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Nonlinear analytical solution for one-dimensional consolidation of soft soil under cyclic loading*

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Abstract: This paper presents an analytical solution for one-dimensional consolidation of soft soil under some common types of cyclic loading such as trapezoidal cyclic loading, based on the assumptions proposed by Davis and Raymond (1965) that the decrease in permeability is proportional to the decrease in compressibility during the consolidation process of the soil and that the distribution of initial effective stress is constant with depth. It is verified by the existing analytical solutions in special cases. Using the solution obtained, some diagrams are prepared and the relevant consolidation behavior is investigated.

Key words: One-dimensional (1D) consolidation, Nonlinear consolidation theory, Analytical solution, Cyclic loading
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INTRODUCTION

Conventional consolidation analysis based on Terzaghi's one-dimensional theory neglects the non-linearity of soil for the sake of mathematical expediency, which is considered to be one of the theory's shortcomings. Many researchers have proposed one-dimensional (1D) consolidation models where the coefficient of volume compressibility m_v, coefficient of permeability k_v, and coefficient of consolidation c_v, vary with either depth or time. Schiffman (1958), Davis and Raymond (1965) and Poskitt (1969) found that m_v and k_v are not constant, and decrease with increase of loading. Davis and Raymond (1965) developed a nonlinear consolidation theory based on the assumptions that the decrease in permeability is proportional to the decrease in compressibility during the consolidation process of soil and that the distribution of initial effective stress does not vary with depth. With the assumptions, (Xie *et al*., 1996a; 2002; Xie and Pan, 1995; Xie and Leo, 1999) deduced some solutions for the time-dependent loading and layered soil. Recently Zhuang *et al*.(2005) presented a more generalized theory for 1D consolidation of soil with variable compression and permeability taking into account the empirical e-lg$p$ and e-lgk_v relationships. Several solutions have been developed for linear consolidation (Wilson and Elgohary, 1974; Baligh and Levadoux, 1978; Favaretti and Soranzo, 1995; Chen *et al*., 1996; Xie *et al*., 1996b; Zhuang and Xie, 2005; Gao *et al*., 1999).

However, it seems that research on nonlinear consolidation theory for soil under cyclic loading is very limited. In this paper, based on (Davis and Raymond, 1965)'s assumptions, an analytical solution is derived for one-dimensional nonlinear consolidation of soil under low-frequency cyclic loading including trapezoidal cyclic loading. To our knowledge, it is the most general solution of the problem so far, and is suitable for small strain and 1D nonlinear consolidation. All the existing solutions for nonlinear consolidation are its particular cases. Some diagrams are prepared from the analytical solution, and thereafter the nonlinear consolidation behavior of soil under cyclic loading is discussed.

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ASSUMPTIONS AND MATHEMATICAL MODELLING

Assumptions

The assumptions of 1D nonlinear consolidation theory proposed by Davis and Raymond (1965) are as follows:

(1) During the consolidation process, the decrease in permeability is proportional to the decrease in compressibility, so $c_v=k_v/(m_v\gamma_w)=k_{v0}/(m_{v0}\gamma_w)=$ const., where c_v is coefficient of consolidation; m_v, m_{v0} are coefficient and initial coefficient of volume compression respectively; k_v, k_{v0} are coefficient and initial coefficient of permeability respectively, and γ_w is bulk density of water.

(2) The initial effective stress (i.e. self-weight stress) does not vary with depth during the consolidation process.

Furthermore, the basic assumptions in reference to classical Terzaghi's One-dimensional Consolidation Theory are:

(3) Saturated and homogeneous soil with impressible soil grains and pore fluid.

(4) Small strain and no creep.

(5) Validity of Darcy's Law.

