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A new approach for target motion analysis with signal time delay*

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Abstract: A new approach is proposed in this paper for the problem of the target motion analysis (TMA) with signal propagation time delay. This problem is an unobservable tracking problem in which the acoustic signal transmits with time delay. We present an intelligent range parameterized unscented Kalman filter (IRPUKF) algorithm to estimate the state of the nonlinear unobservable tracking system and propose a recursive model parameter online adjustment method to deal with the time delay in signal propagation. In a simulation of tracking target using a maneuvering acoustic sensor with signal time delay case study, the effectiveness and efficiency of the proposed algorithm is testified to perform better, compared with the range parameterized extended Kalman filter (RPEKF) algorithm.

Key words: Target motion analysis (TMA), Signal time delay, IRPUKF
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INTRODUCTION

In a standard target motion analysis (TMA) problem, we want to estimate the target kinematics parameter using the bearings only from measurement of one maneuvering sensor platform or sensor array (Kronhamn, 1998). The motion of the target is assumed to be constrained to a straight line, constant-speed segments separated by maneuvers in course and speed (Peach, 1995). The range parameterized extended Kalman filter (RPEKF) algorithm (Peach, 1995) can be applied to this case with satisfying result. Here we focus on a special TMA issue, in which the signal transmits with time delay. This problem arises in many important practical applications, such as submarine tracking with sonar or aircraft surveillance with radar in a passive mode (Ristic *et al*., 2004). Some batch type estimation algorithms for this case have been presented, such as the nonlinear square (NLS) method (Lo *et al*., 2000), the maximum likelihood (ML) method (Dommermuth and Schiller, 1984) and the Taylor series (TS) method (Foy, 1976), etc. These approaches all need measurement data from sensor array or sensor network. In this paper, we propose a recursive intelligent range parameterized unscented Kalman filter (IRPUKF) algorithm which only needs data from a single maneuvering sensor. It combines the merit of the RPEKF and the variable structure multiple model (VSMM) algorithm (Li *et al*., 2005) and can deal with the bearings only target tracking with signal time delay.

PROBLEM DESCRIPTION

System model

In general, we consider the TMA problem in 2D space for simplicity (Passerieux and van Cappel, 1998). Assume the target kinematics parameter in Cartesian coordinate at time k is $\boldsymbol{X}_k^{\mathrm{t}}=[x_k^{\mathrm{t}}, v_{x,k}^{\mathrm{t}}, y_k^{\mathrm{t}}, v_{y,k}^{\mathrm{t}}]^{\mathrm{T}}$, where $(x_k^{\mathrm{t}}, y_k^{\mathrm{t}})$ is the target position and $(v_{x,k}^{\mathrm{t}}, v_{y,k}^{\mathrm{t}})$ is the target velocity. Because of the time

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delay in the signal propagation time delay—the speed of the target is comparable with the sound in air—the acoustic signal emitted by the target at time k will arrive at the sensor at time k'. The sensor state at time k' is denoted as $\boldsymbol{X}_k^{\mathrm{s}}=[x_k^{\mathrm{s}}, v_{x,k}^{\mathrm{s}}, y_k^{\mathrm{s}}, v_{y,k}^{\mathrm{s}}]^{\mathrm{T}}$. k and k' satisfy the following time delay function:

$$k' = k + R_{\mathrm{t,s}}(k)/c, \tag{1}$$

where $R_{\mathrm{t,s}}(k)$ is the distance between the target (at time k) and the sensor (at time k'), c is the sound speed in air. It is easy to conclude that if the sensor scan period ΔT is constant, the signal emitted interval T_k will be variable. Define the system state as $\boldsymbol{X}_k = \boldsymbol{X}_k^{\mathrm{t}} - \boldsymbol{X}_k^{\mathrm{s}} = [x_k, v_{x,k}, y_k, v_{y,k}]^{\mathrm{T}}$ and $\boldsymbol{z}_{k'}$ as the noise-corrupted bearing measurement, we can get the following dynamic model:

$$\boldsymbol{X}_{k+1} = \boldsymbol{F}_k \boldsymbol{X}_k + \boldsymbol{G}_k \boldsymbol{v}_k - \boldsymbol{a}_k^{\mathrm{s}}, \tag{2}$$

$$\boldsymbol{z}_{k'} = \boldsymbol{h}(\boldsymbol{X}_k) + \boldsymbol{w}_{k'}, \tag{3}$$

where

$$\boldsymbol{F}_k = \begin{bmatrix} 1 & T_k & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T_k \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ \boldsymbol{G}_k = \begin{bmatrix} T_k & 0 \\ T_k^2/2 & 0 \\ 0 & T_k \\ 0 & T_k^2/2 \end{bmatrix}, \tag{4}$$

