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Method and application of wavelet shrinkage denoising based on genetic algorithm*

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Abstract: Genetic algorithm (GA) based on wavelet transform threshold shrinkage (WTS) and translation-invariant threshold shrinkage (TIS) is introduced into the method of noise reduction, where parameters used in WTS and TIS, such as wavelet function, decomposition levels, hard or soft threshold and threshold can be selected automatically. This paper ends by comparing two noise reduction methods on the basis of their denoising performances, computation time, etc. The effectiveness of these methods introduced in this paper is validated by the results of analysis of the simulated and real signals.

Key words: Wavelet transform, Translation-invariant wavelet transform, Genetic algorithm (GA), Correlation function

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INTRODUCTION

Most underwater acoustic signals received by sonar are corrupted inevitably by unpredictable noise sources. In most cases, sonar system is adversely influenced by noise so disturbing that target signals cannot be detected and classified correctly so that pre-processing is necessary to reduce the noise as much as possible.

Methods of noise reduction based on wavelet transform have been developed extensively in previous literatures (Xu *et al*., 1994; Sun, 1998), and include wavelet transform modulus maxima method (WTMM), spatially selective noise filtration technique (SSNF), WTS and TIS, etc. In this paper, denoising methods based on WTS and TIS and their applications in underwater acoustic signals processing are discussed in detail.

Good selection of parameters, such as wavelet function, decomposition levels and threshold, etc., is critical to the success of WTS and TIS.

Parameters listed above are usually selected empirically or semi-empirically in practical application, which cannot ensure that the denoising performance is optimal in some sense. To overcome this shortcoming, this paper introduces genetic algorithm (GA) into WTS and TIS, and proposes two methods for noise reduction: GA-WTS and GA-TIS. Tu and Jiang (2004) discussed it originally. At the end of this paper, the effectiveness of these methods is validated by results of analysis of the simulated and real data.

THEORY OF WAVELET SHRINKAGE DENOISING

Wavelet transform

Let $f(t)$ denote a target signal, and $\psi(t)$ denote wavelet basis function. Suppose $\hat{\psi}(\omega)$ is Fourier transform of $\psi(t)$ and the following equation is satisfied

$$C_\psi = \int_0^\infty \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < +\infty . \tag{1}$$

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The wavelet transform of $f(t)$ is written

$$Wf(u,s)=\int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{s}}\psi^*\left(\frac{t-u}{s}\right)\mathrm{d}t, \quad (2)$$

where $s>0$ denotes scale, u denotes position. "*" denotes conjugation. The wavelet transform depends on two parameters s and u that vary continuously over a set of real numbers. For practical applications these parameters must be discretized. For a particular class of wavelets, the scale parameter s can be sampled along the dyadic sequence $(2^j)_{j\in\mathbb{Z}}$, and position parameter can be sampled as $(k2^j)_{k,j\in\mathbb{Z}}$. Then the discrete wavelet transform can be obtained as

$$Wf(k,j)=2^{j/2}\int_{-\infty}^{+\infty} f(t)\psi^*(2^j t-k)\mathrm{d}t \, . \quad (3)$$

Eq.(3) can be computed by the rapid pyramidal filtering algorithm proposed by Mallat (1989; 2002), and discussed in previous literatures.

Denoising method of WTS and TIS

WTS was implemented using the difference of the statistical properties of the signal and noise present in the wavelet domain. WTS involves shrinking in the wavelet transform domain, and consists of three steps (Taswell, 2000): a linear forward wavelet transform, a nonlinear shrinkage denoising, and a linear inverse wavelet transform.