(6) Empirical law shown by results of oedometer tests on normally consolidated soils:

$$e=e_0-C_c\log(\sigma'/\sigma_0'),$$

where e and e_0 are void ratio and initial void ratio of soil subjected to pressure σ' and initial pressure σ_0', respectively; C_c is compression index of soil. Thus m_{v0} can be given by

$$m_{v0}=-\frac{1}{1+e_0}\frac{\partial e}{\partial\sigma'}\Big|_{\sigma'=\sigma_0'}=\frac{0.434C_c}{(1+e_0)\sigma_0'}.$$

Mathematical modelling

In Fig.1, H is the thickness of the soil layer; $q(t)$ is a vertical uniform loading varying with time on the top of the soil surface. t and z are variables of time and space respectively.

Based on the hypotheses above, the consolidation differential equation is given by:

$$c_v\left[\frac{\partial^2 u}{\partial z^2}+\frac{1}{\sigma'}\left(\frac{\partial u}{\partial z}\right)^2\right]=\left(\frac{\partial u}{\partial t}-\frac{dq}{dt}\right), \tag{1}$$

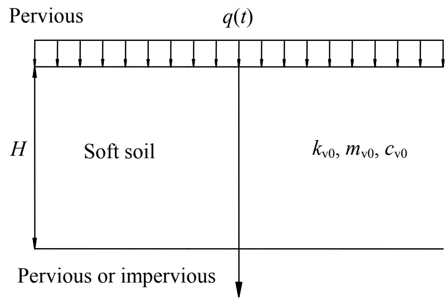

Fig.1 Consolidation model of the soft soil

where u is the excess water pressure, σ' is the effective stress.

Using Terzaghi's principle of effective stress, σ' can be calculated as:

$$\sigma'=q+\sigma_0'-u. \tag{2}$$

Let

$$\omega=\ln[\sigma'/(\sigma_0'+q)], \tag{3}$$

in terms of the function ω, Eq.(1) can be simplified as

$$\frac{\partial\omega}{\partial t}=c_v\frac{\partial^2\omega}{\partial z^2}+R(t), \tag{4}$$

in which,

$$R(t)=-\frac{1}{\sigma_0'+q}\frac{dq}{dt}. \tag{5}$$

The solution conditions of Eq.(4) in terms of u and ω can be given by the equations:

t=0: $u=q(0)=q_0$, or ω=0, (6)

z=0: u=0, or ω=0, (7)

$z=H$: $\partial u/\partial z=0$, or $\partial\omega/\partial z=0$ (for single-drainage situation), (8)

$z=H$: u=0, or ω=0 (for double-drainage situation). (9)

ANALYTICAL SOLUTIONS FOR THE CASE OF TRAPEZOIDAL CYCLIC LOADING

Excess pore water pressure

One period of the trapezoidal cyclic loading shown in Fig.2 can be divided into four stages and expressed as:

$$q(t)=\begin{cases} q_{\mathrm{u}}[t-(N-1)\beta t_0]/\alpha t_0, \\ \qquad (N-1)\beta t_0 \le t \le [(N-1)\beta+\alpha]t_0, \\ q_{\mathrm{u}}, \\ \quad [(N-1)\beta+\alpha]t_0 \le t \le [(N-1)\beta+(1-\alpha)]t_0, \\ -q_{\mathrm{u}}[t-(N-1)\beta t_0-t_0]/\alpha t_0, \\ \quad [(N-1)\beta+(1-\alpha)]t_0 \le t \le [(N-1)\beta+1]t_0, \\ 0, \qquad [(N-1)\beta+1]t_0 \le t \le N\beta t_0, \end{cases} \tag{10}$$

where q_{u} is the maximum loading during the cycle period, βt_0 is the time period of one loading cycle, α, β are the loading factors of the trapezoidal cyclic loading.