and

$$\boldsymbol{a}_k^{\mathrm{s}} = \begin{bmatrix} x_{(k+1)'}^{\mathrm{s}} - x_{k'}^{\mathrm{s}} - \Delta T \cdot v_{x,k'}^{\mathrm{s}} \\ v_{x,(k+1)'}^{\mathrm{s}} - v_{x,k'}^{\mathrm{s}} \\ y_{(k+1)'}^{\mathrm{s}} - y_{k'}^{\mathrm{s}} - \Delta T \cdot v_{y,k'}^{\mathrm{s}} \\ v_{y,(k+1)'}^{\mathrm{s}} - v_{y,k'}^{\mathrm{s}} \end{bmatrix}, \tag{5}$$

$$\boldsymbol{h}(\boldsymbol{X}_k) = \begin{cases} \tan^{-1}(y_k/x_k), & x_k>0, y_k>0; \\ \tan^{-1}(y_k/x_k)+\pi, & x_k<0, y_k>0; \\ \tan^{-1}(y_k/x_k)+\pi, & x_k<0, y_k<0; \\ \tan^{-1}(y_k/x_k), & x_k>0, y_k<0, \end{cases} \tag{6}$$

$\boldsymbol{v}_k$, $\boldsymbol{w}_{k'}$ are i.i.d. zero-mean Gaussian white noise vectors with covariance matrices $\boldsymbol{Q}$, $\boldsymbol{R}$. $\boldsymbol{a}_k^{\mathrm{s}}$ caters for the sensor accelerations (Ristic *et al*., 2004).

The time delay function can be converted into the following equation:

$$\frac{R_{\mathrm{t,s}}(k)}{c} + \Delta T = \frac{R_{\mathrm{t,s}}(k+1)}{c} + \frac{R_{k,k+1}}{c_{\mathrm{t}}},$$

where $R_{k,k+1}$ is the displacement of the target from time k to time k+1, c_{t} is the target velocity. Denoting the state estimation at time k as $\hat{\boldsymbol{X}}_{k|k}$, yields the following formula:

$$\frac{\sqrt{\hat{x}_{k|k}^2 + \hat{y}_{k|k}^2}}{c} + \Delta T = \frac{\sqrt{\hat{x}_{k+1|k}^2 + \hat{y}_{k+1|k}^2}}{c} + \frac{\sqrt{(\hat{x}_{k+1|k} - \hat{x}_{k|k})^2 + (\hat{y}_{k+1|k} - \hat{y}_{k|k})^2}}{\sqrt{\hat{v}_{x,k|k}^2 + \hat{v}_{y,k|k}^2}}. \tag{7}$$

Eqs.(2), (3) and (7) form the TMA model with signal time delay.

IRPUKF ALGORITHM

Division and intelligent management

The division strategy of the range interval is like the RPEKF algorithm. Suppose the target valid range interval is ($R_{\min}$,$R_{\max}$), divide it into $N_{F,k}$ sub-intervals according to geometrical progression as follows (Gai *et al*., 2005; Julier *et al*., 2000):

$$\rho = (R_{\max}/R_{\min})^{1/N_{F,k}}, \ r^{(i)} = \frac{R_{\min}}{2}(\rho^i + \rho^{i-1}), \tag{8}$$

$$C_R = \frac{\sigma^{(i)}}{r^{(i)}} = \frac{2(\rho-1)}{\sqrt{12}(\rho+1)} \Rightarrow \sigma^{(i)} = r^{(i)} C_R, \tag{9}$$

where ρ is the ratio, $r^{(i)}$ and $\sigma^{(i)}$ are the range and standard deviation of the ith subinterval, and C_R is the variance. Each subinterval is endowed with an unscented Kalman filter and its probability $w_k^{(i)}$ is updated according to Baye's rule:

$$w_k^{(i)} = p(i\,|\,\boldsymbol{Z}^{k'}) = \frac{p(\boldsymbol{z}_{k'}\,|\,i)p(i\,|\,\boldsymbol{Z}^{(k-1)'})}{c_{k'}} = \frac{p(\boldsymbol{z}_{k'}\,|\,i)w_{k-1}^{(i)}}{\sum_{j=1}^{N_{F,k}} p(\boldsymbol{z}_{k'}\,|\,j)w_{k-1}^{(j)}}, \tag{10}$$

$p(\boldsymbol{z}_{k'}\,|\,i)$ is the likelihood of the measurement.

