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Three dimensional free convection couette flow with transpiration cooling

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Abstract: Free convection flow between two vertical parallel plates with transverse sinusoidal injection of the fluid at the stationary plate and its corresponding removal by constant suction through the plate in uniform motion has been analyzed. Due to this type of injection velocity, the flow becomes three-dimensional. Analytical expressions for the velocity, temperature, skin friction and rate of heat transfer were obtained. The important characteristics of the problem, namely the skin friction and the rate of heat transfer are discussed in detail with the help of graphs.

Key words: Couette flow, Free convection, Three-dimensional flow, Transpiration cooling, Transverse sinusoidal injection
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INTRODUCTION

In recent years, the subject of free convection flow has attracted the attention of a number of scholars because of its possible applications to several geophysical problems. In view of these applications, a series of investigations were made to study the flow past a vertical wall.

Transpiration cooling can very effectively protect certain structural elements in turbojet and rocket engines, like combustion chamber walls, exhaust nozzles or gas turbine blades from hot gases. Eckert and Drake (1958) and Jain and Bansal (1973) described the reduction of heat transfer in couette flow for the case of an incompressible fluid by injecting the fluid into the flow field from the stationary plate and corresponding removable of heat from the moving plate. The problem is two dimensional due to the uniform injection and suction applied at the porous plates. Flow and heat transfer along a plane wall with periodic suction velocity was studied by Gersten and Gross (1974). Effects of such a suction velocity on various flow and heat transfer problems along horizontal and vertical plates were also been studied extensively by Singh *et al.*(1988), Ahmed and Sharma (1997), Chaudhary and Chand (2002), Singh and Sharma (2002), and Gehlot and Tak (2002).

However, the transverse sinusoidal injection or suction velocity in the problem of transpiration cooling has not attracted much attention. Singh (1999) studied three dimensional couette flow with transpiration cooling. In geothermal region a situation may arise when slip of particles at the boundary may occur. Keeping this in mind, Gupta and Goyal (1995), Jothimani and Anjali Devi (2001), Jain and Taneja (2002) and others have solved their problems considering first order velocity slip conditions (Street, 1960).

This work is aimed at studying the effects of transverse sinusoidal injection velocity distribution on the free convective flow of a viscous incompressible fluid in slip flow regime under the influence of heat source. It was observed that when slip parameter (h) is increased the skin friction along the main flow (τ_x) increases up to the middle of the channel and after that it decreases, and that increase in the heat source strength parameter (α) increases Nusselt number (Nu).

FORMULATION OF THE PROBLEM

The couette flow of a viscous incompressible fluid between two parallel flat plates is shown in Fig.1. One of the plates has transverse sinusoidal injection velocity distribution of the form

$$v^*(z)=V_0\left(1+\varepsilon\frac{\cos\pi z^*}{d}\right), \tag{1}$$

where ε is a positive constant quantity (<<1) and another plate is in uniform motion U_0 and subjected to constant suction and slip boundary conditions.

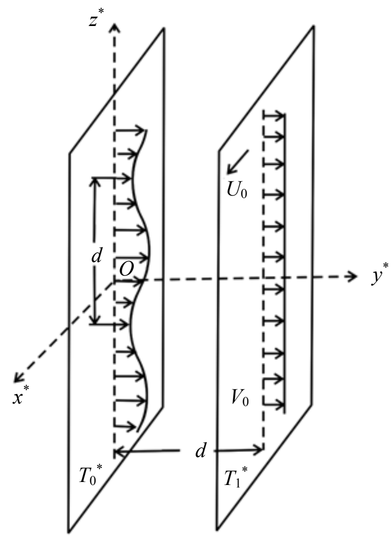


Fig.1 Couette flow with periodic injection and constant suction

Without loss of generality the distance d between the plates is taken equal to the wavelength of the injection velocity. The plates are assumed to be at constant temperature T_0^* and T_1^*, respectively. All the physical quantities are independent of x^* for this problem of fully developed laminar flow but the flow remains three-dimensional due to the injection velocity Eq.(1). Denoting the velocity components u^*, v^*, w^* in the x^*, y^*, z^* directions respectively and the temperature by T^*, the problem is governed by the following equations:

$$\frac{\partial v^*}{\partial y^*}+\frac{\partial w^*}{\partial z^*}=0, \tag{2}$$

$$v^*\frac{\partial u^*}{\partial y^*}+w^*\frac{\partial u^*}{\partial z^*}=g\beta(T^*-T_e^*)+\upsilon\left(\frac{\partial^2 u^*}{\partial y^{*2}}+\frac{\partial^2 u^*}{\partial z^{*2}}\right), \tag{3}$$

$$v^*\frac{\partial v^*}{\partial y^*}+w^*\frac{\partial v^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\upsilon\left(\frac{\partial^2 v^*}{\partial y^{*2}}+\frac{\partial^2 v^*}{\partial z^{*2}}\right), \tag{4}$$

