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Predictive control of a class of bilinear systems based on global off-line models*

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Abstract: A new multi-step adaptive predictive control algorithm for a class of bilinear systems is presented. The structure of the bilinear system is converted into a simple linear model by using nonlinear support vector machine (SVM) dynamic approximation with analytical control law derived. The method does not need on-line parameters estimation because the system's internal model has been transformed into an off-line global model. Compared with other traditional methods, this control law reduces on-line parameter estimating burden. In addition, its overall linear behavior treating method allows an analytical control law available and avoids on-line nonlinear optimization. Simulation results are presented in the article to illustrate the efficiency of the method.

Key words: Bilinear systems, Model predictive control (MPC), Adaptive control, Support vector machine (SVM)
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INTRODUCTION

Bilinear systems are a kind of important nonlinear systems with relatively simple structure, and many industrial processes can be described as a bilinear system. Thus research on the control of this kind of systems is very important. On the other hand, model predictive control (MPC) (Clarke *et al*., 1987) has been widely used in industrial applications and many predictive control methods focusing on bilinear systems are emerging (Bloemen *et al*., 2001; Fontes *et al*., 2004; He *et al*., 1999; Lakhdari *et al*., 1995; Liu and Li, 2004; Yao and Qian, 1997). These methods have solved some problems facing bilinear systems and resulted in good control performance. However, there are still some problems remained to be solved, for instance: (1) If direct use of nonlinear models are adopted, the controller design may result in on-line solving a high order nonlinear optimization problem which may get stuck in some local minimum area; (2) Piecewise linearization method results in the easy solution of a (or a set of) quadratic programming problem(s), but on-line estimation of many linear models in only small regions is difficult in real applications; (3) Some methods take bilinear systems as time-varying linear systems and on-line estimating methods are applied, however, these bilinear systems are nonlinear systems in nature and quick and accurate on-line estimating of parameters is not easy.

In many cases, a lot of nonlinear systems can be regarded as a kind of systems in which operating points vary with operating conditions (Peng *et al*., 2002). This means that nonlinear systems can be locally linearized at any operating points. By doing so, linear time-varying model may be used as the internal model for MPC, however, this method needs on-line parameter estimating.

In this paper, a MPC design method for a class of bilinear systems is presented, which is based on an off-line support vector machine ARX model (SVM-ARX). This SVM-ARX model is identified

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off-line and used as the global model to represent the system dynamics within the whole range of operation. By doing so, the tiring on-line estimation of parameters is not required anymore, and as the obtained model is a global linear model, nonlinear optimization is not required, either. Thus the obtained control law is analytical. To illustrate this method, two examples are given. The rest of the paper is organized as follows: Section 2 deals with the system model and gives a new global off-line model design method. Controller design method is presented in Section 3. Section 4 gives comparison studies on different types of bilinear systems. Conclusion is in Section 5.

SYSTEM MODEL AND ITS PRESENTATION

Model structure

Consider the following SISO discrete time-invariant bilinear system

$$A(z^{-1})y(t)=B(z^{-1})u(t-1)+D(z^{-1})u(t-1)y(t-1)+e(t)/\Delta, \tag{1}$$

where $y(t)$ is the output, $u(t)$ is the input, $e(t)$ denotes white noise independent of the observations. $\Delta=1-z^{-1}$ is the difference operator. z is the z transfer operator.

$$A(z^{-1})=1+\sum_{i=1}^{n_a}a_i z^{-i},\ B(z^{-1})=\sum_{i=0}^{n_b}b_i z^{-i},$$
$$D(z^{-1})=\sum_{i=0}^{n_b}\sum_{j=0}^{n_a}d_{ij}z^{-i}z^{-j}.$$

The bilinear term is

$$D(z^{-1})u(t-1)y(t-1)=\sum_{i=0}^{n_b}\sum_{j-0}^{n_a}d_{ij}u(t-1-i)y(t-1-j).$$

