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3DTS: A 3D tolerancing system based on mathematical definition*

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Abstract: Tolerance is almost ubiquitous during the whole product lift cycle and is imperative for seamless integration of CAD and CAM. Based on the mathematical definition of tolerance, a 3D tolerancing system, 3DTS, is presented with its design principle, system architecture and key functions. The following functional modules, tolerance modeling, semantics interpretation, 3D tolerance analysis, are described in detail. To make the tolerancing system robust and efficient, many techniques such as hierarchical tolerance representation, rule-based evaluation and non-intersection determination of tolerance zone have been devised. Tested by many samples, this system shows good robustness and practicability.

Key words: Tolerance, Mathematical definition, CAD/CAM
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INTRODUCTION

Tolerance is imperative for seamless integration of computer-aided design (CAD) and computer aided manufacturing (CAM) (Bjorke, 1989; Wu and Yang, 1999) and influences greatly the quality, process planning, measurement, cost and assembly of the product. Many researches have been conducted on tolerance (Baer, 1979; Requicha, 1982; 1983; 1992; Clement, 1991). Most of these researches are focused on tolerance analysis and tolerance synthesis. In some commercial CAD/CAM systems the tolerance module has also been imbedded (EDA/VisVSAs in IDEAS, CelTol in Pro/E, 3DCS in CATIA, eMTolMate in eMPOWER, etc.). Furthermore, in those systems tolerance information has already been correspondingly used with entity element. However there are still some issues. The tolerances are just analyzed numerically in most of those modules, such as statistical tolerance analysis and worst-case tolerance analysis. Tolerance analysis can just be conducted for 2D engineering drawings and many preparation works. For example, generation of the dimensional chain must be completed by hand. Moreover, tolerance information is just a text attribute in essence and lacks the interpretation of the engineering semantics. It is very difficult to carry out effectiveness evaluation of the given tolerance specification. Additionally in above mentioned imbedded tolerance modules, the used tolerance type is still very limited. And only the dimensional tolerance is analyzed. The geometric tolerances, which are widely used in mechanical design, are still not included in these tolerance modules.

In this study, a 3D tolerancing system is developed based on the mathematical definition of tolerance (ASME, 1994). The objectives of this work are: (1) Constructing a computer understandable tolerance representation model; (2) Interpreting the tolerance semantics in 3D environment; (3) Conducting tolerance analysis in 3D environment.

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SYSTEM STRUCTURE

Function demand analysis of the 3D tolerancing system

As an important part of the whole product modeling, the 3D tolerancing system must have the following attributes in order to satisfy the requirement of the industry:

(1) Three-dimension. The system should deal with all the related tolerance functionalities in 3D environment.

(2) Wholeness. The system should correctly deal with all kinds of tolerance types, especially the complex tolerance types.

(3) Rationality. The system should give a rational representation of tolerance. On the one hand, all kinds of tolerance information should be organized and expressed independently and the semantic difference of all kinds of tolerance type should also be reflected. On the other hand, a fundamental framework for tolerance information integrated in CAD systems should be established.

(4) Interactivity. The system should offer the function with which the users can revise the tolerance interactively. If the users are not satisfied with the given tolerance type and size, they can change those conveniently.

(5) Effectiveness. The system should verify the correctness and validity of tolerance. Meanwhile, suggestions and methods for modifying the tolerance should also be given to the unreasonable part.

(6) Interpretability. The system should give the semantic interpretation according to the engineering semantics for different tolerance types.

(7) Seamlessness. The system should be integrated with the fundamental CAD system seamlessly.

System structure

According to above analysis of the function tolerance, a 3D tolerancing system is developed based on mathematical definition as shown in Fig.1. There are four layers: the kernel layer, functionality layer, application interface layer and user interface layer.

(1) Kernel module. It is composed of the kernel algorithms of tolerance representation, interpretation and analysis based on mathematical definition with the ACIS geometric modeling kernel, tolerance type database and rule database. This module is fundamental for the other parts of the system.

(2) Functionality module. It is the executive part of the whole system, and includes tolerance representation module, tolerance correctness evaluation module, tolerance semantics interpretation module, variational geometry generation module and 3D tolerance analysis module. The main functionality of each module will be introduced in detail in the following parts.

(3) Application interface. It provides the interface between the 3D tolerancing system and the subsequent application such as process planning, manufacturability evaluation, virtual manufacturing, virtual inspection and other applications.

(4) User interfaces. Users can call the functionality module and carry out the tasks of tolerance representation, modeling, evaluation and analysis through the user interface. It can also be used to show the generated variational geometry.