Suppose we want to estimate f_n from noisy observation signal x_n

$$x_n=f_n+e_n, \quad n=1, \ldots, N, \quad (4)$$

where f_n denotes target signal, and $e=(\ldots e_n\ldots)$ is independent and uniformly distributed $N_N(0,\sigma^2 I)$. Denoising uses an operator Th which acts on the noisy observation signal x_n to minimize the mean square error (MSE) (Donoho, 1995; Hsung *et al*., 2005)

$$MSE=\frac{1}{N}\left\|\hat{\boldsymbol{f}}-\boldsymbol{f}\right\|^2=\frac{1}{N}\sum_{n=0}^{N-1}(\hat{f}_n-f_n)^2 \quad (5)$$

subject to the condition that $\hat{\boldsymbol{f}}$ is at least as smooth as $\boldsymbol{f}$, where $\boldsymbol{f}=(f_0,f_1,\ldots,f_{N-1})^{\mathrm{T}}$, and $\hat{\boldsymbol{f}}=Th(\boldsymbol{x})$ is the signal estimated from observation signal $\boldsymbol{x}$.

Usually, in the wavelet domain (Sun, 1998), signal energy is dominated by a few large amplitude wavelet coefficients, whereas noise energy is distributed uniformly over all wavelet coefficients with small amplitudes, especially in detail components. Re-considering Eq.(4), we modify it as follows

$$\boldsymbol{x}=\boldsymbol{f}+\boldsymbol{e}\, , \quad (6)$$

where $\boldsymbol{x}$ is an observation sequence consisting of the target signal $\boldsymbol{f}$ and additive noise $\boldsymbol{e}$ as functions in time t to be sampled. Let $w(\cdot)$ and $w^{-1}(\cdot)$ denote the forward and inverse wavelet transform operators. Let $D(\cdot,\lambda)$ denote the denoising operator with hard or soft threshold λ. Then the three-step denoising procedure already described can be expressed as follows (Taswell, 2000)

$$\boldsymbol{y}=w(\boldsymbol{x})\, , \quad (7)$$

$$\boldsymbol{z}=D(\boldsymbol{y},\lambda)\, , \quad (8)$$

$$\hat{\boldsymbol{f}}=w^{-1}(\boldsymbol{z})\, . \quad (9)$$

We can see from Eqs.(7)~(9) that many factors affect denoising performance, including $w(\cdot)$, $D(\cdot,\lambda)$, λ, etc., among which, threshold λ is a key parameter. Donoho (1995) and Chang *et al*.(2005) proposed many methods to guide the selection of threshold. VisuShrink is one of them. Theories and experiments proved that WTS can obtain good denoising performance, but in some cases (Sun, 1998), pseudo Gibbs phenomena may appeared in the estimated signal, therefore, Donoho *et al*.(1995) proposed TIS on the basis of WTS. TIS not only restrains pseudo Gibbs phenomena, but also reduces MSE between target and estimated signal.

Suppose x_n is a noisy observation signal, for $n=0$, $1, \ldots, N-1$. Let S_h denote an operator which shifts a signal h sample to the right circularly, viz $(S_h x)_n = x_{(n+h)\mathrm{mod}N}$. Let $D(\cdot,\lambda)$ denote the denoising operator with hard or soft threshold λ. TIS can be written as

$$\overline{T}(x)=\frac{1}{N}\sum_{h=0}^{N-1} S_{-h}(D(S_h x,\lambda))\, . \quad (10)$$

Sun (1998) introduced an algorithm for TIS.

Denoising performance of WTS and TIS may be affected by many parameters, such as wavelet function, decomposition levels, threshold, hard or soft threshold, etc., in practical applications. At present, there are still no methods to guide the selection of these parameters besides experience. To solve this problem, we propose two methods of noise reduction: GA-WTS and GA-TIS, by which, parameters affecting denoising performance can be determined automatically so that the optimal denoising performance in some sense can be reached.

APPLICATION OF GA IN WTS AND TIS

GA is a stochastic search method that mimics the metaphor of natural biological evolution. GA operates on a population of potential solutions applying the principle of survival of the fittest to produce better and better approximations to a solution, and has been gaining popularity in many applications requiring optimization of a solution without requiring theoretical knowledge of the actual problem, or where relationships are too complex to model or understand analytically (Jack and Nandi, 2002), which suggests the probability of GA being used in WTS and TIS.