Lemma 1 (Peng *et al.*, 2002; Priestley, 1980) Nonlinear system

$$y(t)=f(\boldsymbol{Y}_{t-1}^{t-p},\boldsymbol{U}_{t-1}^{t-q},\boldsymbol{V}_{t-1}^{t-r})+e(t), \tag{2}$$

where $f(\cdot)$ is the unknown nonlinear function,

$$\boldsymbol{Y}_{t-1}^{t-p}=[y(t-1),y(t-2),\cdots,y(t-p)],$$
$$\boldsymbol{U}_{t-1}^{t-q}=[u(t-1),u(t-2),\cdots,u(t-q)],$$
$$\boldsymbol{V}_{t-1}^{t-r}=[v(t-1),v(t-2),\cdots,v(t-r)],$$

can be described by the following global AR model:

$$\begin{aligned}&y(t)=\varphi_0(\boldsymbol{X}(t-1))+\boldsymbol{\Phi}(\boldsymbol{X}(t-1))\boldsymbol{X}(t-1)+e(t),\\&\boldsymbol{X}(t-1)=[\boldsymbol{Y}_{t-1}^{t-p},\boldsymbol{U}_{t-1}^{t-q},\boldsymbol{V}_{t-1}^{t-r}]^{\mathrm{T}}.\end{aligned} \tag{3}$$

Eq.(3) is the global linear model of the original nonlinear model shown in Eq.(2); the coefficients of Eq.(3) are expressed by some functions of the state $\boldsymbol{X}(t-1)$. Based on Lemma 1, considering the bilinear system Eq.(1) to be controlled, introduce the following method to construct a global linear model for the bilinear system.

Define

$$\boldsymbol{W}(t-1)=[\boldsymbol{Y}_{t-1}^{t-n_a},\boldsymbol{U}_{t-1}^{t-n_b-1}]^{\mathrm{T}}. \tag{4}$$

Eq.(1) can be rewritten as

$$A(z^{-1})y(t)=[B(z^{-1})+D(z^{-1})\,y(t-1)]\,u(t-1)+e(t)/\Delta. \tag{5}$$

Introduce a SVM (Suykens and Vandewalle, 1999) to construct the global linear model for Eq.(5), and the derived SVM-ARX model is

$$\begin{aligned}y(t)=\varphi_0(\boldsymbol{W}(t-1))+\sum_{r=1}^{n_a}\phi_{y,r}(\boldsymbol{W}(t-1))y(t-r)\\+\sum_{r=0}^{n_b}\phi_{u,r}(\boldsymbol{W}(t-1))u(t-r-1)+e(t)/\Delta.\end{aligned} \tag{6}$$

Compare Eq.(5) with Eq.(6), let

$$\varphi_0(\boldsymbol{W}(t-1))=0,$$
$$\phi_{y,r}(\boldsymbol{W}(t-1))=a_r\ (r=1,2,\ldots,n_a),$$
$$\phi_{u,r}(\boldsymbol{W}(t-1))=\sum_{i=1}^{N}\alpha_{i,r}\exp\{-\|\boldsymbol{W}(t-1)-\boldsymbol{W}_i(t-1)\|_2^2/\sigma^2\}+\beta$$
$$(r=0,\ 1,\ \ldots,\ n_b), \tag{7}$$

where $\alpha_{i,r}$, σ^2, β are the parameters of SVM, $\boldsymbol{W}_i(t-1)$ is the ith item of training data, N is the number of training data.

Note that the parameters of the global linear model shown in Eq.(6) are functions of $\boldsymbol{W}(t-1)$, and local linearization of the system can also be obtained at any operating point by fixing $\boldsymbol{W}(t-1)$ into Eq.(6), it means that the evolution of the process at time $t-1$ is governed by a set of coefficients $\{\phi_{y,r}(\boldsymbol{W}(t-1)),\ \phi_{u,r}(\boldsymbol{W}(t-1))\}$, all of which depend on the operating point of the process at time $t-1$.

