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Simulation of game analysis based on an agent-based artificial stock market re-examined

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Abstract: This work re-examined the simulation result of game analysis (Joshi *et al*., 2000) based on an agent-based model, Santa Fe Institute Artificial Stock Market. Allowing for recent research work on this artificial model, this paper's modified game simulations found that the dividend amplitude parameter is a crucial factor and that the original conclusion still holds in a not long period, but only when the dividend amplitude is large enough. Our explanation of this result is that the dividend amplitude parameter is a measurement of market uncertainty. The greater the uncertainty, the greater the price volatility, and so is the risk of investing in the stock market. The greater the risk, the greater the advantage of including technical rules.

Key words: Agent-based model, Technical trading, Asset prices, Simulation
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INTRODUCTION

The Santa Fe Institute Artificial Stock Market (SFI-ASM) is a stochastic discrete event agent based model that simulates a market in which a number of agents choose between investing in a risk-free bond with a fixed interest rate, and a stock with an autoregressive stochastic dividend. In contrast to mathematical models of stock markets based on unrealistic assumptions of agents' knowledge and processing power, the model had been shown to demonstrate features observed in real markets. It was developed in 1989 and had been described in various papers (Palmer *et al*., 1994; Arthur *et al*., 1997; LeBaron *et al*., 1999; 2002). The original software, written in C, then in Objective-C for the NeXTstep architecture, was converted by Johnson (2002) for use with Swarm. Both are open sources and available at http://artstkmkt.sourceforge.net/.

One of SFI-ASM's main results is the identification of a single parameter, i.e., the learning speed of agents, which can shift the model to either a regime that is close to the homogeneous rational expectation equilibrium, or to a more complex regime that better fits the empirical facts. The complex regime emerges for fast learning rates and exhibited technical trading that dominated fundamental strategies. Consequently, Joshi *et al*.(2000; 2002) concluded that financial markets inevitably are at sub-optimal equilibria, and that technical trading is caused by a typical prisoner's dilemma. In recent research works, Ehrentreich (2005) claimed that the original SFI-ASM GA mutation operator was biased. In his corrected version (His version is programmed in Java and uses the Repast library 1.4.1), the emergence of technical trading rules does not occur and the homogeneous rational expectation equilibrium is reached.

Allowing for recent research work on this artificial model, this paper's modified game simulation still found that the use of technical trading bits is a

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dominant strategy and that the prisoner's dilemma holds in a not long period, but only when the dividend amplitude is large enough. Our explanation of this result is that the dividend amplitude parameter can be regarded as a measurement of the dividend uncertainty. This uncertainty is then transferred to agent's expectation of future price and dividend. The greater the uncertainty, the greater the price volatility, and so is the risk of investing in the stock market. The greater the risk, the greater the advantage of including technical rules.

Section 2 below describes the Santa Fe Institute Artificial Stock Market model that we used in our argument, Section 3 explains the game analysis experimental framework, Section 4 and 5 present and explain the results of our experiment, and Section 6 ends with the summary and conclusion.

SANTA FE INSTITUTE ARTIFICIAL STOCK MARKET

Brief introduction

Santa Fe Institute Artificial Stock Market, one of the first agent-based models of a financial market, was developed by Brian Arthur, John Holland, Blake LeBaron, Richard Palmer, Paul Tayler, and Brandon Weber. The basic economic structure of the market draws heavily on existing market setups such as those of Bray (1982) and Grossma and Stiglitz (1980). The model is inhabited by a population of myopic, imperfectly rational, heterogeneous agents who make investment decisions by forecasting the future states of the market, and who also learn from their experience over time. The model illustrates how simple interactions among such agents may lead to the appearance of the realistic structure itself.

The model

Agents, initially endowed with one unit of risky stock and 20000 units of cash, have to decide during each time period of the simulation how much to invest in risky stock and how much to keep in cash assets yielding a risk-free rate of return.