In this paper, we take the application of cross-correlation between observation signal and estimated signal as fitness function, and use GA operators, such as selection, crossover, mutation, etc., to optimize parameters of WTS and TIS to improve denoising performance. Applications of GA in WTS and TIS will be discussed at length in this section.

Fitness function

The most straightforward fitness is the *MSE* between target signal and estimated signal, as shown in Eq.(5), but in general, as the target signal is unknown. *MSE* cannot be obtained. Therefore we take the coefficient of the cross-correlation between observation and estimated signal, which is written as

$$F_{\text{obj}} = \frac{1}{N_{\text{lag}}} \sum_{m=1}^{N_{\text{lag}}} \left| \sum_{n=0}^{N-1} x_n \hat{f}_{n+m} \right|, \tag{11}$$

where m is lags, and N_{lag} is total lags.

Usually, the cross-correlation coefficient is directly proportional to SNR. Fig.1 shows four simulated signals that Donoho and Johnston originally called Blocks, Bumps, HeavySine, and Doppler. The relation between the cross-correlation coefficient and SNR of these four simulated signals is shown in Fig.2 showing that the cross-correlation coefficient increases with SNR. It is reasonable to take the cross-correlation coefficient as fitness function.

Encoding

Encoding is the key of GA and largely depends on the property of the problem to be solved, and may affect the design of GA operators. In this paper, parameters of WTS and TIS, including wavelet function, decomposition levels, threshold and hard or soft threshold, are encoded with real numbers, and are explained in Table 1.

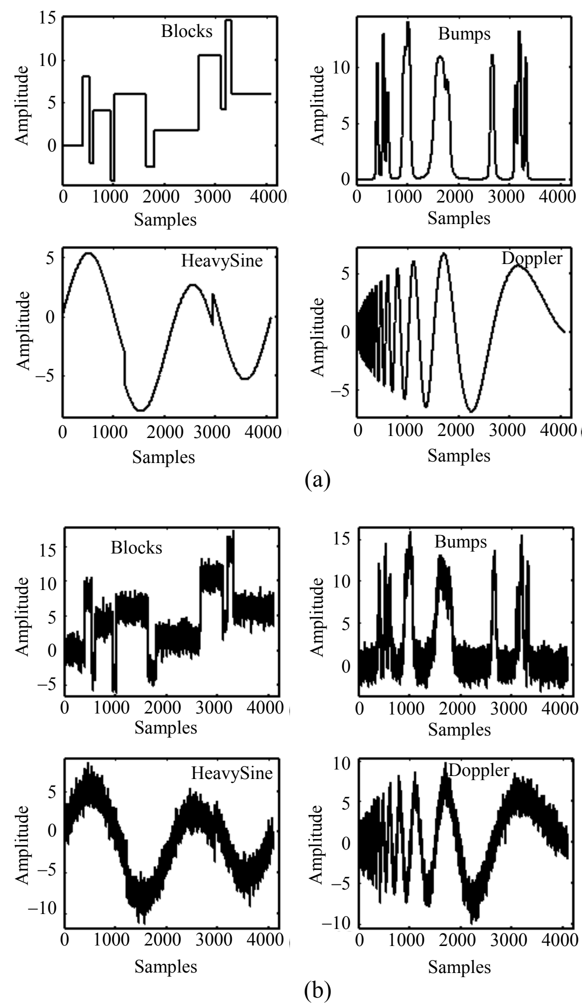

Fig.1 Four simulated signals used in the paper. (a) Standardized simulated signals; (b) Noisy simulated signals