Note that in Eq.(5)

$$[B(z^{-1})+D(z^{-1})y(t-1)]u(t-1) = \left[\sum_{i=0}^{n_b} b_i z^{-i} + \sum_{i=0}^{n_b} z^{-i} \sum_{j=0}^{n_a} d_{ij} y(t-1-j)\right] u(t-1). \quad (8)$$

And the ith term of Eq.(8) is

$$\left(b_i + \sum_{j=0}^{n_a} d_{ij} y(t-1-j)\right) u(t-1-i).$$

Using the operating point dependent coefficients of Eq.(6), especially in view of the perfect properties of SVM in function approximation, makes the SVM-ARX model very efficient in representing the behaviour of the system at each operating point. In this paper, Eq.(6) is taken as the internal model for the predictive controller design. All the parameters of Eq.(6) are identified off-line, which avoids the problems of on-line parameter estimation, such as divergence and computing burden, etc.

Off-line parameter identification

The parameter estimation of SVM model shown in Eq.(7) is a nonlinear parameter optimization problem. For the SVM model Eq.(7), select the input-output data within the whole operating area, and take $\boldsymbol{X}_k = [\boldsymbol{Y}_{t-1}^{t-n_a}, \boldsymbol{U}_{t-1}^{t-n_b-1}]^{\mathrm{T}}$ $(k=1,2,\cdots,N)$ as the input set, $d_k = b_i + \sum_{j=0}^{n_a} d_{ij} y(t-1-j)$ (i=0, 1, …, n_b; k=1, 2, …, N) as the output set.

The cost function is selected as

$$\min_{w,\beta,e} J(\boldsymbol{w},e) = \frac{1}{2}\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w} + \frac{1}{2}\gamma\sum_{k=1}^{N} e_k^2, \quad (9)$$

such that $d_k=\boldsymbol{w}^{\mathrm{T}}\varphi(\boldsymbol{X}_k)+\theta+e_k$ (k=1, 2, …, N) with $\varphi(\cdot)$: $\mathbb{R}^n \rightarrow \mathbb{R}^{n_h}$ a function which maps the input space into a so-called higher dimensional (possibly infinite dimensional) feature space, weight vector $\boldsymbol{w}\in\mathbb{R}^{n_h}$ in primal weight space, error variables $e_k\in\mathbb{R}$ and bias term θ.

After elimination of $\boldsymbol{w}$ and e, the solution is

$$\begin{bmatrix}\theta \\ \boldsymbol{\alpha}\end{bmatrix} = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{1}^{\mathrm{T}} \\ \boldsymbol{1} & \boldsymbol{\Omega}+\gamma^{-1}\boldsymbol{I}\end{bmatrix}^{-1} \begin{bmatrix}\boldsymbol{0} \\ \boldsymbol{D}\end{bmatrix}, \quad (10)$$

where $\boldsymbol{D}=[d_1,d_2,\ldots,d_N]^{\mathrm{T}}$, $\boldsymbol{1}=[1,1,\ldots,1]^{\mathrm{T}}$, $\boldsymbol{\alpha}=[\alpha_{1,r},\alpha_{2,r},\ldots,\alpha_{N,r}]^{\mathrm{T}}$, $\Omega_{ij}=\varphi(\boldsymbol{X}_i)^{\mathrm{T}}\varphi(\boldsymbol{X}_j)=K(\boldsymbol{X}_i,\boldsymbol{X}_j)$, i, j=1, 2, …, N, $\boldsymbol{I}$ is the unit matrix with proper dimension.

PREDICTIVE CONTROL BASED ON SVM-ARX MODEL

Multi-step-ahead prediction

For simplicity, let $n=n_a$, $m=n_b+1$, then Eq.(6) can be written as

$$y(t)=A_1y(t-1)+\ldots+A_{n+1}y(t-n-1)+B_{1,0}\Delta u(t-1) +\ldots+B_{1,m-1}\Delta u(t-m)+e(t), \quad (11)$$

where $A_1=1+\phi_{y,1}$, $A_i=\phi_{y,i}-\phi_{y,i-1}$ (i=2, 3, …, n), $A_{n+1}=-\phi_{y,n}$, $B_{1,i}=\phi_{u,i}$ (i=0, 1, …, m−1).