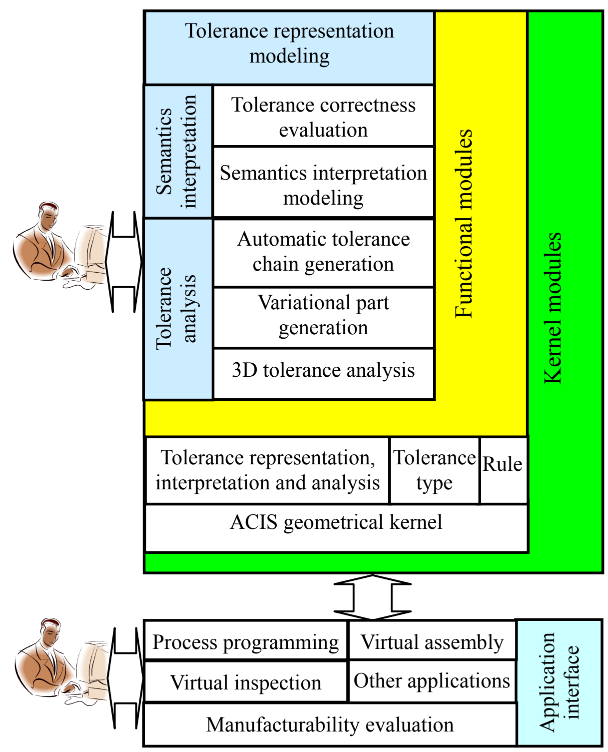


Fig.1 Functional modules of 3DTS

DESIGN AND REALIZATION OF PRIMARY FUNCTIONS

Hierarchical tolerance representation model

This system has three tolerance representation layers: (1) Layer of Feature-based Topologically and

Technologically Related Surfaces (FTTRS), (2) Layer of Minimum Feature Datum Elements (MFDE), (3) Layer of Constraint Primitive (CP). Noticeably, FTTRS is made up of two surfaces belonging to the same feature (or two child-FTTRS) in a given order. It is designed as a tree structure during implementation. Two surfaces (or two child-FTTRS) are the left sub-tree and right sub-tree respectively. MFDE is the minimum element used to locate the feature. For example, if the hole feature needs to be located, only the central axis needs to be located. Similarly only the center point needs to be located when the sphere feature needs to be located. Generally, the central elements are MFDEs. The CP is used to restrict the dimension constraints of the MFDE whose rotational degrees of freedom (RDOFs) are determined impliedly. Liu (2000) gave detailed methods for constructing the tolerance layers.

According to tolerance semantics, the form tolerance is mainly specified for the surface elements (including the surfaces and boundary edges). It should be attached on the FTTRS layer. The location tolerance should be attached on the MFDE layer because it is closely related to the location and orientation of the feature. The dimension tolerance should be attached on the CP layer used to determine the extreme variational range of the dimension constraint between the features. Consequently, as shown in Fig.2, the feature-based hierarchical tolerance model has FTTRS layer, MFDE layer and CP layer. There are close relationships between those three layers. On the one hand, dimension tolerance is always greater than location tolerance and location tolerance is always greater than form tolerance. On the other hand, through the tolerance representation models of this kind of layer type it is easy to evaluate the rationality, abundance, and validity of tolerance conveniently.

On the other hand, the tolerance should be organized itself. Here object-oriented technology is used to define all kinds of tolerance with a hierarchical structure. In order to represent tolerance semantics more explicitly, the notion of non-datum tolerance class and of-datum tolerance class are also introduced. Non-datum tolerance class is the tolerance that is only related to the toleranced element itself, i.e. there is no any reference datum for this type of tolerance. While of-datum tolerance class is the tolerance that is not only related to the toleranced element, but also related to the referenced datum elements.

In our implementation, the ATTRIB class offered by ACIS geometrical kernel is used to establish