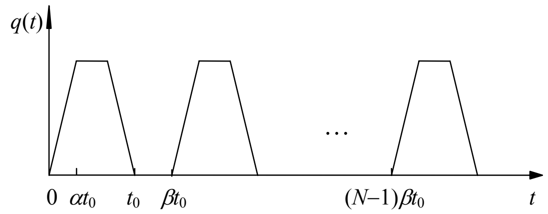

Fig.2 Trapezoidal cyclic loading

Single-drainage situation is considered first. In order to find the general solution of Eq.(4) subjected to the solution conditions of Eqs.(6)~(8), we assume that the solution of Eq.(4) may have the form below:

$$\omega=\sum_{m=1}^{\infty}\sin(\lambda_m z/H)C_m T_m(t)\mathrm{e}^{-\beta_m t}, \tag{11}$$

where β_m, B_m, C_m, λ_m are unknown coefficients to be found; $T_m(t)$ is the function of t, resulting from the item $R(t)$.

Eq.(11) has to satisfy Eqs.(4), (6), (7) and (8). Obviously Eq.(7) can always be satisfied by Eq.(11). Substituting Eq.(11) into Eq.(8) yields: $\lambda_m=M$, $M=(2m-1)\pi/2$, m=1, 2, 3, …, and similarly substituting Eq.(11) into Eq.(4) yields

$$\beta_m=c_{\mathrm{v}}M^2/H^2, \tag{12}$$

$$\sum_{m=1}^{\infty}\sin(Mz/H)\mathrm{e}^{-\beta_m t}C_m T_m'(t)=R(t). \tag{13}$$

From Eq.(13), the orthogonal characteristic of circular function is used to obtain:

$$C_m=2/M, \tag{14}$$

$$T_m=\int_0^t \mathrm{e}^{\beta_m\tau}R(\tau)\mathrm{d}\tau. \tag{15}$$

Substituting all the coefficients and $T_m(t)$ into Eq.(11), the integrated expression for ω can be obtained as:

$$\omega=\sum_{m=1}^{\infty}\frac{2}{M}\sin(Mz/H)T_m(t)\mathrm{e}^{-\beta_m t}. \tag{16}$$

By transforming Eq.(3), the excess pore water stress can be expressed as:

$$u=(\sigma_0'+q)(1-\mathrm{e}^{\omega}). \tag{17}$$

So far, an analytical solution to 1D nonlinear consolidation under trapezoidal cyclic loading for single-drainage condition has been completely deduced.

Similarly, an analytical solution for double-drainage condition can also be easily expressed by replacing H with $H/2$, while the rest are the same.

Average degree of consolidation

Average degree of consolidation can be defined either by settlement or by effective stress. The differences between them are: the former indicates the rate of settlement development, while the latter shows the rate of dissipation of excess pore water pressure or increase of effective pressure.

Average degree of consolidation defined by settlement can be expressed as:

$$U_{\mathrm{s}}=\frac{\int_0^H \varepsilon\mathrm{d}z}{\int_0^H \varepsilon_{\mathrm{f}}\mathrm{d}z}=\frac{\int_0^H \lg(\sigma'/\sigma_0')\mathrm{d}z}{\int_0^H \lg(\sigma_{\mathrm{f}}'/\sigma_0')\mathrm{d}z}=\frac{\ln\frac{\sigma_0'+q}{\sigma_0'}+\frac{1}{H}\int_0^H\omega\mathrm{d}z}{\ln\frac{\sigma_0'+q_{\mathrm{u}}}{\sigma_0'}}, \tag{18}$$

where the vertical strain $\varepsilon=\frac{e_0-e}{1+e_0}=\frac{C_{\mathrm{c}}}{1+e_0}\log\left(\frac{\sigma'}{\sigma_0'}\right)$; the final vertical strain $\varepsilon_{\mathrm{f}}=\frac{e_0-e_{\mathrm{f}}}{1+e_0}=\frac{C_{\mathrm{c}}}{1+e_0}\log\left(\frac{\sigma_{\mathrm{f}}'}{\sigma_0'}\right)$; the final void ratio $e_{\mathrm{f}}=e_0-C_{\mathrm{c}}\lg(\sigma_{\mathrm{f}}'/\sigma_0')$; the final effective stress $\sigma_{\mathrm{f}}'=\sigma_0'+q_{\mathrm{u}}$.