The optimal predictive output $\boldsymbol{Y}$ consists of three parts: the first part is determined by past inputs and outputs, which is denoted as $\boldsymbol{Y}_{\text{past}}$; the second part is determined by present and future inputs, which is denoted as $\boldsymbol{GU}$; the third part is prediction error, denoted as $\boldsymbol{E}$. Note that for simplicity, take both the prediction horizon and control horizon as p, and $\boldsymbol{E}$ is just the feedback correction part whose elements are $y(t)-\hat{y}(t)$, where $\hat{y}(t)$ is the estimated output of the process.

$$\boldsymbol{Y}=\boldsymbol{Y}_{\text{past}}+\boldsymbol{GU}+\boldsymbol{E}, \quad (12)$$

where

$\boldsymbol{Y}=(\hat{y}(t+1/t), \hat{y}(t+2/t), \ldots, \hat{y}(t+p/t))^{\mathrm{T}}$,

$\boldsymbol{Y}_{\text{past}}=(y_{\text{past}}(t+1), y_{\text{past}}(t+2), \ldots, y_{\text{past}}(t+p))^{\mathrm{T}}$,

$\boldsymbol{U}=(\Delta u(t), \Delta u(t+1), \ldots, \Delta u(t+p-1))^{\mathrm{T}}$,

$\boldsymbol{E}=(y(t)-\hat{y}(t), y(t)-\hat{y}(t), \ldots, y(t)-\hat{y}(t))^{\mathrm{T}}$,

$$\boldsymbol{G} = \begin{bmatrix} B_{1,0} & & & \\ B_{2,0} & B_{1,0} & 0 & \\ \ldots & \ldots & & \\ B_{p,0} & B_{p-1,0} & \ldots & B_{1,0} \end{bmatrix}.$$

$\boldsymbol{Y}_{\text{past}}$ can be calculated through Eq.(6), the elements in $\boldsymbol{G}$ are calculated as follows (Jin and Gu, 1990):

$$B_{1,0}=\phi_{u,0},$$

$$B_{k,0} = \phi_{u,k-1} + \sum_{j=1}^{k-1} A_j B_{k-j,0}, \quad k=2, \ldots, p. \quad (13)$$

Predictive control law

The reference trajectory is

$$y_r(t)=y(t),$$
$$y_r(t+k)=\mu^k y(t)+(1-\mu^k)y_s,\quad k=1,2,\ldots,p,\qquad(14)$$

where μ is the smoothing factor, y_s is the set point.

Let us form a trajectory vector and a cost function as

$$\boldsymbol{Y}_r=(y_r(t+1), y_r(t+2), \ldots, y_r(t+p))^T,$$
$$J=\min\{(\boldsymbol{Y}_r-\boldsymbol{Y})^T(\boldsymbol{Y}_r-\boldsymbol{Y})+\lambda^2\boldsymbol{U}^T\boldsymbol{U}\},\qquad(15)$$

where λ^2 is the weighting part on control increments, from $\partial J/\partial \boldsymbol{U}=0$, the control increments are

$$\boldsymbol{U}=(\boldsymbol{G}^T\boldsymbol{G}+\lambda^2\tilde{\boldsymbol{I}})^{-1}\boldsymbol{G}^T(\boldsymbol{Y}_r-\boldsymbol{Y}_{past}-\boldsymbol{E}),\qquad(16)$$

where $\tilde{\boldsymbol{I}}$ is the unit matrix with appropriate dimension.

Define $\boldsymbol{q}^T$ as the first row of $(\boldsymbol{G}^T\boldsymbol{G}+\lambda^2\tilde{\boldsymbol{I}})^{-1}\boldsymbol{G}^T$, then

$$u(t)=u(t-1)+\boldsymbol{q}^T(\boldsymbol{Y}_r-\boldsymbol{Y}_{past}-\boldsymbol{E}).\qquad(17)$$

This algorithm is summarized as follows:

Step 1: Get the process output $y(t)$ and substitute it into Eq.(6) to gain the linear model at the present operating point;

Step 2: Calculate $\boldsymbol{Y}_{past}$ using Eq.(6);

Step 3: Calculate the reference trajectory using Eq.(14);

Step 4: Calculate the elements of $\boldsymbol{G}$ using Eq.(13);

Step 5: Get the control law $u(t)$ using Eq.(17);

Step 6: Return to Step 1.