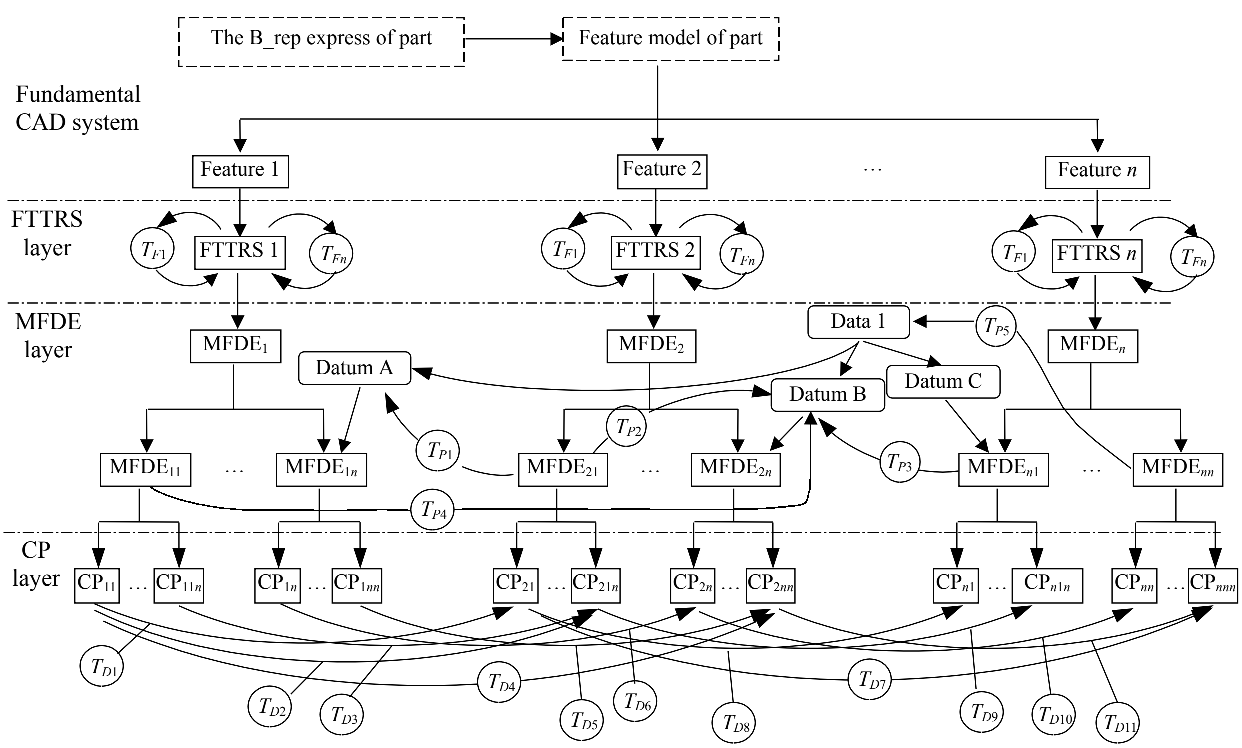


Fig.2 The layers of tolerance representation model based on feature

the hierarchical structure of all the necessary tolerance classes as illustrated in Fig.3. In this way the tolerance information can become a natural part of CAD systems.

Tolerance correctness evaluation

The tolerance specification is closely related to the functional requirement of a part. In practical design only the important sections of a part are specified with tolerance requirements. Furthermore, the tolerance type and size are determined according to engineering practice analysis and standard and the existing design instances can also be referred to. For other tolerance requirements, they are determined with the default values of manufacture procedure. A complete tolerance specification is constructed with the default tolerance specification and the specified tolerance. For such a tolerance specification, the correctness cannot be guaranteed and the following three aspects should be considered:

(1) Is the given tolerance type and size reasonable?

(2) Is the given tolerance complete?

(3) Is the given tolerance specification valid?

So it is necessary to evaluate the tolerance correctness when conducting computer-aided tolerancing. In this system the tolerance completeness is evaluated through determining whether the DOFs are correctly constrained with defaulted tolerance or the specified tolerance. The tolerance validity is evaluated by generating variational geometry. Moreover, how to evaluate the tolerance reasonability is discussed here.

Tolerance reasonability includes two aspects: tolerance type reasonability and tolerance value reasonability. Tolerance type reasonability refers to whether the given tolerance type is reasonable for the toleranced feature and whether correct semantics can be explained. Tolerance value reasonability refers to whether tight tolerance is given for a feature that has loose tolerance requirements, which can lead to inconsistence between the tolerance allowed variation and another tolerance allowed variation. Or, on the other hand, very tight tolerance is given to the feature with too loose tolerance requirements. Both situations lead to high manufacturing cost. In this system tolerance reasonability is evaluated based on rules. The general described form for a rule is as follows:

Rule # IF Condition THEN Action

where Rule # is rule number that represents the serial number in rule database; Condition is called the conditional part, front item or the left part of the expression; Action is called as the conclusion (operation) part, back item or the right part of the expression.