Average degree of consolidation defined by effective stress can be given by:

$$U_{\mathrm{p}}=\frac{\int_0^H(\sigma'-\sigma_0')\mathrm{d}z}{\int_0^H(\sigma_{\mathrm{f}}'-\sigma_0')\mathrm{d}z}=\frac{\int_0^H(q-u)\mathrm{d}z}{\int_0^H q_{\mathrm{u}}\mathrm{d}z}=\frac{q-\frac{1}{H}\int_0^H u\mathrm{d}z}{q_u}. \tag{19}$$

Complete solution

The complete analytical solution to 1D nonlinear consolidation problem under trapezoidal cyclic loading can now be expressed as follows:

$$u=\begin{cases}\dfrac{q_{\mathrm{u}}T_1}{N_\sigma-1}(1-\mathrm{e}^{-B_1}), & (N-1)\beta t_0\le t\le[(N-1)\beta+\alpha]t_0,\\ \dfrac{q_{\mathrm{u}}N_\sigma}{N_\sigma-1}(1-\mathrm{e}^{-B_2}), & [(N-1)\beta+\alpha]t_0\le t\le[(N-1)\beta+(1-\alpha)]t_0,\\ \dfrac{q_{\mathrm{u}}T_2}{N_\sigma-1}(1-\mathrm{e}^{-B_3}), & [(N-1)\beta+(1-\alpha)]t_0\le t\le[(N-1)\beta+1]t_0,\\ \dfrac{q_{\mathrm{u}}}{N_\sigma-1}(1-\mathrm{e}^{-B_4}), & [(N-1)\beta+1]t_0\le t\le N\beta t_0,\end{cases}\tag{20}$$

$$U_{\mathrm{s}}=\begin{cases}\dfrac{1}{\ln N_\sigma}\left(\ln T_1-\sum_{m=1}^{\infty}\dfrac{2C_1}{M^2}\mathrm{e}^{-M^2T_{\mathrm{v}}}\right), & (N-1)\beta t_0\le t\le[(N-1)\beta+\alpha]t_0,\\ \dfrac{1}{\ln N_\sigma}\left(\ln N_\sigma-\sum_{m=1}^{\infty}\dfrac{2C_2}{M^2}\mathrm{e}^{-M^2T_{\mathrm{v}}}\right), & [(N-1)\beta+\alpha]t_0\le t\le[(N-1)\beta+(1-\alpha)]t_0,\\ \dfrac{1}{\ln N_\sigma}\left(\ln T_2-\sum_{m=1}^{\infty}\dfrac{2C_3}{M^2}\mathrm{e}^{-M^2T_{\mathrm{v}}}\right), & [(N-1)\beta+(1-\alpha)]t_0\le t\le[(N-1)\beta+1]t_0,\\ -\dfrac{1}{\ln N_\sigma}\left(\sum_{m=1}^{\infty}\dfrac{2C_4}{M^2}\mathrm{e}^{-M^2T_{\mathrm{v}}}\right), & [(N-1)\beta+1]t_0\le t\le N\beta t_0,\end{cases}\tag{21}$$