$$v^*\frac{\partial w^*}{\partial y^*}+w^*\frac{\partial w^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}+\upsilon\left(\frac{\partial^2 w^*}{\partial y^{*2}}+\frac{\partial^2 w^*}{\partial z^{*2}}\right), \tag{5}$$

$$v^*\frac{\partial T^*}{\partial y^*}+w^*\frac{\partial T^*}{\partial z^*}=\frac{k}{\rho C_p}\left(\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{\partial^2 T^*}{\partial z^{*2}}\right)+\frac{Q}{\rho C_p}(T^*-T_e^*), \tag{6}$$

where g is the acceleration due to gravity, β is the coefficient of volume expansion. The last term on the right hand side of Eq.(6) denotes the heat generation varying directly with the temperature difference. T_e^* is the equilibrium temperature and remaining symbols have their usual meanings.

The boundary conditions relevant to the problem are taken as:

$$\left.\begin{aligned} &y^*=0:\ u^*=0,\ v^*=V_0\left(1+\varepsilon\frac{\cos\pi z^*}{d}\right),\ w^*=0,\ T^*=T_0^*\\ &y^*=d:\ u^*=U_0+L_1\frac{\partial u^*}{\partial y^*},\ v^*=V_0,\quad w^*=0,\ T^*=T_1^* \end{aligned}\right\} \tag{7}$$

where $L_1=((2-m_1)/m_1)L$, L being mean free path and m_1 the Maxwell's reflection coefficient.

Introducing the following non-dimensional quantities

$$y=y^*/d,\ z=z^*/d,\ u=u^*/U_0,\ v=v^*/V_0,$$

$$w=\frac{w^*}{V_0},\ p=\frac{p^*}{\rho V_0^2},\ \theta=\frac{T^*-T_e^*}{T_0^*-T_e^*},\ m=\frac{T_1^*-T_e^*}{T_0^*-T_e^*},$$

$P=\mu C_p/k$ (The Prandtl number),

$G=\dfrac{g\beta d(T_0^*-T_e^*)}{V_0U_0}$ (The Grashof number),

$\alpha=Qd\upsilon/V_0k$ (The heat source parameter),

$h=L_1/d$ (The slip parameter),

$\lambda=V_0d/\upsilon$ (The suction parameter),

and get the equations as:

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{8}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=G\theta+\frac{1}{\lambda}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right), \tag{9}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{\lambda}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right), \tag{10}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{\lambda}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right), \tag{11}$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{P\lambda}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)+\frac{\alpha\theta}{P}, \tag{12}$$

The corresponding boundary conditions become:

$$\left.\begin{array}{l} y=0:\ u=0,\ v(z)=1+\varepsilon\cos\pi z,\ w=0,\ \theta=1 \\ y=1:\ u=1+h\dfrac{\partial u}{\partial y},\quad v=1,\quad w=0,\ \theta=m \end{array}\right\}. \tag{13}$$

SOLUTION OF THE PROBLEM

Since the amplitude of the injection velocity ε (<<1) is very small, we now assume the solution in the following form:

$$f(y,z)=f_0(y)+\varepsilon f_1(y,z)+\varepsilon^2 f_2(y,z)+... \tag{14}$$

where f stands for u, v, w, p and θ. When ε=0 the problem is reduced to the well known 2D flow with constant injection and suction at both plates and are governed by the following equations:

$$\frac{\partial v_0}{\partial y}=0, \tag{15}$$

$$v_0\frac{\partial u_0}{\partial y}=G\theta_0+\frac{1}{\lambda}\frac{\partial^2 u_0}{\partial y^2}, \tag{16}$$

$$v_0\frac{\partial \theta_0}{\partial y}=\frac{1}{P\lambda}\frac{\partial^2 \theta_0}{\partial y^2}+\frac{\alpha\theta_0}{P}, \tag{17}$$

with boundary conditions:

$$\left.\begin{array}{l} y=0:\ u_0=0,\qquad v_0=1,\ \ w_0=0,\ \ \theta_0=1 \\ y=1:\ u_0=1+h\dfrac{\partial u_0}{\partial y},\ v_0=1,\ w_0=0,\ \theta_0=m \end{array}\right\}. \tag{18}$$