An example rule is given as follows:

Rule 1 IF the object that is given tolerance specification is planar

THEN the specified form tolerance must be flatness

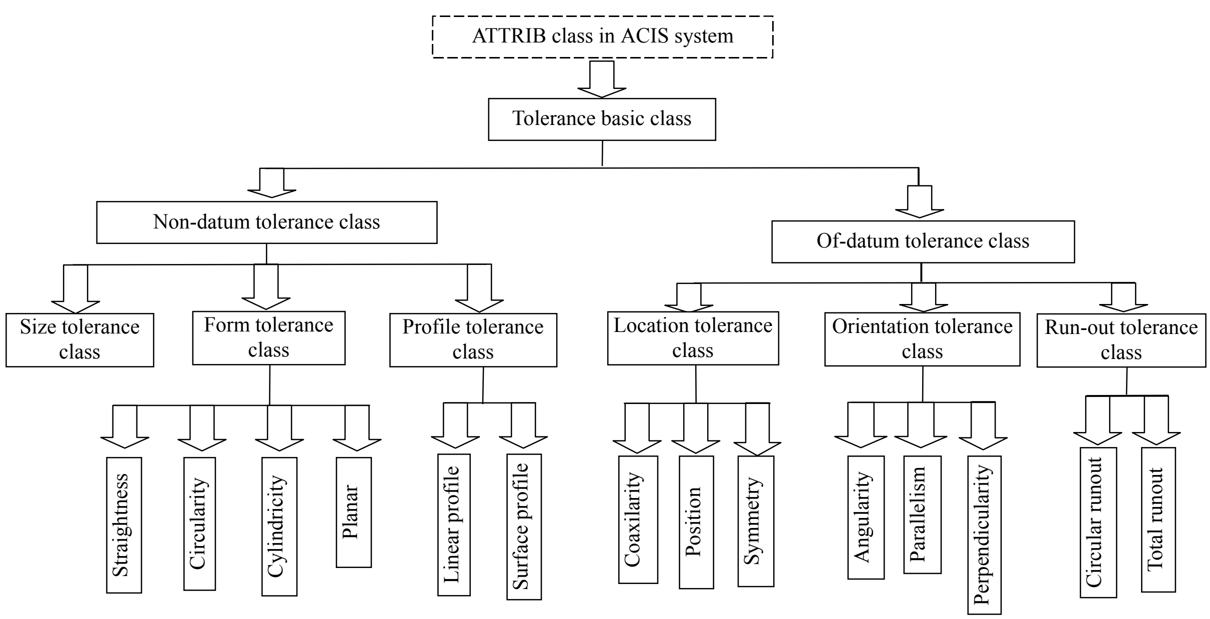


Fig.3 The hierarchical tolerance class

Tolerance semantics interpretation

In essence, the tolerance semantics can be embodied in the tolerance zone, which has four basic attributes: position, direction, form and size. The form of the tolerance zone is decided although different tolerance types have the same or different forms of tolerance zone. In the other three attributes of tolerance zone, the size of tolerance zone is specified by the designers. Therefore the key to exactly represent the tolerance semantics is to determine the position and direction of the tolerance zone. Analyzing of the position and direction of the tolerance zone revealed that it can be divided into three types: (1) The tolerance that tolerance zone is fixed; (2) The tolerance that tolerance zone can be translated; (3) The tolerance that tolerance zone can be floated. The traditional tolerance types are reclassified as shown in Table 1.

Table 1 Classification of tolerance

New tolerance type	Traditional tolerance type
Tolerance that tolerance zone is fixed	Dimensional tolerance*, coaxiality, symmetry, positional tolerance*, linear profile, planar profile
Tolerance that tolerance zone is translated	Dimensional tolerance*, positional tolerance*, parallelism, perpendicular, angularity
Tolerance that tolerance zone is floated	Straightness, flatness, circularity, cylindricity

*Dimension tolerance and positional tolerance belong to different type under different conditions

Different interpretation model can be used to explain the tolerance semantics for the tolerance types (Liu and Gao, 2004; Liu *et al*., 2003a; 2003b). Meanwhile the effect of tolerance principles should also be considered. In this system, maximum material condition (MMC) and envelope principle are both considered.

Automatic generation of dimensional chain

The generation of dimensional chain is imperative for the subsequent tasks of computer-aided tolerancing such as tolerance analysis and synthesis. Dimensional chain is made up of the connected dimensions that constitute a closed dimensional link. So the key issue for automatic generation of dimensional chain is how to search the related dimension belonging to the same dimensional link.