The stock pays a stochastic dividend per period which is generated by a stationary AR(1)-process

$$d_t = \bar{d} + \rho(d_{t-1} - \bar{d}) + \mu_t, \tag{1}$$

where $\mu_t \sim N(0, \sigma_\mu^2)$ (Its behavior is denoted by the dividend amplitude parameter *amplitude* $=\sigma_\mu/(\bar{d}\times\sqrt{1-\rho^2})$ in the source code and also in this paper), and $\bar{d}$ =10, ρ=0.95 for all experiments. It is well known that assuming CARA utility functions and Gaussian distributions for dividend and prices, the demand for holding shares of the risky asset by agent i is given by

$$s_{t,i} = \frac{E_{t,i}(p_{t+1} + d_{t+1}) - p_t(1+r)}{\gamma\sigma_{t,i,p+d}^2}, \tag{2}$$

where p_t is the price of the risky asset at t, $\sigma_{t,i,p+d}^2$ is the conditional variance of "$p+d$" at time t for agent i, γ is the coefficient of absolute risk aversion, and $E_{t,i}(p_{t+1}+d_{t+1})$ is the expectation for agent i at time t.

Agents do this by forecasting the price of the stock, and assessing its risk (measured by the variance of the prices). Forecasting rules are IF-THEN statements: IF (a certain market state occurs) THEN (a certain forecast is made).

Agents can recognize two different kinds of market states: technical and fundamental. A market state detected by an agent is "technical" if it identifies a pattern in the past price history, and is fundamental if it identifies an immediate over- or under-valuation of the stock. The market states are summarized in a binary state vector 12 bits long. Each element corresponds to whether the conditions in Table 1 are true or false.

Table 1 Condition bits

Bit	Conditions
0	*Price*×*interest*/*dividend*>1/4
1	*Price*×*interest*/*dividend*>1/2
2	*Price*×*interest*/*dividend*>3/4
3	*Price*×*interest*/*dividend*>7/8
4	*Price*×*interest*/*dividend*>1
5	*Price*×*interest*/*dividend*>9/8
6	*Price*×*interest*/*dividend*>4
7	*Price*>5-*period MA*
8	*Price*>20-*period MA*
9	*Price*>100-*period MA*
10	*Price*>500-*period MA*
11	On: 1
12	Off: 0

MA: moving average

One forecasting rule is said to be activated if the market state in a period matches its descriptor. Because a number of forecasting rules may be activated at a given time, the agent must make a choice from them and decide which of the active forecasts to use by choosing at random among the active forecasts with a probability proportional to its accuracy, a measure that indicates how well the rule has performed in the past. Once the agent has chosen a specific rule to use, an investment decision follows to determine how much stock to buy, sell or hold, using a standard risk-aversion calculation. Agents then submit their decisions to the market specialist, an extra agent in the market whose work is to declare a market-cleaning price.

Genetic Algorithm (GA) (Holland, 1975; Goldberg, 1989; Mitchell, 1996) provides for the evolution of the population of forecasting rules over time. Whenever the GA is invoked, it substitutes new forecasting rules for a fraction of the least-fit forecasting rules in each agent's pool of rules. A rule's success or "fitness" is determined by its accuracy and by how complex it is (the GA is biased against complex rules). New rules are created by applying the genetic operators of mutation and crossover to the bit strings of the more successful rules in the agent's rule pool.

It is important to note that agents in this model learn in two ways: First, as each rule's accuracy varies from time period to time period, each agent preferentially uses the more accurate of the rules available to it; second, on an evolutionary time scale, the pool of rules as a whole improves through the action of the genetic algorithm.

Experimental results

The most significant early finding was that this market exhibits two quite different kinds of behavior, corresponding to different rates at which market-forecasting rules are being revised by the genetic algorithm. When the GA-invocation interval is large (between 1000 and 10000) resulting in forecasting rules evolving relatively slowly, prices are more stable; evolved forecasting rules are simple; levels of technical trading are low; trading volumes are low; and there is little evidence of nonlinearity. Since this kind of behavior resembles the predictions of the theory of efficient markets, this regime has been termed the "Rational Expectation Regime" (SFI Bulletin, 1999).