$$U_{\mathrm{p}}=\begin{cases}\dfrac{T_1}{N_\sigma-1}\left(\dfrac{1}{H}\int_0^H\mathrm{e}^{-B_1}\mathrm{d}z-\dfrac{1}{T_1}\right), & (N-1)\beta t_0\le t\le[(N-1)\beta+\alpha]t_0,\\ \dfrac{N_\sigma}{N_\sigma-1}\left(\dfrac{1}{H}\int_0^H\mathrm{e}^{-B_2}\mathrm{d}z-\dfrac{1}{N_\sigma}\right), & [(N-1)\beta+\alpha]t_0\le t\le[(N-1)\beta+(1-\alpha)]t_0,\\ \dfrac{T_2}{N_\sigma-1}\left(\dfrac{1}{H}\int_0^H\mathrm{e}^{-B_3}\mathrm{d}z-\dfrac{1}{T_2}\right), & [(N-1)\beta+(1-\alpha)]t_0\le t\le[(N-1)\beta+1]t_0,\\ \dfrac{1}{N_\sigma-1}\left(\dfrac{1}{H}\int_0^H\mathrm{e}^{-B_4}\mathrm{d}z-1\right), & [(N-1)\beta+1]t_0\le t\le N\beta t_0,\end{cases}\tag{22}$$

where $N_\sigma=\dfrac{q_{\mathrm{u}}+\sigma_0'}{\sigma_0'}$, $T_{\mathrm{v}}=\dfrac{c_{\mathrm{v}}t}{H'^2}$,

$$T_1=\frac{T_{\mathrm{vc}}+(N_\sigma-1)(T_{\mathrm{v}}-T_{\mathrm{vb}})}{T_{\mathrm{vc}}},$$

$$T_2=\frac{T_{\mathrm{vc}}+(N_\sigma-1)(T_{\mathrm{vf}}-T_{\mathrm{v}})}{T_{\mathrm{vc}}},$$

$$T_{\mathrm{vc}}=\frac{c_{\mathrm{v}}\alpha t_0}{H'^2},\quad T_{\mathrm{vb}}=\frac{c_{\mathrm{v}}(N-1)\beta t_0}{H'^2},$$

$$T_{\mathrm{vf}}=\frac{c_{\mathrm{v}}[(N-1)\beta t_0+t_0]}{H'^2},$$

$$B_i=\sum_{m=1}^{\infty}\left[\frac{2C_i}{M}\sin\left(Mz/H'\right)\mathrm{e}^{-M^2T_{\mathrm{v}}}\right],\quad i=1, 2, 3, 4,$$

$$C_1=D(N-1)+D_1,\ C_2=D(N-1)+D_2,$$

$$C_3=D(N-1)+D_2+D_3,\ C_4=D(N),$$

$$D_1=\mathrm{e}^{\frac{M^2[(N_\sigma-1)T_{\mathrm{vb}}-T_{\mathrm{vc}}]}{N_\sigma-1}}\left[\ln T_1+\sum_{k=1}^{\infty}\frac{(M^2T_{\mathrm{vc}})^k(T_1^k-1)}{k!k(N_\sigma-1)^k}\right],$$

$$D_2=\mathrm{e}^{\frac{M^2[(N_\sigma-1)T_{\mathrm{vb}}-T_{\mathrm{vc}}]}{N_\sigma-1}}\left[\ln N_\sigma+\sum_{k=1}^{\infty}\frac{(M^2T_{\mathrm{vc}})^k(N_\sigma^k-1)}{k!k(N_\sigma-1)^k}\right],$$

$$D_3=\mathrm{e}^{\frac{M^2[(N_\sigma-1)T_{\mathrm{vf}}+T_{\mathrm{vc}}]}{N_\sigma-1}}\left[\ln\frac{T_2}{N_\sigma}+\sum_{k=1}^{\infty}\frac{(-M^2T_{\mathrm{vc}})^k(T_2^k-N_\sigma^k)}{k!k(N_\sigma-1)^k}\right],$$

$$D_4=\mathrm{e}^{\frac{M^2[(N_\sigma-1)T_{\mathrm{vf}}+T_{\mathrm{vc}}]}{N_\sigma-1}}\left[\ln\frac{1}{N_\sigma}+\sum_{k=1}^{\infty}\frac{(-M^2T_{\mathrm{vc}})^k(1-N_\sigma^k)}{k!k(N_\sigma-1)^k}\right],$$

$$D(n)=\sum_{N=1}^{n}(D_2+D_4),$$

$H'=H$ (for single-drainage),
$H'=H/2$ (for double-drainage).