Generally there are three methods for generating dimensional chain: coordinate method, chain method, and synthesis method. For chain method, there always exist two CPs in the dimensional model, which only serve as constraining CP or constrained CP respectively. And they are called as head CP and end CP. The other CPs are constrained CP of one dimensional constraint and the constraining CP of the next dimensional constraint simultaneously. So in the chain method, the dimensional chain can be formed naturally between head CP and end CP and search is not needed anymore. The coordinate method always has a common datum CP. It is convenient to construct dimensional chain between any two CPs even if there is no direct constraint relationship between any two dimensions. A simple dimensional chain of three links (including closing) is formed only by finding out the 2D CPs linked by two CPs. In the synthesis method, the generation of dimensional chain is more complex due to lack of regular rules. A systematic algorithm is given here for generating dimensional chain automatically. Without loss of generality, a dimensional design plan of a part's x-direction is shown in Fig.4a where $x\Sigma_1$ is regarded as the closing. The process is as follows:

(1) Find CPs f_4 and f_5 that formed $x\Sigma_1$. Also find all the dimensional constraints related to f_4 (marked as x_3 and x_5 as shown in Fig.4a), which serve as constraining CP or constrained CP in the dimension constraint f_4. Meanwhile the attribute of increasing link or decreasing link should also be determined.

(2) The CP related to x_3 (x_5), such as f_2 and f_6 is found as shown in Fig.4.

(3) Find the dimensional constraints and related

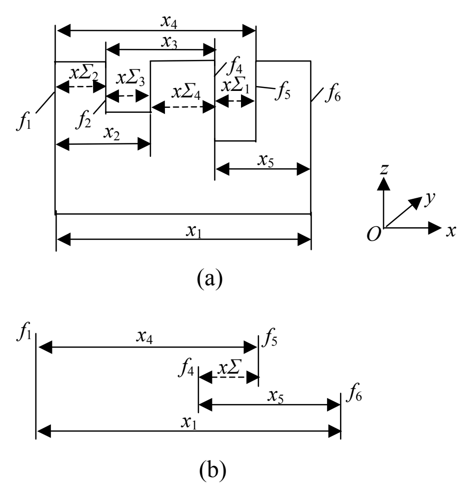


Fig.4 Tolerance chain generate automatically. (a) Scheme of dimension design; (b) Chart of tolerance chain generate automatically

CPs (expected to be found CP) regarding f_2 and f_6 as the constraining CP or constrained CP in the dimensional model until another CP (f_5) in $x\Sigma_1$ is encountered. The dimensional chain is shown in Fig.4b.

Generation of variational geometry

In this system the generation of variational geometry is completed with the help of modeling tools of the geometric modeling engine ACIS 6.0. The variational geometry can be generated with two steps: The first step is to generate all the variational elements such as the variational surface and variational central axis; the second step is to regenerate the whole variational part through the surface-to-surface intersection, curve-to-curve intersection and surface-to-curve intersection. A general generation algorithm of the variation geometry is given as follows:

Step 1: Establish the local coordinate system;

Step 2: Determine the position of locational dimension tolerance zone ZL (or locational tolerance zone);

Step 3: Determine the position of orientational tolerance zone ZO in ZL (if there is orientational tolerance specification) and obtain the resultant tolerance zone (RTZ) ZLO by conducting the intersection between the tolerance ZO and ZL;

Step 4: Determine the orientation and position of the form tolerance zone in the above RTZ ZLO (if there is form tolerance specification);

Step 5: Generate RTZ ZLOF. Noticeably for the central elements, if there exists tolerance principle for the tolerance specification, the compensation between the dimensional tolerance of surface and the tolerance of central elements should be considered.

Step 6: Generate the corresponding variational elements satisfying the tolerance specification according to different interpretation of the tolerance engineering semantics. After that, and the needed variational body is generated according to different cases:

(1) For the plane, the variational plane can be simulated with a planar surface with variational position and direction if there no form tolerance requirement for the plane. Otherwise it is simulated with NURBS curved surface where the variational elements are the discrete points satisfying the tolerance requirement.

(2) For the line, the variational line can be simulated directly with a straight line with variational position and direction if there is no form tolerance requirement for the line. Otherwise it is simulated with NURBS curve where the generation variational elements are the discrete points satisfying the tolerance requirement.

(3) For cylinder surface, it is sliced equally into many equal segments along the direction perpendicular to the axis. The variational cylinder surface of each slide is generated which satisfies the tolerance requirement. Then the whole variational cylinder surface is generated based on skinning operation of ACIS geometric engine.