SIMULATION STUDIES

Example 1 A minimum phase system

$$y(t)-1.5y(t-1)+0.7y(t-2)$$
$$=u(t-1)-0.7u(t-2)+0.3u(t-1)y(t-1)$$
$$+0.1u(t-2)y(t-2)+e(t)/\Delta.$$

The global off-line SVM-ARX model of bilinear system Eq.(18) is derived through the method described in Section 2, the control parameters are p=5, λ^2=1, μ=0.65, $e(t)$ is white noise independent of the observations ranging from −0.01 to 0.01. This paper gives a comparison of the proposed method with Liu's method (Liu and Li, 2004) which is recognized widely in the area. The simulation procedure is as follows:

A unit step change is added to the setpoint input at time t=0 s and a step change of load disturbance with magnitude −0.2 is added to the process at time t=150 s. Simulation results are illustrated in Fig.1. It can be seen that the proposed method results in the improved tracking performance (Fig.1a) and disturbance rejection (Fig.1b).

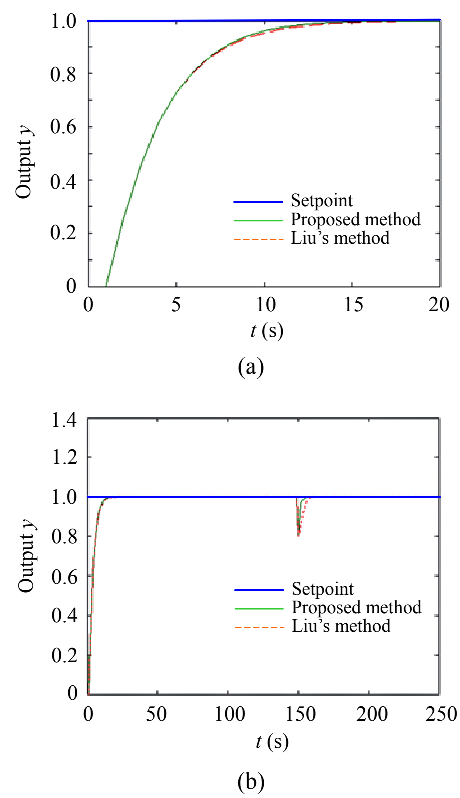

Fig.1 Tracking performance (a) and disturbance rejection (b) for Example 1

Example 2 A nonminimum phase system (Lakhdari *et al*., 1995)

Simulation conditions remain the same as those in Example 1, tracking performance is shown in Fig.2 and disturbance rejection performance is shown in Fig.3. It is seen that the proposed method provides satisfactory tracking performance (Fig.2a) and disturbance rejection (Fig.3a).

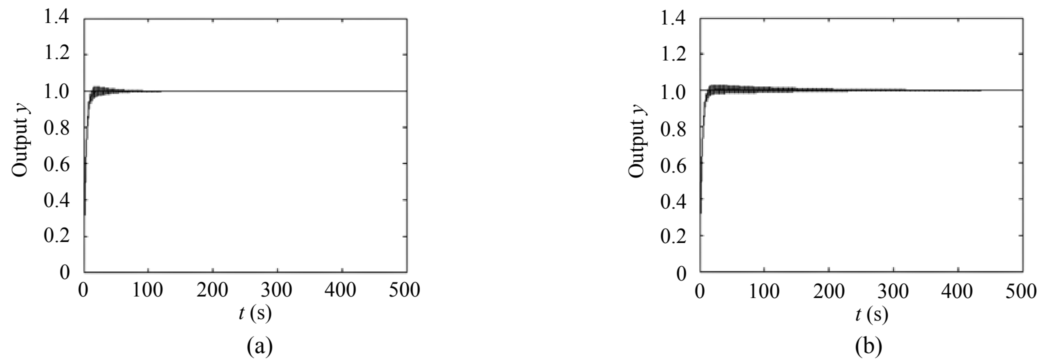

Fig.2 Tracking performance of proposed method (a) and Liu's method (b) for Example 2

