low transmission loss.

Transmittance and reflectance of 1D Bragg reflector structures consisting of alternate LHM and right-handed material (RHM) layers were theoretically investigated (Gerardin and Lakhtakia, 2002; Wu *et al.*, 2003). In these studies the effects of a reflective Bragg region and Fabry-Perot resonance were observed. In the present paper we study the phenomenon of unusual narrow transmission bands produced by RHTL-LHTL multilayered Bragg reflectors. We study reflectors based on both microstrip TLs and L-C lumped elements.

#### TL BRAGG REFLECTORS

A distributed Bragg reflector (DBR) consisting of a stack of alternate layers of materials having different index of refraction (Fig.1) exhibits very high reflectance in the Bragg region. A very flat-toped Bragg region can be attained if the condition in Eq.(1) is satisfied,

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$$n_1 d_1 \cos \theta_1 + n_2 d_2 \cos \theta_2 = p \lambda/2, p = \pm 1, \pm 2, \dots$$
 (1)

However, in case the Fabry-Perot resonating condition is also satisfied this resonance will instead be dominant. In this case the Bragg region will be deteriorated and sometimes disappears altogether. The Fabry-Perot resonating condition can be written as

$$n_2 d_2 \cos \theta_2 = q\lambda/2, q = \pm 1, \pm 2, \cdots$$
 (2)

Note that for the normal incidence case, both  $\theta_1$  and  $\theta_2$  are 0.



Fig.1 Structure of an RHM-LHM multilayered Bragg reflector

To investigate the unusual transmission property of multilayered periodic structures containing LHMs (Wu *et al.*, 2003), we proposed a periodic RHTL-LHTL structure to form a Bragg reflector. Fig.2 depicts the two kinds of (lossless) TL units. The voltage



Fig.2 Transmission line units: (a) Right-handed transmission line (RHTL) unit and (b) Left-handed transmission line (LHTL) unit

wave propagation equation of the two kinds of TL units can be written as

$$\partial^2 u / \partial z^2 + \gamma^2 u = 0.$$
 (3)

The propagation constant  $\gamma$  is given by

$$\gamma = j\beta = \sqrt{ZY} , \qquad (4)$$

where the per-unit length impedance Z and the per-unit length admittance Y are defined by

$$Z_{\rm R}(\omega) = j\omega L_{\rm R}\Delta z, \ Y_{\rm R}(\omega) = j\omega C_{\rm R}\Delta z;$$
  

$$Z_{\rm L}(\omega) = -j\Delta z / (\omega C_{\rm L}), \ Y_{\rm L}(\omega) = -j\Delta z / (\omega L_{\rm L}).$$
(5)

From Eqs.(3)~(5), permittivity  $\varepsilon$ , permeability  $\mu$  and wave impedance  $\eta$  can be written as

$$\varepsilon_{\rm R} = L_{\rm R} \Delta z, \ \mu_{\rm R} = C_{\rm R} \Delta z, \ \eta_{\rm R} = \sqrt{L_{\rm R} / C_{\rm R}};$$
  

$$\varepsilon_{\rm L} = -\Delta z / (\omega^2 C_{\rm L}), \ \mu_{\rm L} = -\Delta z / (\omega^2 L_{\rm L}), \ \eta_L = \sqrt{L_{\rm L} / C_{\rm L}}.$$
(6)

## SIMULATION AND ANALYSIS

We consider a Bragg reflector consisting of a multilayered periodic structure based on alternate ideal microstrip RHTL and LHTL layers (Fig.1). The reflector consists of 40 layers in total. Using Eqs.(1) and (2), the electrical lengths (*EL*) of the RHTL and LHTL layers are designed to be half of the reference wavelength ( $\lambda_0/2$ ). At the reference frequency 2.0 GHz, the phase delay will be  $\pi$  and  $-\pi$  for the RHTL and LHTL layers respectively when the voltage wave propagates through the two different layers. This ensures that both the Bragg condition and Fabry-Perot resonating condition are satisfied at the reference frequency. To simplify the calculation, the wave impedances  $\eta$  of the RHTL and LHTL layers are chosen to be 1  $\Omega$  and 2  $\Omega$  respectively.