The solution of this two dimensional problem is

$$u_0=C_3+C_4\mathrm{e}^{\lambda y}+Z_1\mathrm{e}^{X_{11}y}+Z_2\mathrm{e}^{X_{22}y}, \tag{19}$$

$$\theta_0=C_1\mathrm{e}^{X_{11}y}+C_2\mathrm{e}^{X_{22}y}, \tag{20}$$

with

$$v_0=1,\ w_0=0,\ \ p_0=\text{constant}, \tag{21}$$

where

$$C_1=1-C_2,\ \ C_2=\frac{\mathrm{e}^{X_{11}}-m}{\mathrm{e}^{X_{11}}-\mathrm{e}^{X_{22}}},\ \ C_3=-(C_4+Z_1+Z_2),$$

$$C_4=\frac{1-Z_1[\mathrm{e}^{X_{11}}(1-hX_{11})-1]-Z_2[\mathrm{e}^{X_{22}}(1-hX_{22})-1]}{[\mathrm{e}^{\lambda}(1-h\lambda)-1]},$$

$$Z_1=\frac{-\lambda GC_1}{X_{11}(X_{11}-\lambda)},\ \ Z_2=\frac{-\lambda GC_2}{X_{22}(X_{22}-\lambda)},$$

$$X_{11}=\frac{1}{2}\left[\lambda P+\sqrt{\lambda^2P^2-4\lambda\alpha}\right],$$

$$X_{22}=\frac{1}{2}\left[\lambda P-\sqrt{\lambda^2P^2-4\lambda\alpha}\right].$$

When $\varepsilon\neq 0$, substituting Eq.(14) in Eqs.(8)~(12) and comparing the coefficients of identical powers of ε, neglecting those of ε^2, ε^3, etc., the following first order equations are obtained with the help of Eqs.(19)~(21):

$$\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}=0, \tag{22}$$

$$\frac{\partial u_1}{\partial y}+v_1\frac{\partial u_0}{\partial y}=G\theta_1+\frac{1}{\lambda}\left(\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial^2 u_1}{\partial z^2}\right), \tag{23}$$

$$\frac{\partial v_1}{\partial y}=-\frac{\partial p_1}{\partial y}+\frac{1}{\lambda}\left(\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial z^2}\right), \tag{24}$$

$$\frac{\partial w_1}{\partial y}=-\frac{\partial p_1}{\partial z}+\frac{1}{\lambda}\left(\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial z^2}\right), \tag{25}$$

$$\frac{\partial \theta_1}{\partial y}+v_1\frac{\partial \theta_0}{\partial y}=\frac{1}{P\lambda}\left(\frac{\partial^2 \theta_1}{\partial y^2}+\frac{\partial^2 \theta_1}{\partial z^2}\right)+\frac{\theta_1\alpha}{P}, \tag{26}$$

with boundary conditions:

$$\left.\begin{array}{l} y=0:\ u_1=0,\quad v_1=\cos\pi z,\ \ w_1=0,\ \ \theta_1=0 \\ y=1:\ u_1=h\dfrac{\partial u_1}{\partial y},\quad v_1=0,\ w_1=0,\ \ \theta_1=0 \end{array}\right\}. \tag{27}$$

These are the linear partial differential equations describing the three-dimensional cross flow.

In order to solve these equations, we shall first consider Eqs.(22), (24) and (25) for cross flow, being independent of the main flow component u_1 and the temperature field θ_1.

We assume v_1, w_1, and p_1 in the following manner:

$$v_1(y,z)=v_{11}(y)\cos\pi z, \tag{28}$$

$$w_1(y,z)=-\{v_{11}{}'(y)\sin\pi z\}/\pi, \tag{29}$$

$$p_1(y,z) = p_{11}(y)\cos \pi z\,, \tag{30}$$

where a prime denotes differentiation with respect to y. Eqs.(28) and (29) have been so chosen that the continuity Eq.(22) is satisfied. Substituting these equations into Eqs.(24), (25) and applying the transformed boundary conditions, we get the solutions of v_1, w_1, and p_1 as

$$v_1(y,z) = [A_3 e^{X_1 y} + A_4 e^{X_2 y} - A_1 e^{\pi y} - A_2 e^{-\pi y}]\cos \pi z\,, \tag{31}$$

$$w_1(y,z) = -\frac{1}{\pi}[A_3 X_1 e^{X_1 y} + A_4 X_2 e^{X_2 y} - A_1 \pi e^{\pi y} + A_2 \pi e^{-\pi y}]\sin \pi z, \tag{32}$$

$$p_1(y,z) = [A_1 e^{\pi y} + A_2 e^{-\pi y}]\cos \pi z\,, \tag{33}$$

where

$$A_1 = \frac{b_8 - (b_9 + b_7)A_2}{b_5 - b_6},$$

$$A_2 = \frac{b_4(b_5 - b_6) - b_8(b_0 - b_2)}{\left[(b_1 - b_3)(b_5 - b_6) - (b_0 - b_2)(b_9 + b_7)\right]},$$

$$A_3 = \frac{-\left[X_2 + A_1(X_2 - \pi) + A_2(X_2 + \pi)\right]}{(X_1 - X_2)},$$