Table 1 Method of chromosome encoding in WTS and TIS

Variable	Variable description	Definition domain	Remark
v_1	Wavelet function	$[1,N_{name}]$	N_{name} is the number of wavelet function in definition domain, and round up v_1 when it is decoded
v_2	Hard or soft threshold	[1,2]	Hard or soft threshold. Round up v_2 when it is decoded
v_3	Decomposition levels	$[1,N_{max}]$	N_{max} is the maximum of decomposition levels, and round up v_3 when it is decoded
v_4~v_M	Threshold of every decomposition level	$[0,Q_i]$	M–3=N_{max}, Q_i is the upper limit of threshold for every decomposition level

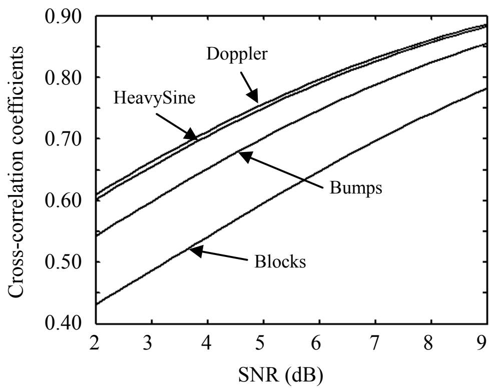

Fig.2 Relation between cross-correlation coefficients and SNR of four simulated signals

Selection, crossover and mutation

The GA here uses a population size of 80 individuals, starting with randomly generated genomes. The objective function is evaluated for these individuals and the fitness of every individual is assigned by rank-based fitness order. Parents are then selected according to their fitness by means of roulette-wheel selection. When all parents participating in evolution have been selected, crossover will be started, which is the kernel of GA and search optimal solution in the whole definition domain (Xiao and Xue, 2003). Line crossover is used in this paper, viz

$$\begin{cases} c_1 = P_1 a + P_2(1-a), \\ c_2 = P_1(1-a) + P_2 a, \end{cases} \tag{12}$$

where a is a random number distributed uniformly in [0,1]. P_i and c_i (i=1, 2) denote parents and offspring chromosome respectively, and c_i is limited to its definition domain. After crossover every offspring undergoes mutation. Variables of offspring are mutated by small perturbations, at low probability, with search solution being in a small part of the definition domain and local optima converging is avoided. Non-uniform mutation operator (Yang and Zheng, 2003) is used in the paper, and mutation rate P_m is set to 0.2. Let P denote a chromosome participating in mutation. Variable v to be mutated is randomly determined uniformly, and its definition domain is $[v_{min}, v_{max}]$. The operation of mutation can be expressed as follows

$$\text{if } \chi \le P_m \Rightarrow$$
$$v' = \begin{cases} v + (v_{max} - v)\left(\sigma\left(1 - \dfrac{gen_c}{gen_{max}}\right)^b\right), & \gamma > 0.5, \\ v - (v - v_{min})\left(\sigma\left(1 - \dfrac{gen_c}{gen_{max}}\right)^b\right), & \gamma \le 0.5, \end{cases} \tag{13}$$

where χ, σ, γ are uniform random variables in [0,1]. v' is the variable after mutation. gen_c is the number of current generation. gen_{max} is the maximum number of generations. b is the parameter for mutation. After evolution operations of selection, crossover and mutation, a new generation is produced. If the optimization criteria are not met, the evolution process is repeated until the optimization criteria are reached. We take the number of maximum evolution generations (80) as criteria to stop GA in this paper.