ANALYTICAL SOLUTIONS FOR SPECIAL CASE

It is easy to demonstrate that the solution for stage I and stage II of the trapezoidal cyclic loading can be transformed into the analytical solution to 1D consolidation of soil under constant loading, proposed firstly by Davis and Raymond (1965), and into the one under linear loading, proposed by Xie *et al.*(1996b). Hence, the known solutions for nonlinear consolidation of soil are special cases of the solution obtained in this study.

Constant loading

When N=1, α=0 and β=1, trapezoidal cyclic loading reduces to constant loading, i.e., $q(t)=q_u$, $dq/dt=0$. Therefore, $R(t)=0$.

The solution is then given by:

$$u=\frac{q_u N_\sigma}{N_\sigma-1}(1-N_\sigma^{-B}), \tag{23}$$

$$U_s=1-\sum_{m=1}^{\infty}\frac{2}{M^2}e^{-M^2T_v}, \tag{24}$$

$$U_p=\frac{N_\sigma}{N_\sigma-1}\left[\frac{1}{H}\int_0^H N_\sigma^{-B}dz-\frac{1}{N_\sigma}\right], \tag{25}$$

where $B=\sum_{m=1}^{\infty}\left[\frac{2}{M}\sin(Mz/H')e^{-M^2T_v}\right]$.

Linear loading

When $N=1$, and t_0 tends to infinity, trapezoidal cyclic loading reduces into linear loading. Then,

$$q(t)=\begin{cases} q_u t/(\alpha t_0), & 0\le t\le \alpha t_0, \\ q_u, & t\ge \alpha t_0. \end{cases} \tag{26}$$

Therefore,

$$R(t)=\begin{cases} \dfrac{-1}{t+\alpha t_0 \sigma_0'/q_u}, & 0\le t\le \alpha t_0^{-}, \\ 0, & t\ge \alpha t_0^{+}. \end{cases} \tag{27}$$

The solution can be shown as follows:

$$u=\begin{cases} \dfrac{q_u T}{N_\sigma-1}(1-e^{-B_1}), & t\le t_c, \\ \dfrac{q_u N_\sigma}{N_\sigma-1}(1-e^{-B_2}), & t\ge t_c, \end{cases} \tag{28}$$

$$U_s=\begin{cases} \dfrac{1}{\ln N_\sigma}\left(\ln T-\sum_{m=1}^{\infty}\dfrac{2C_1}{M^2}e^{-M^2T_v}\right), & t\le t_c, \\ \dfrac{1}{\ln N_\sigma}\left(\ln N_\sigma-\sum_{m=1}^{\infty}\dfrac{2C_2}{M^2}e^{-M^2T_v}\right), & t\ge t_c, \end{cases} \tag{29}$$

$$U_p=\begin{cases} \dfrac{T}{N_\sigma-1}\left(\dfrac{1}{H}\int_0^H e^{-B_1}dz-\dfrac{1}{T}\right), & t\le t_c, \\ \dfrac{N_\sigma}{N_\sigma-1}\left(\dfrac{1}{H}\int_0^H e^{-B_2}dz-\dfrac{1}{N_\sigma}\right), & t\ge t_c, \end{cases} \tag{30}$$

where construction time $t_c=\alpha t_0$,

$$B_1=\sum_{m=1}^{\infty}\left[\frac{2C_1}{M}\sin(Mz/H')e^{-M^2T_v}\right],$$

$$B_2=\sum_{m=1}^{\infty}\left[\frac{2C_2}{M}\sin(Mz/H')e^{-M^2T_v}\right],$$

$$C_1=e^{-\frac{M^2T_{vc}}{N_\sigma-1}}\left[\ln T+\sum_{k=1}^{\infty}\frac{(M^2T_{vc})^k(T^k-1)}{k!k(N_\sigma-1)^k}\right],$$