$$A_4 = 1 - A_3 + A_1 + A_2\,,$$

$$b_0 = -e^{X_1}(X_2 - \pi) + e^{X_2}(X_1 - \pi)\,,$$

$$b_1 = -e^{X_1}(X_2 + \pi) + e^{X_2}(X_1 + \pi)\,,$$

$$b_2 = e^{\pi}(X_1 - X_2),\ \ b_3 = e^{-\pi}(X_1 - X_2)\,,$$

$$b_4 = X_2 e^{X_1} - X_1 e^{X_2}\,,$$

$$b_5 = -e^{X_1}X_1(X_2 - \pi) + e^{X_2}X_2(X_1 - \pi)\,,$$

$$b_6 = \pi e^{\pi}(X_1 - X_2),\ \ b_7 = \pi e^{-\pi}(X_1 - X_2)\,,$$

$$b_8 = (e^{X_1} - e^{X_2})X_2 X_1\,,$$

$$b_9 = -X_1 e^{X_1}(X_2 + \pi) + X_2 e^{X_2}(X_1 + \pi)\,,$$

$$X_1 = \frac{1}{2}\left[\lambda + \sqrt{\lambda^2 + 4\pi^2}\right],\ \ X_2 = \frac{1}{2}\left[\lambda - \sqrt{\lambda^2 + 4\pi^2}\right].$$

In order to solve the differential equations for u_1 and θ_1, we assume

$$u_1(y,z) = u_{11}(y)\cos \pi z, \tag{34}$$

$$\theta_1(y,z) = \theta_{11}(y)\cos \pi z, \tag{35}$$

and substitution of Eqs.(34) and (35) into the partial differential Eqs.(23) and (26) reduce them to the following ordinary differential equations as

$$\frac{\partial^2 u_{11}}{\partial y^2} - \lambda\frac{\partial u_{11}}{\partial y} - \pi^2 u_{11} = v_{11}\lambda\frac{\partial u_0}{\partial y} - \lambda G\theta_{11}\,, \tag{36}$$

$$\frac{\partial^2 \theta_{11}}{\partial y^2} - \lambda P\frac{\partial \theta_{11}}{\partial y} - (\lambda\alpha - \pi^2)\theta_{11} = v_{11}\lambda P\frac{\partial \theta_0}{\partial y}\,, \tag{37}$$

with the corresponding boundary conditions:

$$\left.\begin{aligned} &y=0:\ u_{11}=0, && \theta_{11}=0\\ &y=1:\ u_{11}=h_1\frac{\partial u_{11}}{\partial y}, && \theta_{11}=0\end{aligned}\right\}. \tag{38}$$

Substitution of u_{11} and θ_{11} obtained from Eqs.(36) and (37) under boundary conditions Eq.(38) in Eqs.(34) and (35), we get

$$\begin{aligned} u_1(y,z) = [&A_7 e^{X_1 y} + A_8 e^{X_2 y} + h_9 e^{(X_1+\lambda)y} + h_{10}e^{(X_1+X_{11})y}\\ &+ h_{11}e^{(X_1+X_{22})y} + h_{12}e^{(X_2+\lambda)y} + h_{13}e^{(X_2+X_{11})y} + h_{14}e^{(X_2+X_{22})y}\\ &- h_{15}e^{(\lambda-\pi)y} - h_{16}e^{(X_{11}-\pi)y} - h_{17}e^{(X_{22}-\pi)y} - h_{18}e^{(\lambda+\pi)y}\\ &- h_{19}e^{(X_{11}+\pi)y} - h_{20}e^{(X_{22}+\pi)y} - h_{21}e^{s_1 y} - h_{22}e^{s_2 y}]\cos \pi z, \end{aligned} \tag{39}$$

$$\begin{aligned} \theta_1(y,z) = [&A_5 e^{s_1 y} + A_6 e^{s_2 y} + h_1 e^{(X_1+X_{11})y} + h_2 e^{(X_2+X_{11})y}\\ &+ h_3 e^{-(\pi-X_{11})y} - h_4 e^{(\pi+X_{11})y} + h_5 e^{(X_1+X_{22})y} + h_6 e^{(X_2+X_{22})y}\\ &- h_7 e^{-(\pi-X_{22})y} - h_8 e^{(\pi+X_{22})y}]\cos \pi z, \end{aligned} \tag{40}$$

where

$$s_1 = \frac{1}{2}\left[\lambda P + \sqrt{\lambda^2 P^2 - 4(\lambda\alpha - \pi^2)}\right],$$

$$s_2 = \frac{1}{2}\left[\lambda P - \sqrt{\lambda^2 P^2 - 4(\lambda\alpha - \pi^2)}\right],$$