ANALYSIS OF SIMULATED AND REAL SIGNAL

Before the analysis, it is necessary to list all the wavelet functions used in our method, viz definition domain of variable v_1 in Table 1, which is [Haar Db2 Db3 Db4 Db5 Db8 Db10 Db12 Db15 Db18 Db20 Db30 Sym2 Sym4 Sym7 Sym8 Sym10 Bior1.1 Bior1.3 Bior2.2 Bior2.4 Bior2.6 Bior3.1 Bior3.3

Bior3.5 Bior3.7 Bior3.9 Bior4.4 Bior5.5 Bior6.8 Rbio1.1 Rbio1.5 Rbio2.2 Rbio3.3 Rbio3.5 Rbio3.7 Rbio3.9 Rbio4.4 Rbio5.5 Rbio6.8 Dmey]. Means of the abbreviations of the wavelet functions are explained in many wavelet literatures (Sun, 1998; Mallat, 2002). Here we only explain a few important wavelet functions of them. "Haar" wavelet is discontinuous, and resembles a step function. "Db*N*" denote Daubechies family, which is the one of the brightest stars in the world of wavelet research. "Sym*N*" denote symlets, which are nearly symmetrical wavelets proposed by Daubechies as modifications to "Db" family. "Bior*N.M*" denotes biorthogonals, which exhibit the property of linear phase. "Rbio*N.M*" denote the reverse biorthogonal wavelets, and "Dmey" denote discrete Meyer wavelet.

Analysis of simulated signal

We discuss applications of GA in WTS and TIS using four simulated signals shown in Fig.1. Figs.3a and 3b display the results obtained with GA-WTS and GA-TIS, and Fig.3c shows the results obtained with a general method introduced in many literatures. Eq.(14) is used to evaluate the performance of denoising, and the results are listed in Table 2.

$$SNR = 10\log 10 \sum_{n=0}^{N-1} |f_n|^2 \bigg/ \sum_{n=0}^{N-1} |f_n - \hat{f}_n|^2 . \quad (14)$$

Analysis of simulated signals led us to conclude that: (1) Parameters of WTS and TIS can be optimized and selected automatically with GA, and aimless selecting of parameters may be avoided. (2) From the improvement of SNR, we can see that the denoising performances of GA-WIS and GA-TIS (besides Blocks) are better than those of the general denoising methods based on WTS. The best generation of the GA method is obtained in the aspect of optimizing fitness function. (3) Denoising performances of GA-TIS are better than those of GA-WTS whether in the improvement of SNR or in visual quality, although GA-TIS involves more computation than GA-WIS, as shown in Table 2 (The algorithms were implemented with Matlab 6.5 on computer with CPU P4 2.8 G and Memory 1 G. We should point out that in Table 2, the computation time of VisuShrink-WTS denotes a single running time of the algorithm,