3D tolerance analysis

Tolerance analysis is an important step in computer-aided tolerancing used to determine whether the specified tolerance type and tolerance value satisfy the functional requirement. Here variational geometries conduct the tolerance analysis in 3D environment. Moreover, the geometric tolerances are also considered in the 3D tolerance analysis. In this system, the statistical method is adopted for tolerance analysis. The process is described as follows:

(1) Determine the distribution function of the link errors. Here asymmetry distributions function, β-distribution, is used as the distribution function of the errors. The possibility density function of β-distribution is given as follows:

$$f(\mu,\alpha,\beta)=\begin{cases}\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)}\mu^{\alpha-1}(1-\mu)^{\alpha-1}, \\ \qquad\qquad 0\le\mu\le 1, \alpha\ge 0, \beta\ge 0; \\ 0, \qquad\qquad \text{others.}\end{cases}$$

Different values of α, β will generate different forms of distribution as shown in Fig.5.

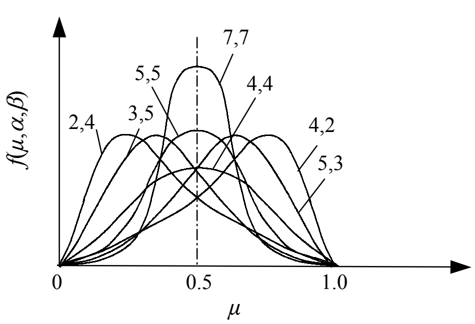


Fig.5 Different form of β-distribution

(2) Estimate the statistical parameters α_r, β_r of the closing which are unknown before the calculation. It should be estimated to avoid meaningless tolerance analysis. The estimation method for α_r, β_r is given as (Liu, 2000):

$$\alpha_r = \frac{\bar{\mu}_r^2 - \bar{\mu}_r^3 - \bar{\mu}_r \cdot v'_r}{v'_r},$$

$$\beta_r = \frac{(1-\bar{\mu}_r)(\bar{\mu}_r^2 - \bar{\mu}_r^3 - \bar{\mu}_r \cdot v'_r)}{\bar{\mu}_r \cdot v'_r},$$

where, $\bar{\mu}_r$ and v'_r are the mean value and standard variance of the closing for unit β-distribution. They are calculated as follows:

$$\bar{\mu}_r = \frac{1}{n}\mu_r = \frac{1}{n}\sum_{i=1}^{n}\mu_i,$$

$$v'_r = \frac{1}{n^2}v_r = \frac{1}{n^2}\sum_{i=1}^{n}v_i,$$

where μ_i and v_i are the mean value and standard variance of the composing link i respectively.

(3) Generate the random variational geometries for each composing link according to the distribution parameters and models. And then conduct direct simulated measurement of the dimension of the closing.

(4) Repeat Step 3 the required times and calculate the possibility of satisfying the functional requirements to check if the tolerance specification is satisfied.

IMPROVEMENT OF THE COMPUTATIONAL EFFICIENCY

The 3D tolerancing system has many Boolean intersection operations which must be conducted to get the resultant tolerance zone and be very time consuming. This problem may even make it unacceptable to perform tolerance analysis in 3D CAD systems. In this system, we propose a direct and analytical method that can improve the efficiency dramatically in two steps: (1) Converting 3D Boolean intersection operations into 2D ones; (2) Avoiding the 2D Boolean intersection operations by obtaining the intersection analytically.

In most cases, RTZ can be degenerated from 3D to 2D as shown in Fig.6. If the form tolerance is specified, the situation will be changed because of the uncertain position and uncertain orientation. RTZ is complicated as shown in Fig.7. When the form tolerance is specified, we slice the 3D tolerance zone into a series of 2D flakes in order to simplify the representation and modeling of RTZ so that RTZ can be uniformly treated in 2D environment.