$$C_2=e^{-\frac{M^2T_{vc}}{N_\sigma-1}}\left[\ln N_\sigma+\sum_{k=1}^{\infty}\frac{(M^2T_{vc})^k(N_\sigma^k-1)}{k!k(N_\sigma-1)^k}\right],$$

$$T=\frac{T_{vc}+(N_\sigma-1)T_v}{T_{vc}}.$$

DISCUSSION

The computation program based on the solution shown above has been developed to calculate the excess pore water pressure and degree of consolidation. The curves of consolidation U_p and U_s versus time corresponding to different values of parameters can be obtained accordingly. Some important diagrams shown in Figs.3~5 are prepared to analyze the influence of parameters on consolidation behavior of soil under trapezoidal cyclic loading.

In Fig.3, curves of average degrees of consolidation U_p and U_s versus time factor T_v are shown, corresponding to q_u=100 kPa, N_σ=3.0, β=1.5, T_{vc}=0.4, α=0.1, 0.3 and 0.4 respectively.

It shows that the value of average degree of consolidation during the relevant cycle will be greater with smaller α which represents the rate of loading or unloading.

Fig.4 shows the variations of average degree of consolidation U_p and U_s with T_{vc}, corresponding to q_u=100 kPa, N_σ=3.0, β=1.5, α=0.25, T_{vc}=0.01, 0.1 and 1.0 respectively.

It can be found that the peak values of average degrees of consolidation U_p and U_s tend to be steady after several cycles. Moreover, as T_{vc} increases, fewer cycles are needed to attain stable state. That is to say, the rate of consolidation of foundation soil will become stable after some few cycles.

Figs.3~4 also show that U_s is different from U_p in a nonlinear consolidation, and is a little greater than U_p at the same time.

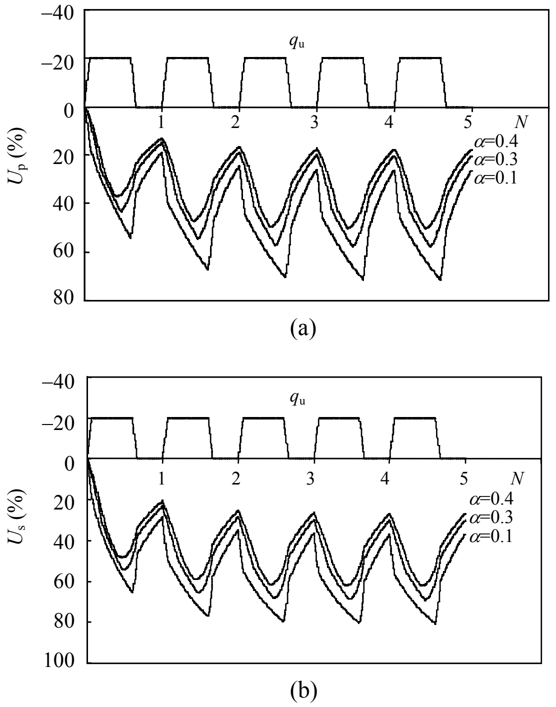

Fig.3 Influence of α on U_p~T_v curves (a) and on U_s~T_v curves (b). q_u=100 kPa, N_σ=3.0, β=1.5, T_{vc}=0.4