On the other hand, when the GA-invocation interval is small (between 10 and 100) it results in forecasting rules evolving relatively quickly, and the variance of the price time series is relatively high; the evolved rules are complex; levels of technical trading are high; trading volumes are higher; and there is strong evidence of nonlinearity. This regime is called the "Complex Regime" (SFI Bulletin, 1999).

Since the 1990s, Joshi, Parker and Bedau have been further studying the dynamics of the ASM (Joshi *et al*., 2000; 2002). Using a simple game theoretical model together with the Santa Fe Institute Stock Market, they showed that the market has only one symmetric Nash equilibrium, and that this equilibrium lays in the "Complex Regime". Most important of all, they concluded that financial markets inevitably are at sub-optimal equilibria, and that technical trading is caused by a typical prisoner's dilemma.

In recent research works, Ehrentreich (2005) claimed that the original SFI-ASM GA mutation operator was biased. In his corrected version, the emergence of technical trading rules does not occur and the homogeneous rational expectation equilibrium is reached. What is more, he claimed that the game analysis conclusion (Joshi *et al*., 2000; 2002) can be partly replicated with his modified version only at a not long simulation period.

GAME ANALYSIS SIMULATION

Most of the original experimental framework (Joshi *et al*., 2000) is followed in this paper except for the implementing details. To investigate whether or not including technical trading rules is advantageous for traders, we contemplate a single agent confronted with a choice between two strategies: either to include technical trading rules in the agent's repertoire of trading rules, or to exclude them entirely and instead use only fundamental rules. The agent assumes that other traders in the market all follow one or the other of these two strategies (either include all technical trading rules or exclude of them), but the agent does not know which of these two possibilities occurs.

Thus, the agent confronts a classic 2×2 decision problem. To make a rational decision, the agent needs to know the relative value or payoff of each choice in each situation. This paper's criterion for social and individual welfare is relative wealth[1]. So, to determine the payoffs in the decision matrix, we observed the relative wealth of the agent in four different conditions: (1) The agent includes technical rules and all other traders include them; (2) The agent includes technical rules and all other traders exclude them; (3) The agent excludes technical rules and all other traders include them; (4) The agent excludes technical rules and all other traders exclude them.

By comparing the agent's payoffs in these four possible situations, we can determine whether there is a dominant strategy for this decision[2].

Since all agents in the market act independently and simultaneously, each time period in the market can be considered to be a multi-person simultaneous-move game. Furthermore, each agent's decision can be construed in exactly the form of the single agent considered above. So, if the single-agent decision considered above has a dominant strategy, it will be rational for all agents to use it and the simultaneous-move game will reach a symmetric Nash equilibrium (Bierman and Fernandez, 1993). Thus, situations (1) and (4) above are the only potential symmetric Nash equilibria in our context.

Expected payoffs in situations (1)~(4) were determined by simulating the artificial market[3] 30 times in the four corresponding circumstances. In each simulation, there were 26 agents in the market: one agent following a given strategy and 25 other agents all following another given strategy (possibly the same strategy as that of the single agent). Each simulation was run for 10000 time periods[4]. The same 30 random sequences for dividends and initial distributions of rule descriptors among agents were used for all four experiments. And the crucial GA-invocation interval 100 is followed (Joshi *et al*., 2000; 2002).

[1]The relative wealth of an agent in the market is the ratio of final wealth-interest payments from the risk free asset, returns from stocks, and cash holdings (money not invested) over each initial cash holding (all with 20000 units in our experiment).

[2]A dominant strategy is defined as one that outperforms all other strategies regardless of the strategies being used by other agents (Bierman *et al*., 1993).

[3]In this paper, ASM2.4, a Java version of SFI-ASM is used combined with Swarm2.2, JDK 1.4.7 and Fedora Core 3 OS platform.

[4]Allowing for Ehrentreich's research, Joshi *et al*.(2000)'s simulation time periods 300 000 is not followed.

RESULTS

Tables 2~4 shows the expected payoffs to the agent in the situations (1)~(4) at different dividend amplitude mentioned in Eq.(1). These payoffs were calculated by averaging the agent's final relative wealth in repeated simulations of each of the four situations. These decision matrixes support two conclusions.