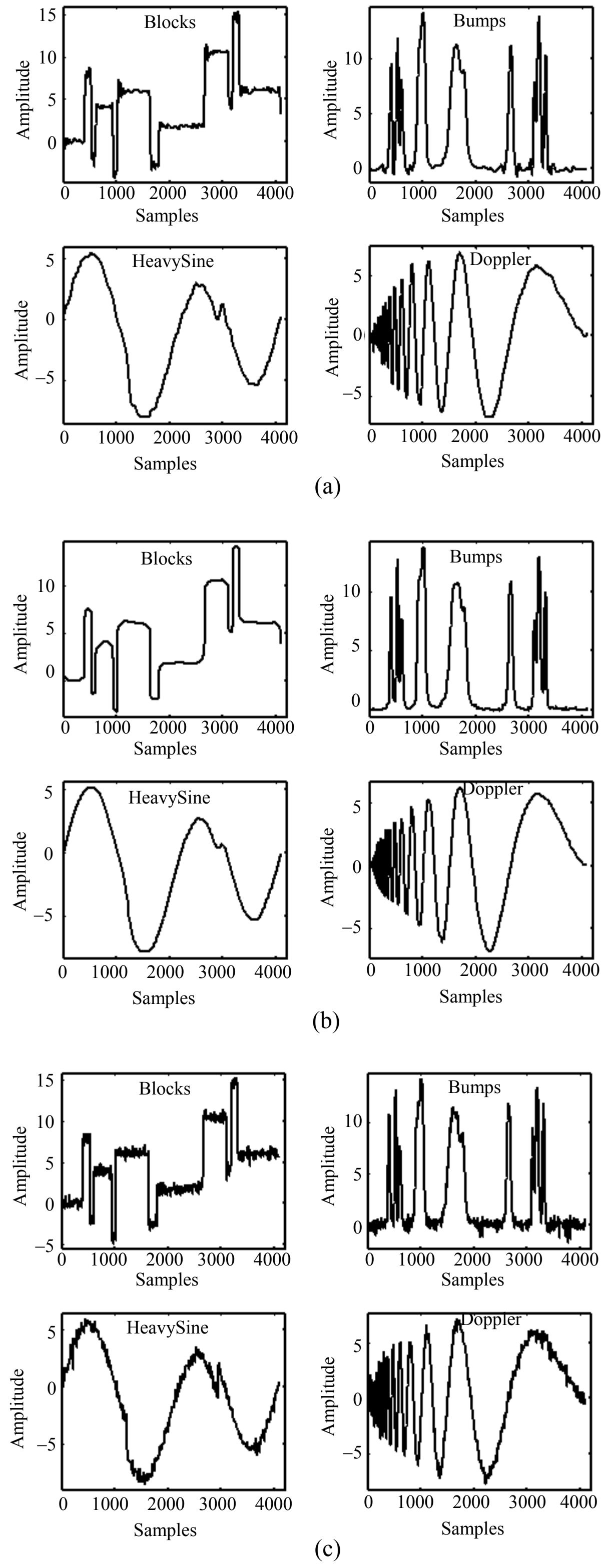

Fig.3 (a) Denoising results of GA-WTS; (b) Denoising results of GA-TIS; (c) Denoising results of WTS whose threshold is determined by VisuShrink and other parameters are determined empirically

and that the performances were obtained experimentally for the case of knowing the target signal, which is not a fact in practical applications so that the computation efficiency advantage of VisuShrink-WTS cannot be obtained). We should find a tradeoff between denoising performances and computation efficiency when we use GA-WTS and GA-TIS.

Analysis of real signal

In this experiment, a real signal received by sonar was used for noise reduction. The signal was sampled at 15 kHz, and contained 32768 simples, whose waveforms of time domain and power spectrum are shown in Fig.4. We applied GA-WTS and GA-TIS to the real signal (results are shown in Fig.5). As the target signal is unknown, denoising performances can only be evaluated by the coefficient of cross-correlation between the real and estimated signal. Suppose the total lags are 5 samples. The auto-

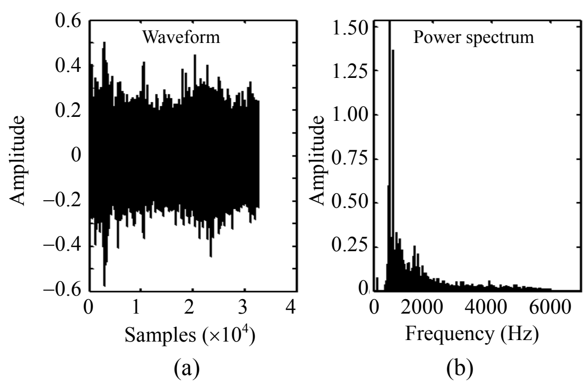

Fig.4 Waveform of time domain (a) and power spectrum of real signal (b)