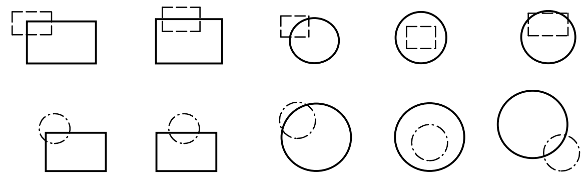


Fig.6 Some degenerated resultant tolerance zone

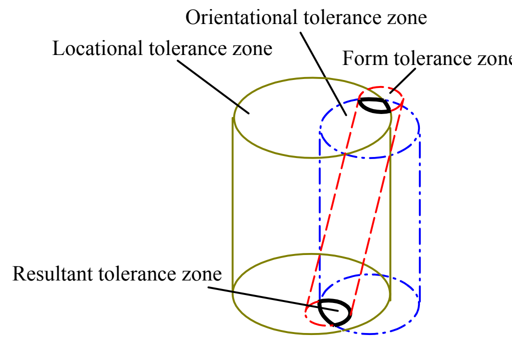
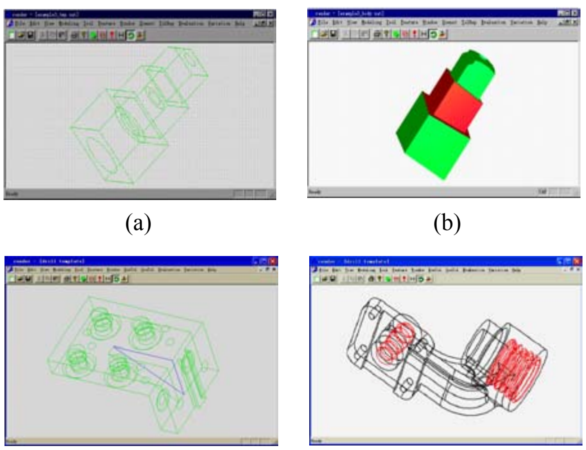


Fig.7 An example of complicated resultant tolerance zone

Furthermore, we found that the determined elements involved in the RTZ are circle and straight line. There exist only three types of Boolean intersection operations: circle-to-circle intersection, circle-to-line intersection and line-to-line intersection. Considering the different location relation of the tolerance zones, the possible number of Boolean intersection operation for the above three types can be 8 for line-to-line intersection, 28 for line-to-circle intersection and 1 for circle-to-circle intersection. It is obvious that for all kinds of the enumerated cases, the intersection points can all be analytically calculated and parameterized with the position of tolerance zone and tolerance value. Given the tolerance value and the position of the resultant tolerance zone, the intersection points can be calculated easily and efficiently.

IMPLEMENTATION AND EXAMPLES

3DTS system was developed based on geometry model engine ACIS 6.0 with Microsoft Visual C++ 6.0. The interface of the system is shown in Fig.8a. The input of 3DTS can be the production model created by other CAD systems such as MDT, Pro/E, SolidWorks and so on. The interactive interface for tolerance input is shown in Fig.9. We tested the system by using many parts with different type of features as shown in Figs.8b~8d. The tested feature types included planar feature, hole feature, straight-line feature and pattern of holes. For testing the robustness of the system, different tolerance specifications were imposed on the parts. The test results showed that the system can deal with complicated parts with different type of features and different tolerance requirements and has good robustness and practicability.

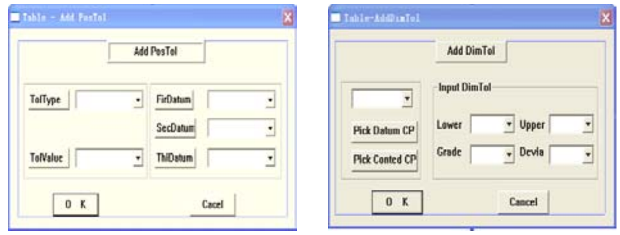


(a) (b) (c) (d)

Fig.8 System interface and test examples. (a) Prototype system and main interface; (b) Variational geometry 1; (c) Variational geometry 2; (d) Variational geometry 3

3D tolerance analysis was conducted on the part given in Fig.10. The tolerance requirements are given in Table 2 and the virtual measurement results for the closing are given in Table 3 (The number N is 240). Here for the β-distribution, the value of α and β is 1.5 and 3 respectively. Obviously the tolerance analysis here is easy to conduct and just need to directly measure the dimension of the closing. From Table 3, we can easily get the probability of satisfying the requirement calculated as follows:

$$p = (240 - 5)/240 = 97.92\%.$$



(a) (b)

Fig.9 Input interface of dimension tolerance information (a) and position tolerance information (b)

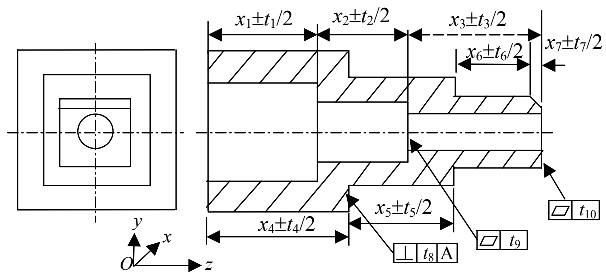


Fig.10 An example part for tolerance analysis

Table 2 The tolerance requirement of the part in Fig.10

Dimension	Dimension value	Tolerance	Tolerance value
x_1	40	t_1	0.12
x_2	35	t_2	0.12
x_3	50	t_3	0.22
x_4	50	t_4	0.08
x_5	40	t_5	0.08
x_6	30	t_6	0.07
x_7	5	t_7	0.03
–	–	t_8	0.03
–	–	t_9	0.04
–	–	t_{10}	0.03

x_3 is the closing

CONCLUSION

The system structure, functional module, kernel functionality and implementation of the developed 3D tolerancing system are presented in this paper. The major contributions of the work include:

(1) A hierarchical tolerance definition is given for all kinds of tolerance type and the tolerance principle is also considered in our systems. By this way, all types of tolerance defined in the mathematical definition as well as the compound tolerances can be

Table 3 The virtual measurement error results for the closing (mm)

0.014	–0.105	–0.163	–0.189	0.017	–0.201	–0.114	–0.088	0.048	–0.078	–0.045	0.017
–0.152	–0.098	–0.062	–0.125	0.056	0.108	0.103	0.104	0.019	–0.156	–0.044	–0.119
–0.227	0.147	–0.012	–0.066	–0.182	0.023	–0.041	0.004	–0.092	–0.022	–0.025	0.147
–0.199	–0.116	–0.127	0.018	0.115	–0.081	0.021	–0.005	–0.033	0.136	–0.191	–0.028
–0.155	0.061	–0.076	–0.144	–0.172	–0.233	–0.077	0.090	0.021	–0.013	–0.051	–0.099
0.093	0.005	–0.141	–0.189	–0.127	–0.018	0.014	–0.118	–0.131	–0.081	–0.094	–0.112
0.040	0.019	–0.001	–0.089	–0.024	–0.135	0.036	0.013	–0.026	0.014	0.047	–0.210
–0.044	–0.183	0.052	–0.221	0.065	–0.133	0.087	–0.176	–0.196	–0.089	0.055	0.111
0.045	–0.183	0.088	0.075	–0.023	0.161	–0.194	–0.033	–0.092	0.127	0.049	–0.134
–0.139	0.072	–0.178	–0.036	0.029	–0.154	0.051	–0.125	–0.172	–0.141	–0.095	0.101
0.073	–0.003	–0.037	0.118	–0.142	–0.095	–0.102	0.032	0.055	–0.144	0.012	0.095
–0.146	–0.161	–0.217	–0.185	–0.029	0.101	–0.143	0.042	–0.127	0.022	–0.114	–0.099
–0.144	0.067	–0.045	–0.110	–0.070	–0.089	0.080	–0.120	–0.032	0.042	0.023	–0.044
–0.052	–0.125	0.083	–0.095	0.062	–0.014	–0.122	–0.090	0.089	–0.094	–0.136	0.030
0.091	–0.117	–0.145	–0.080	–0.061	0.055	0.087	0.066	0.016	–0.117	–0.136	–0.239
–0.161	0.013	–0.012	0.002	–0.071	–0.113	–0.072	0.088	–0.022	0.038	–0.028	–0.088
0.015	–0.046	0.031	–0.067	0.021	–0.028	0.079	–0.091	–0.068	–0.035	–0.046	0.007
–0.103	–0.043	–0.029	–0.073	–0.091	–0.089	–0.170	0.153	–0.151	–0.105	0.118	–0.068
–0.089	–0.116	–0.201	–0.071	–0.057	–0.228	–0.023	–0.042	–0.179	0.018	–0.039	–0.155
0.044	–0.030	0.065	0.101	–0.091	–0.169	–0.124	–0.014	0.023	–0.148	–0.144	0.101

dealt with robustly in the system.

(2) A hierarchical tolerance representation model is devised which can represent the essence of tolerance semantics and facilitate the integration of the tolerance with the CAD systems.

(3) A rule-based evaluation method is given for the correctness of tolerance reasonability. Meanwhile, the completeness and validity is also evaluated to guarantee the tolerance information's reasonability, completeness and effectiveness.

(4) A complete interpretation of the engineering semantics of the tolerance is given based on mathematical definition.

(5) An efficient method is proposed for the generation of variational geometry. Based on this method, there is no need for any intersection when generating the resultant tolerance zone, which is necessary for generating the variational geometry.

(6) A method is given for automatic generation of the dimensional chain.

(7) 3D tolerance analysis is conducted with the help of the variational geometry.

Future work will be focused on: (1) Further strengthening the robustness of the system; (2) Extending the function of the system according to industry requirements; (3) Integrating the 3DTS with other CAD and CAPP systems.

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