$$A_5 = -A_6 - h_1 - h_2 - h_3 + h_4 - h_5 - h_6 + h_7 + h_8\,,$$

$$\begin{aligned} A_6 = \frac{1}{(e^{s_1} - e^{s_2})}\Big[&h_1\{e^{(X_1+X_{11})} - e^{s_1}\} + h_2\{e^{(X_2+X_{11})} - e^{s_1}\}\\ &+ h_3\{e^{s_1} - e^{-(\pi-X_{11})}\} + h_4\{e^{s_1} - e^{(\pi+X_{11})}\} + h_5\{e^{(X_1+X_{22})} - e^{s_1}\}\\ &+ h_6\{e^{(X_2+X_{22})} - e^{s_1}\} + h_7\{e^{s_1} - e^{-(\pi-X_{22})}\} + h_8\{e^{s_1} - e^{(\pi+X_{22})}\}\Big], \end{aligned}$$

$$A_7 = \frac{h_{23} + h_{24}(1 - hX_2)e^{X_2}}{[(1 - hX_1)e^{X_1} - (1 - hX_2)e^{X_2}]},$$

$$\begin{aligned} A_8 = -A_7 - (&h_9 + h_{10} + h_{11} + h_{13} + h_{14} + h_{15} + h_{16}\\ &+ h_{17} - h_{18} - h_{19} - h_{20} - h_{21} - h_{22}), \end{aligned}$$

$$h_1 = \frac{\lambda P A_3 C_1 X_{11}}{[(X_1 + X_{11})^2 - \lambda P(X_1 + X_{11}) + (\lambda\alpha - \pi^2)]},$$

$$h_2 = \frac{\lambda P A_4 C_1 X_{11}}{[(X_2 + X_{11})^2 - \lambda P(X_2 + X_{11}) + (\lambda\alpha - \pi^2)]},$$

$$h_3 = \frac{\lambda P A_2 C_1 X_{11}}{[(\pi - X_{11})^2 - \lambda P(\pi - X_{11}) + (\lambda\alpha - \pi^2)]},$$

$$h_4=\frac{\lambda PA_1C_1X_{11}}{[(\pi+X_{11})^2-\lambda P(\pi+X_{11})+(\lambda\alpha-\pi^2)]},$$
$$h_5=\frac{\lambda PA_3C_2X_{22}}{[(X_1+X_{22})^2-\lambda P(X_1+X_{22})+(\lambda\alpha-\pi^2)]},$$
$$h_6=\frac{\lambda PA_4C_2X_{22}}{[(X_2+X_{22})^2-\lambda P(X_2+X_{22})+(\lambda\alpha-\pi^2)]},$$
$$h_7=\frac{\lambda PA_2C_2X_{22}}{[(\pi-X_{22})^2+\lambda P(\pi-X_{22})+(\lambda\alpha-\pi^2)]},$$
$$h_8=\frac{\lambda PA_1C_2X_{22}}{[(\pi+X_{22})^2-\lambda P(\pi+X_{22})+(\lambda\alpha-\pi^2)]},$$
$$h_9=\frac{\lambda^2A_3C_4}{[(X_1+\lambda)^2-\lambda(X_1+\lambda)-\pi^2]},$$
$$h_{10}=\frac{\lambda[Z_1A_3X_{11}-Gh_1]}{[(X_1+X_{11})^2-\lambda(X_1+X_{11})-\pi^2]},$$
$$h_{11}=\frac{\lambda[Z_2A_3X_{22}-Gh_5]}{[(X_1+X_{22})^2-\lambda(X_1+X_{22})-\pi^2]},$$
$$h_{12}=\frac{\lambda^2A_4C_4}{[(X_2+\lambda)^2-\lambda(X_2+\lambda)-\pi^2]},$$
$$h_{13}=\frac{\lambda[Z_1A_4X_{11}-Gh_2]}{[(X_2+X_{11})^2-\lambda(X_2+X_{11})-\pi^2]},$$
$$h_{14}=\frac{\lambda[Z_2A_4X_{22}-Gh_6]}{[(X_2+X_{22})^2-\lambda(X_2+X_{22})-\pi^2]},$$
$$h_{15}=\frac{\lambda^2A_2C_4}{[(\lambda-\pi)^2-\lambda(\lambda-\pi)-\pi^2]},$$
$$h_{16}=\frac{\lambda[Z_1A_2X_{11}-Gh_3]}{[(\pi-X_{11})^2+\lambda(\pi-X_{11})-\pi^2]},$$
$$h_{17}=\frac{\lambda[Z_2A_2X_{22}-Gh_7]}{[(X_{22}-\pi)^2-\lambda(X_{22}-\pi)-\pi^2]},$$
$$h_{18}=\frac{\lambda^2A_1C_4}{[(\lambda+\pi)^2-\lambda(\lambda+\pi)-\pi^2]},$$
$$h_{19}=\frac{\lambda[Z_1A_1X_{11}-Gh_4]}{[(X_{11}+\pi)^2-\lambda(X_{11}+\pi)-\pi^2]},$$
$$h_{20}=\frac{\lambda[Z_2A_1X_{22}-Gh_8]}{[(X_{22}+\pi)^2-\lambda(X_{22}+\pi)-\pi^2]},$$
$$h_{21}=\frac{\lambda GA_5}{[s_1^2-\lambda s_1-\pi^2]},\quad h_{22}=\frac{\lambda GA_6}{[s_2^2-\lambda s_2-\pi^2]},$$
$$\begin{aligned}h_{23}&=h_9e^{(X_1+\lambda)}\{h_0(X_1+\lambda)-1\}+h_{10}e^{(X_1+X_{11})}\{h_0(X_1\\&+X_{11})-1\}+h_{11}e^{(X_1+X_{22})}\{h_0(X_1+X_{22})-1\}+h_{12}e^{(X_2+\lambda)}\\&\times\{h_0(X_2+\lambda)-1\}+h_{13}e^{(X_2+X_{11})}\{h_0(X_2+X_{11})-1\}\\&+h_{14}e^{(X_2+X_{22})}\{h_0(X_2+X_{22})-1\}-h_{15}e^{(\lambda-\pi)}\{h_0(\lambda-\pi)\\&-1\}-h_{16}e^{(X_{11}-\pi)}\{h_0(X_{11}-\pi)-1\}-h_{17}e^{(X_{22}-\pi)}\{h_0(X_{22}\\&-\pi)-1\}-h_{18}e^{(\lambda+\pi)}\{h_0(\lambda+\pi)-1\}-h_{19}e^{(X_{11}+\pi)}\{h_0(X_{11}+\pi)\\&-1\}-h_{20}e^{(X_{22}+\pi)}\{h_0(X_{22}+\pi)-1\}-h_{21}e^{s_1}\{h_0s_1-1\}\\&-h_{22}e^{s_2}\{h_0s_2-1\},\end{aligned}$$
$$\begin{aligned}h_{24}&=h_9+h_{10}+h_{11}+h_{12}+h_{13}+h_{14}-h_{15}-h_{16}\\&-h_{17}-h_{18}-h_{19}-h_{20}-h_{21}-h_{22}.\end{aligned}$$