First, although the wealth level difference is not as strong as the former research (Joshi *et al*., 2000) when dividend amplitude equals 0.0879 which is achieved by setting σ_μ^2 in equation *amplitude*= $\sigma_\mu/(\bar{d}\times\sqrt{1-\rho^2})$ equal to 0.07429 (LeBaron *et al*., 1999) (Table 2), their conclusion that technical trading emerges due to a prisoner's dilemma, and that rational traders are technical traders can be upheld. In Table 2, note that since the payoff in situation (1) exceeds that in situation (3) and the payoff in situation (2) exceeds that in situation (4), no matter what strategy the other agents in the market might be using, it is always advantageous for the agent to include technical trading in his market forecasting rules. So including technical trading is the dominant strategy for each agent, and this is the one and only symmetric Nash equilibrium in this multi-person simultaneous-move game. What is more, since the expected payoff in situation (1) is less than that in situation (4), everyone is better off if no one includes technical trading. So, engaging in technical trading, which is apparently rational for each rational trader, leads the market to a sub-optimal state.

Second, when dividend amplitude decreased (Table 3), the difference between relevant payoffs can be ignored allowing for the error bound, which means that the conclusion mentioned above can be hardly upheld. But when this amplitude increased (Table 4), the difference between relevant payoffs ample at the same time, which means that the former conclusion can be supported more steadily.

DISCUSSION

These findings raise an important question: what is the influence of dividend amplitude on agent's wealth level in the SFI-ASM? From Eq.(1), the divid-

Table 2 The decision table when dividend amplitude equals 0.0879 for an agent contemplating whether to include technical trading rules to make market forecasts, when the agent is uncertain whether or not the other traders in the market are doing so. The agent's payoff in each of the four situations (1)~(4) is the agent's expected relative wealth, derived by averaging the results of 30 simulations of each situation. Errors bounds are calculated using standard deviations of the 30 simulations

	Technical rules	All other traders	
		Technical rules included	Technical rules excluded
The agent	Include	(1): 1.966±0.041	(2): 2.064±0.062
	Exclude	(3): 1.930±0.038	(4): 2.006±0.051
Wealth level difference		(1)−(3): 0.036±0.003	(2)−(4): 0.058±0.011

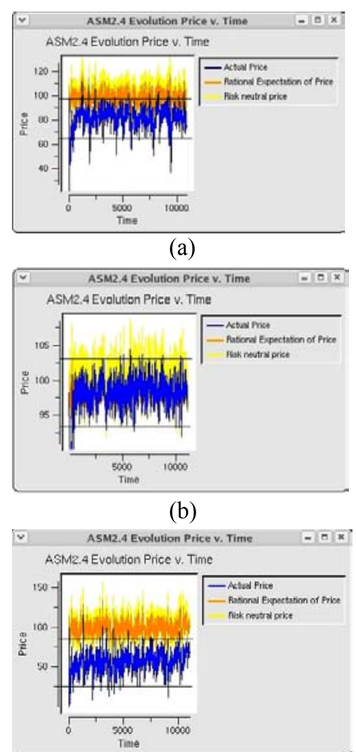
Table 3 The same notes with Table 1 but dividend amplitude equals 0.02727

	Technical rules	All other traders	
		Technical rules included	Technical rules excluded
The agent	Include	(1): 1.139±0.023	(2): 1.141±0.028
	Exclude	(3): 1.141±0.030	(4): 1.131±0.028
Wealth level difference		(1)−(3): −0.002±0.007	(2)−(4): 0.010±0.000

Table 4 The same notes with Table 1 but dividend amplitude equals 0.14178

	Technical rules	All other traders	
		Technical rules included	Technical rules excluded
The agent	Include	(1): 3.265±0.067	(2): 3.454±0.077
	Exclude	(3): 3.155±0.075	(4): 3.311±0.069
Wealth level difference		(1)−(3): 0.110±0.008	(2)−(4): 0.143±0.008

end amplitude parameter actually holds the same influence of μ_t, which can be regarded as a measurement of the dividend uncertainty. This uncertainty is then transferred to agent's expectation in Eq.(2). The greater the uncertainty, the greater the price volatility, and so is the risk of investing in the stock market. It seems that including technical rules benefits much only in a little bit high risky market, which agrees with the empirical realization from practical use of technical trading. This explanation fits well with the results of our experiments. Fig.1 shows the simulation result of running SFI-ASM under the above-mentioned three different dividend amplitudes with all other parameters being same. In Fig.1a the stock price ranges roughly from 65 to 95, in Fig.1b roughly from 93 to 103, and in Fig.1c from 25 to 85. The greater the dividend amplitude, the greater the price volatility range, and so is the advantage of including technical rules.