In Fig.3, Agilent ADS circuit simulation results are shown for the reflectance of the studied reflector. Comparing Figs.3a and 3b, one can clearly see the unusual narrow bands of the RHTL-LHTL based Bragg reflector as opposed to the widespread oscillation in the dual RHTL reflector. This can be explained as follows: If the constitutive layers consist of dual RHTLs (or LHTLs) and both the Bragg condition and the Fabry-Perot condition are satisfied, the transmission will be very large in a comparatively wide band centered at the Fabry-Perot resonating frequency. The Fabry-Perot resonating condition will dominate and thus the transmission will be still large as the frequency shifts away from the center frequency which is why the widespread oscillation is found. However, if the constitutive layers consist of alternate RHTLs and LHTLs, the Bragg condition dominates and the transmission will be small as the frequency shifts away from the centre frequency which is the reason for the unusual narrow transmission bands.



Fig.3 The reflectance as a function of frequency for a DBR consisting of ideal microstrip RHTL and LHTL layers (totally 40 layers). (a) For the case of RHTL-LHTL period with  $EL_{R}=\pi$ ,  $\eta_{R}=1$   $\Omega$  and  $EL_{L}=-\pi$ ,  $\eta_{L}=2$   $\Omega$ ; (b) For the case of dual RHTL period with  $EL_{R1}=\pi$ ,  $\eta_{R1}=1$   $\Omega$  and  $EL_{R2}=\pi$ ,  $\eta_{R2}=1$   $\Omega$ 

A practical LHTL can be implemented using the L-C lumped-element units as discussed in connection with Fig.2b. Therefore, a DBR can be easily fabricated by alternating microstrip RHTL layers and L-C lumped-element LHTL layers (Fig.4a). The microstrip RHTL layers are equivalent to the lumped elements of Fig.2a. By carefully choosing the unit number ( $N_L$ ) and L-C value ( $L_L$ ,  $C_L$ ) of the LHTL layers, both the Bragg condition and Fabry-Perot

resonating condition can be satisfied. This is achieved with  $N_L=10$ ,  $L_L\Delta z'=2.0264$  nH,  $C_L\Delta z'=0.5066$  nF. ADS circuit simulation results on this structure are shown in Fig.4b showing that the edge of the narrow transmission bands is not as steep as the ideal microstrip mode and that side lobes are found around the transmission bands. We attribute this to the fact that the alternate-layered structure is based on two different circuit modes, i.e., that the RHTL layer is based on a continuous TL whereas the LHTL layer is based on discrete L-C lumped elements.



Fig.4 (a) Circuit diagram of the alternate-layered structure; (b) The reflectance as a function of frequency for a DBR consisting of ideal microstrip RHTL and lumped element LHTL layers (40 layers in all), with  $EL_R=\pi$ ,  $\eta_R=1$  $\Omega$  and  $EL_L=-\pi$ ,  $\eta_L=2$   $\Omega$ 

In the two designs discussed above we use ideal microstrip TLs as RHMs. Since the L-C lumpedelement RHTL is also very simple to implement, we also analyse a design consisting of alternate RHTL and LHTL layers both based on L-C lumped element. The electrical lengths of RHTL and LHTL layers are designed to be  $\pi$  and  $-2\pi$  respectively so that  $n_1d_1+n_2d_2\neq 0$ . Calculations yield suitable parameters to be  $N_{\rm R}$ =50,  $L_{\rm R}\Delta z$ =0.1 nH,  $C_{\rm R}\Delta z$ =0.25 pF and  $N_{\rm L}$ =10,  $L_{\rm L}\Delta z'$ =0.5066 nH,  $C_{\rm L}\Delta z'$ = 0.5066 nF. Fig.5a depicts the circuit of the alternate-layered structure and ADS circuit simulation result is shown in Fig.5b. The results indicate that unusual narrow transmission bands are retained even though the electrical length of the LHTL layer does not cancel the electrical length of the RHTL layer as in the ideal case.



Fig.5 (a) The circuit diagram of the alternate-layered structure; (b) The reflectance as a function of frequency for a DBR consisting of L-C lumped-element RHTL (period with  $EL_R=\pi$ ,  $\eta_R=20 \Omega$ , 1 layer of 50 units) and LHTL layers (period with  $EL_L=-2\pi$ ,  $\eta_L=1 \Omega$ , 1 layer of 10 units)

## SUMMARY

New types of Bragg reflectors with multi layered periodic structure based on alternate LHTL and RHTL have been proposed. The new structures, based on the ideal microstrip TLs and the L-C lumped elements, are both designed and analyzed. Simulation results demonstrate that unusual narrow transmission bands can be attained in these kinds of structures. It was also shown that this unusual transmission property exists regardless of whether the electrical length of the LHTL layer cancels the electrical length of the RHTL layer or not making practical implementation feasible.

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