Now after knowing the velocity field, we can calculate the skin friction components τ_x and τ_z in the main flow and transverse directions, respectively as

$$\begin{aligned}\tau_x&=\frac{d\tau_x^*}{\mu U_0}=\left(\frac{du_0}{dy}\right)_{y=0}+\varepsilon\left(\frac{du_{11}}{dy}\right)_{y=0}\cos\pi z\\&=(C_4\lambda+Z_1X_{11}+Z_2X_{22})+\varepsilon\big[A_7X_1+A_8X_2+h_9(X_1+\lambda)\\&+h_{10}(X_1+X_{11})+h_{11}(X_1+X_{22})+h_{12}(X_2+\lambda)+h_{13}(X_2\\&+X_{11})+h_{14}(X_2+X_{22})-h_{15}(\lambda-\pi)+h_{16}(\pi-X_{11})+h_{17}\\&\times(\pi-X_{22})-h_{18}(\lambda+\pi)-h_{19}(\pi+X_{11})-h_{20}(\pi+X_{22})\\&-h_{21}s_1-h_{22}s_2\big]\cos\pi z,\end{aligned}\tag{41}$$

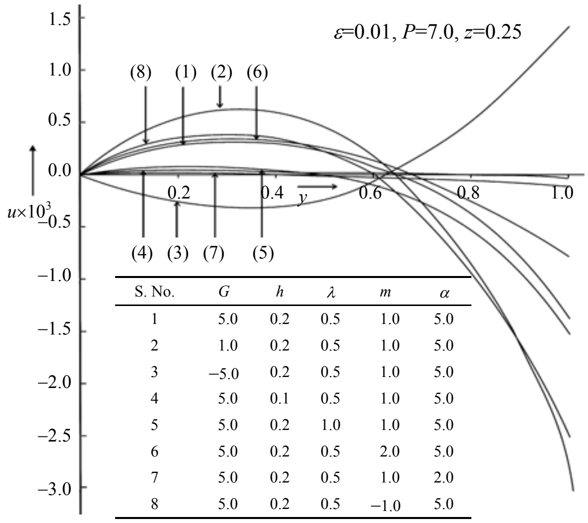
$$\begin{aligned}\tau_z&=\frac{d\tau_z^*}{\mu V_0}=\varepsilon\left(\frac{\partial w_1}{\partial y}\right)_{y=0}\\&=-\frac{\varepsilon}{\pi}\,[A_3X_1^2+A_4X_2^2-A_1\pi^2-A_2\pi^2]\sin\pi z.\end{aligned}\tag{42}$$