(c)

Fig.1 Price volatility (the actual price line) under dividend amplitude equals 0.0879 (a), 0.02727 (b), 0.14178 (c) respectively with other parameters unchanged

SUMMARY AND CONCLUSION

In this work, we re-examined the simulation result of game analysis based on an agent-based model, Santa Fe Institute Artificial Stock Market, which found that the use of technical trading bits is a dominant strategy in the market. Allowing for recent research work on this artificial model, our simulations based on SFI-ASM suggest that the dividend amplitude parameter is a crucial factor on deciding the influence of technical rules. Such parameter can be regarded as a measurement of market uncertainty. The greater the uncertainty, the greater the price volatility, and so is the risk of investing in the stock market. The greater the risk, the greater the advantage of including technical rules, which agrees with the empirical realization from practical use of technical trading. And the prisoner's dilemma conclusion mentioned in former research work (Joshi *et al*., 2000) can be much more firmly supported with large dividend amplitude in a not long period.

References

Arthur, W.B., Holland, J., LeBaron, B., Palmer, R., Tayler, P., 1997. Asset Pricing under Endogenous Expectations in an Artificial Stock Market. *In*: Arthur, W.B., Durlauf, S., Lane, D.(Eds.), The Economy as an Evolving Complex System II. Addison-Wesley, Reading, MA, p.15-44.

Bierman, S.H., Fernandez, L., 1993. Game Theory with Economic Applications. Addison-Wesley, Redwood City, CA.

Bray, M., 1982. Learning, estimation, and the stability of rational expectations. *Journal of Economic Theory*, **26**(2): 318-339. [doi:10.1016/0022-0531(82)90007-2]

Ehrentreich, N., 2005. The Santa Fe artificial stock market re-examined-suggested corrections. *Journal of Economic Behavior and Organization* (in Press).

Goldberg, D.E., 1989. Genetic Algorithms in Search, Optimization and Machine Learning. Addison-Wesley, Reading, MA.

Grossman, S., Stiglitz, J., 1980. On the impossibility of informationally efficient markets. *American Economic Review*, **70**:393-408.

Holland, J., 1975. Adaptation in Natural and Artificial Systems. University of Michigan Press, Ann Arbor.

Johnson, P., 2002. What I learned from the artificial stock market. *Social Science Computer Review*, **20**(2):174-196.

Joshi, S., Parker, J., Bedau, M.A., 2000. Technical Trading Creates a Prisoner's Dilemma: Results from an Agent-Based Model. Computational Finance 99, MIT Press, Cambridge, MA, p.465-479.

Joshi, S., Parker, J., Bedau, M.A., 2002. Financial markets can be at sub-optimal equilibria. *Computational Economics*, **19**(1):5-23. [doi:10.1023/A:1014988805326]

LeBaron, B., 2002. Building the Santa Fe Artificial Stock Market. Working paper, Http://people.brandeis.edu/blebaron/wps/sfisum.pdf.

LeBaron, B., Arthur, W.B., Palmer, R., 1999. Time series properties of an artificial stock market. *Journal of Economic Dynamics and Control*, **23**(9-10):1487-1516. [doi:10.1016/S0165-1889(98)00081-5]

Mitchell, M., 1996. An Introduction to Genetic Algorithms. MIT Press, Cambridge, MA.

Palmer, R., Arthur, W.B., Holland, J., LeBaron, B., Tayler, P., 1994. Artificial economic life: Simple model of a stock market. *Physica*, **D75**:264-274.

SFI Bulletin, 1999. Http://www.santafe.edu/research/publications/bulletin.php, **14**(2):6-7.