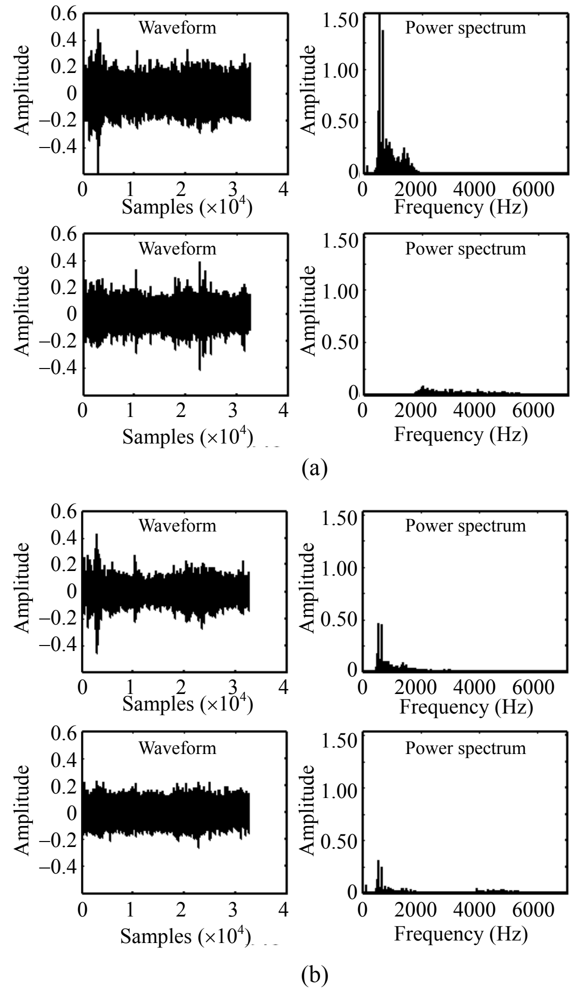

Fig.5 (a) Denoising result of GA-WTS; (b) Denoising result of GA-TIS. In (a) and (b), the top two figures are estimated signals and the bottom two figures are remaining noises, respectively

Table 2 Denoising performances of several methods based on wavelet transform

		Blocks	Bumps	HeavySine	Doppler
Noisy signal	SNR	15.3780	13.2772	12.4602	12.2209
GA-WTS	SNR	20.4963	24.3363	26.4774	22.2311
	Wavelet function	Db10	Bior5.5	Db15	Db20
	Decomposition levels	5	4	6	5
	Computation time (s)	10.5705	7.6880	11.6485	8.3435
GA-TIS	SNR	22.5237	25.3212	25.5930	23.4137
	Wavelet function	Haar	Sym4	Bior2.2	Sym8
	Decomposition levels	8	7	8	8
	Computation time (s)	244.8130	191.6570	254.2340	322.3600
VisuShrink-WTS	SNR	21.7853	24.1279	23.9649	21.1986
	Wavelet function	Haar	Db4	Db4	Db4
	Decomposition levels	3	4	4	3
	Computation time (s)	0.1870	0.2500	0.2500	0.2030

correlation coefficient of the real signal is 0.2440. The cross-correlation coefficients are increased to 0.3500 and 0.3447 respectively after denoising processing by GA-WTS and GA-TIS.

Conclusions that can be draw from Fig.5 and various auto-correlation coefficients are: (1) The higher the value of the coefficients, the less the remaining noise in the denoised signal. The result indicates that the proposed method can achieve more efficient noise reduction. (2) There is much difference between GA-WTS and GA-TIS in denoising performance. Noise can be reduced effectively by GA-WTS, although when we use it, the target signal may be reduced along with noise. (3) The definition domain of variables in chromosome has great effect on the denoising performance of GA-WTS and GA-TIS. We should determine it carefully.

CONCLUSION

In this paper, we introduce GA into WTS and TIS, and propose a method to guide the selection of parameters (wavelet function, decomposition levels, thresholds and hard or soft thresholding) in WTS and TIS. The effectiveness of the method was validated by the analysis of simulated and real signal. The method proposed in this paper is also applicable to other denoising methods based on wavelet transform, such as WTTM and SSNF. But it is necessary to point out that the fitness function used in this paper is not optimal. If we exploit prior knowledge of the input signal as much as possible, a more reasonable fitness function can be designed. Future research will be focused on the selection of the fitness function in view of the idea of pattern recognition and adaptive noise cancelling.

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