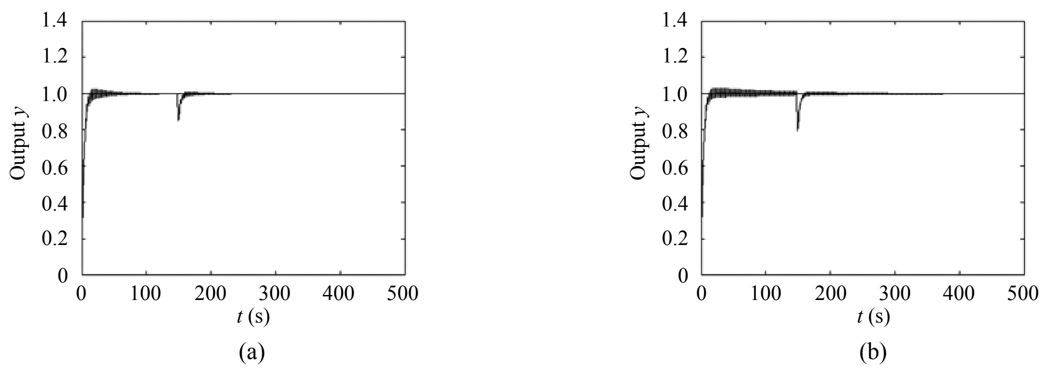

Fig.3 Disturbance rejection performance of proposed method (a) and Liu's method (b) for Example 2

CONCLUSION

This paper proposed a new off-line SVM-ARX model based predictive control method for a class of bilinear systems. The SVM-ARX model has the advantages of linear model and the merit of support vector machine (SVM) in function approximation. The derived off-line model can represent the whole behavior of the bilinear systems and thus on-line parameter estimation is not necessary.

A derived off-line model based MPC is proposed with good performance results. The control law is analytical and on-line nonlinear optimization is not required anymore.

References

Bloemen, H.H.J., van den Boom, T.J.J., Verbruggen, H.B., 2001. An Optimization Algorithm Dedicated to a MPC Problem for Discrete Time Bilinear Models. Proceedings of the American Control Conference, Arlington, VA, p.2371-2381.

Clarke, D.W., Mohtadi, C., Tuffs, P.S., 1987. Generalized predictive control—part I. the basic algorithm. *Automatica*, **23**(2):137-148. [doi:10.1016/0005-1098(87)90087-2]

Fontes, A.B., Maitelli, A.L., Cavalcanti, A.L.O., 2004. Bilinear Compensed Generalized Predictive Control: An Adaptive Approach. 5th Asian Control Conference, Melbourne, Australia, p.1781-1785.

He, J.C., Yang, M.Y., Yu, L., Chen, G.D., 1999. Predictive control of a class of generalized bilinear systems. *Mechatronic Engineering*, **16**(5):225-226 (in Chinese).

Jin, Y.Y., Gu, X.Y., 1990. Improved generalized predictive control. *Information and Control*, (3):8-14 (in Chinese).

Lakhdari, Z., Mokhtari, M., Lecluse, Y., Provost, J., 1995. Adaptive Predictive Control of a Class of Nonlinear Systems—A Case Study. IFAC Proceedings: Adaptive Systems in Control and Signal Processing, Budapest, Hungary, p.209-214.

Liu, G.Z., Li, P., 2004. Generalized Predictive Control for a Class of Bilinear Systems. IFAC 7th Symposium on Advanced Control of Chemical Processes, Hong Kong, China, p.952-956.

Peng, H., Ozaki, T., Toyoda, Y., Haggan-Ozaki, V., 2002. Nonlinear Predictive Control Based on a Global Model Identified Off-line. Proceedings of the American Control Conference, Anchorage, AK, p.8-10.

Priestley, M.B., 1980. State dependent models: a general approach to nonlinear time series analysis. *Journal of Time Series Analysis*, **1**:57-71.

Suykens, J.A.K., Vandewalle, J., 1999. Least square support vector machine classifiers. *Neural Processing Letters*, **9**(3):293-300. [doi:10.1023/A:1018628609742]

Yao, X.Y., Qian, J.X., 1997. Generalized predictive control of algorithm of bilinear system. *Journal of Zhejiang University: Engineering Science*, **31**(2):231-236 (in Chinese).