From the temperature field we can obtain the heat transfer coefficient i.e. Nusselt number as:

$$\begin{aligned}Nu&=\frac{dq_w^*}{K(T_1^*-T_0^*)}=\left(\frac{d\theta_0}{dy}\right)_{y=0}+\varepsilon\left(\frac{d\theta_{11}}{dy}\right)_{y=0}\cos\pi z\\&=(C_1X_{11}+C_2X_{22})+\varepsilon\big[A_5s_1+A_6s_2+h_1(X_1+X_{11})\\&+h_2(X_2+X_{11})+h_3(\pi-X_{11})-h_4(\pi+X_{11})+h_5(X_1+X_{22})\\&+h_6(X_2+X_{22})+h_7(\pi-X_{22})-h_8(\pi+X_{22})\big]\cos\pi z.\end{aligned}\tag{43}$$

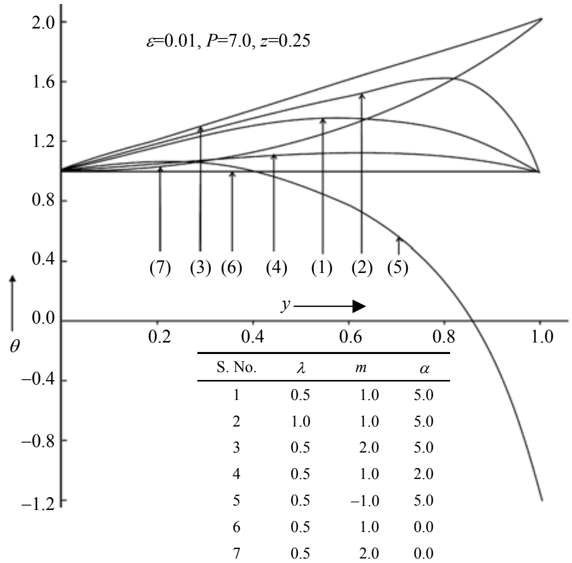
DISCUSSION AND CONCLUSION

For the physical results, we have calculated the numerical values of the velocity distribution (u), temperature field (θ), cross flow velocity (w), skin friction (τ_x, τ_z) and rate of heat transfer (Nu) in Figs.2 to 6 taking water as a fluid (P=7.0) and fixing ε=0.01.



S. No.	G	h	λ	m	α
1	5.0	0.2	0.5	1.0	5.0
2	1.0	0.2	0.5	1.0	5.0
3	−5.0	0.2	0.5	1.0	5.0
4	5.0	0.1	0.5	1.0	5.0
5	5.0	0.2	1.0	1.0	5.0
6	5.0	0.2	0.5	2.0	5.0
7	5.0	0.2	0.5	1.0	2.0
8	5.0	0.2	0.5	−1.0	5.0

Fig.2 Main flow velocity distribution *u* plotted against *y* for different values of λ, α, *G*, *m* and *h*



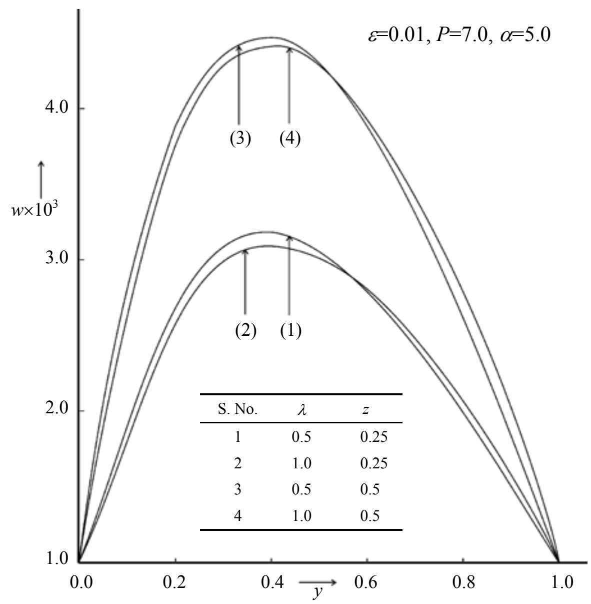
S. No.	λ	m	α
1	0.5	1.0	5.0
2	1.0	1.0	5.0
3	0.5	2.0	5.0
4	0.5	1.0	2.0
5	0.5	−1.0	5.0
6	0.5	1.0	0.0
7	0.5	2.0	0.0

Fig.3 Temperature distribution θ plotted against *y* for different values of λ, α, and *m*

In Fig.2 velocity distribution (u) is plotted against y for z=0.25. It is evident from this figure that the velocity increases with the increase of the slip parameter (h) and m, and that velocity decreases with the increase of the injection parameter (λ). It is interesting to note that when Groshof number (G) and strength of heat source parameter (α) are increased the velocity near the stationary plate is increased while that near the moving plate is decreased.