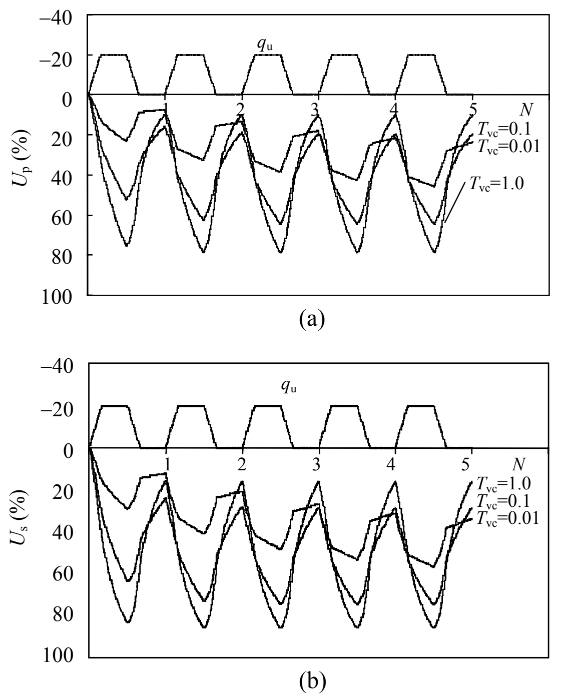

Fig.4 Influence of T_{vc} on U_p~T_v curves (a) and on U_s~T_v curves (b). q_u=100 kPa, N_σ=3.0, β=1.5, α=0.25

Fig.5 presents the variations of excess pore water pressure with dimensionless depth z/H, corresponding to q_u=125 kPa, N_σ=4.0, β=1.5, α=0.25, T_{vc}=0.3, z/H=0.1 and 0.5 respectively.

It can be seen that the greater the value of z/H, the greater the excess pore water pressure, indicating that the effective stress σ' and excess pore water pressure u vary with depth.

Besides, according to the solution given above, the consolidation degree and excess pore water pressure also vary with the parameter β and the ratio of final effective pressure to initial effective pressure N_σ.

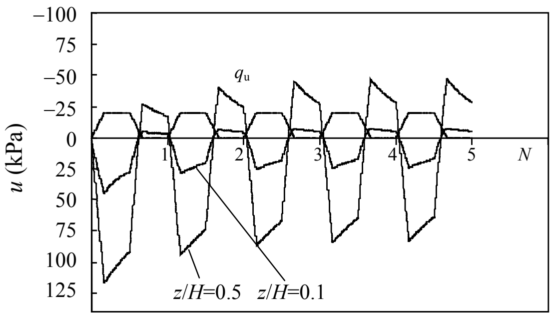

Fig.5 Influence of z/H on u~T_v curves. q_u=125 kPa, N_σ=4.0, α=0.25, β=1.5, T_{vc}=0.3

CONCLUSION

(1) Based on the assumptions proposed by Davis and Raymond (1965) that the decrease in permeability is proportional to the decrease in compressibility during the consolidation process of soil and that the distribution of initial effective stress is constant with depth, an analytical solution is deduced for the one-dimensional nonlinear consolidation of soil under trapezoidal cyclic loading. It is shown that the existing solutions so far are special cases of the solution obtained in this study.

(2) In nonlinear consolidation, U_p and U_s are different. Moreover, U_p is a little greater than U_s at the same time.

(3) Compared with constant and linear loading, when the soil is under cyclic loading, U_s, U_p and effective stress σ' do not increase with time throughout, but fluctuate with the loading and unloading. At the beginning, the difference is small.

After the first unloading, the difference becomes greater and greater. Therefore, the average consolidation degrees of soil under cyclic loading are much lower than those under constant and linear loading with the same maximum loading. The consolidation degree and pore water pressure values increase during the loading, but decrease during the unloading. After certain cycles, the fluctuation will tend to stable conditions. The values reach maximum (i.e. wave crest) when unloading starts and reach minimum (i.e. wave trough) when loading begins. Moreover, the slope of the turning point at each stage changes greatly.

(4) Apart from boundary drainage conditions, the main factors affecting the rate of nonlinear consolidation under cyclic loading are loading factors α, β and T_{vc}, the ratio of final effective pressure to initial effective pressure N_σ, the thickness of soft soil H, the time t_0 and the initial coefficients of volume compressibility m_{v0} and permeability k_{v0}.

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