Fig.3 of temperature distribution (θ) plotted against y for z=0.25 shows that increase in injection parameter (λ), m and strength of heat source parameter (α) increases the temperature field also.

The cross flow velocity component (w) is due to transverse sinusoidal injection velocity distribution applied through the plate at rest. This secondary flow component is shown in Fig.4. It is interesting to note that w increases with the increase in z and decreases with increase in the injection parameter λ, up to the middle of the channel and thereafter increases with increasing λ. This is due to the injection at the stationary plate and suction at the plate in motion, which are two exactly opposite processes.



S. No.	λ	z
1	0.5	0.25
2	1.0	0.25
3	0.5	0.5
4	1.0	0.5

Fig.4 Cross flow velocity *w* plotted against *y* for different values of λ and *z*

The variations of skin friction component τ_x and τ_z in the main flow and transverse directions, respectively are shown in Fig.5. It was found that τ_x decreases with an increase in z, up to the middle of the channel and thereafter increases with increasing z. The phenomena reverses when h decreases, τ_x decreases up to the middle of the channel and thereafter increases with decreasing h. When z increases, τ_z also increases.

Important parameter Nusselt number is plotted against λ in Fig.6 showing that when strength of heat source parameter (α), z and m are increased, the Nusselt number is increased.

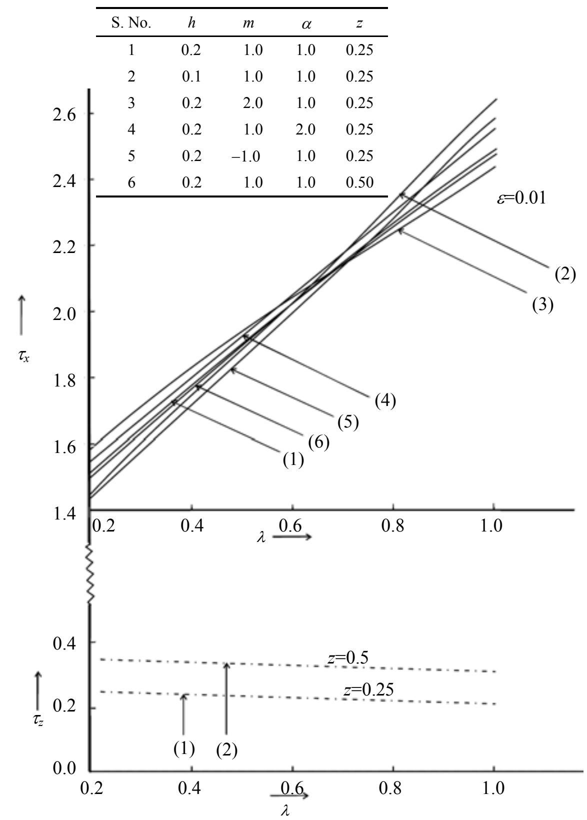


Fig.5 Skin friction τ_x and τ_z plotted against *y* for different values of *z*, α, *h* and *m*

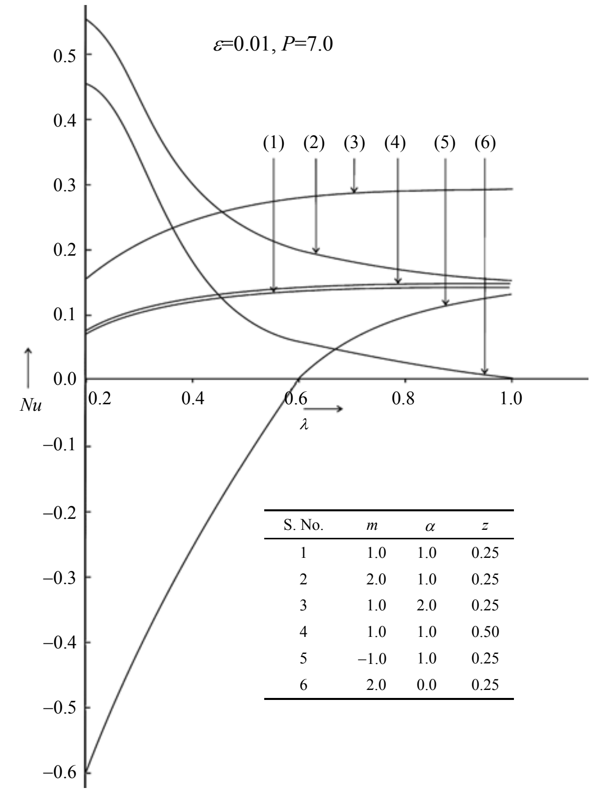


Fig.6 Nusselt number *Nu* plotted against λ for different values of *z*, α and *m*

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