During the recursive course, some tracker weight becomes so small that it has little effect on the state estimation. We design a threshold P_{LB} for the tracker weight and delete the subintervals whose weight is smaller than it. Under the Gaussian assumption, the low-bound is given according to the "3σ rule":

$$p(\hat{X}_{k|k} - 3\sigma \le \hat{X}_{k|k}^{(i)} \le \hat{X}_{k|k} + 3\sigma) \le 1 - P_{\rm LB}, \quad (11)$$

$$P_{\rm LB} = 1 - \Phi(3), \quad (12)$$

$$\Phi(\xi) = \int_{-\infty}^{\xi} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du, \quad (13)$$

where $\Phi(\xi)$ is the standard normal cumulative probability function. If $w_k^{(i)} \le P_{\rm LB}$, the corresponding subinterval is deleted and the other weights are renormalized.

$$\bar{w}_k^j = \frac{w_k^j}{\sum_{1 \le p \le N_{F,k}, p \ne i} w_k^p}. \quad (14)$$

In another case, the maximum weight subinterval may be in the margin of the range interval. We will add a new subinterval close to it and assign an initial weight to the new tracker. See Fig.1.

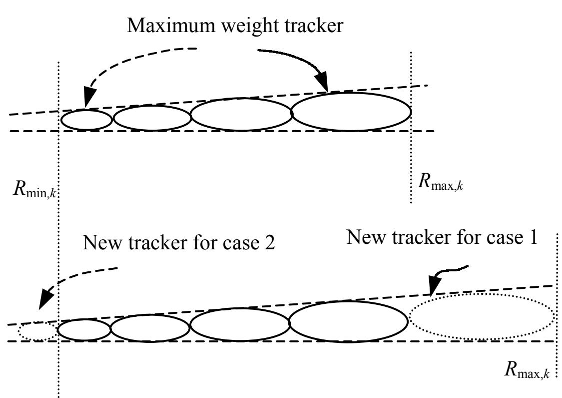

Fig.1 The augmentation of the range interval

The weights are renormalized as follows:

$$\bar{w}_k^{(i)} = \frac{w_k^{(i)} N_{F,k}}{N_{F,k} + 1}, \ 1 \le i \le N_{F,k}, \quad (15)$$

$$\bar{w}_k^{(N_{F,k}+1)} = 1 - \sum_{i=1}^{N_{F,k}} \bar{w}_k^{(i)}. \quad (16)$$

Using the intelligent management method, we reduce the elapsed time and improve the precision of the algorithm.

Algorithm flow

Step 1: Initialization

The range interval is divided into $N_{F,0}$ sub regions with each initial weight $w_0^{(i)} = 1/N_{F,0}$. The posterior probability density of the ith filter is assumed Gaussian $p(X_0^{(i)} | Z^{0'}) = N(X_0^{(i)}; \hat{X}_{0|0}^{(i)}, P_{0|0}^{(i)})$.

$$\hat{X}_{0|0}^{(i)} = [r_0^{(i)} \cos(z_{0'}), c_{t0} \cos(z_{0'}), r_0^{(i)} \sin(z_{0'}), c_{t0} \sin(z_{0'})]^{\rm T}, \quad (17)$$

$$P_{0|0}^{(i)} = \begin{bmatrix} (\sigma_0^{(i)})^2 & & & \\ & \sigma_{v,0}^2 & & \\ & & (\sigma_0^{(i)})^2 & \\ & & & \sigma_{v,0}^2 \end{bmatrix}. \quad (18)$$

Represent the density by a set of $2a$+1 sample points $\{X_0^{(i)(j)}\}_{j=1}^{2a+1}$ and their weights $\{\omega_0^{(i)(j)}\}_{j=1}^{2a+1}$ according to the unscented transform (Ristic *et al.*, 2004) (Julier *et al.*, 2000), where a is the dimension of the system state:

$$\begin{aligned} X_0^{(i)(0)} &= \hat{X}_{0|0}^{(i)}, \quad \omega_0^{(i)(0)} = \frac{\kappa}{(a+\kappa)}, \\ X_0^{(i)(j)} &= \hat{X}_{0|0}^{(i)} + \left(\sqrt{(a+\kappa) P_{0|0}^{(i)}}\right)_i, \ j = 1, \ ..., \ a, \\ \omega_0^{(i)(0)} &= \frac{\kappa}{2(a+\kappa)}, \\ X_0^{(i)(j)} &= \hat{X}_{0|0}^{(i)} - \left(\sqrt{(a+\kappa) P_{0|0}^{(i)}}\right)_i, \ j = a+1, \ ..., \ 2a, \\ \omega_0^{(i)(0)} &= \frac{\kappa}{2(a+\kappa)}, \end{aligned} \quad (19)$$

where κ is a scaling parameter and $\left(\sqrt{(a+\kappa) P_{0|0}^{(i)}}\right)_i$ is the ith row of the matrix square root of $(a+\kappa) P_{0|0}^{(i)}$.

Step 2: Recursive estimation

(1) At time k, represent the density by a set of particles $\{X_k^{(i)(j)}\}_{j=1}^{2a+1}$ with weights $\{\omega_k^{(i)(j)}\}_{j=1}^{2a+1}$ according to the unscented transform;

(2) Estimate the model parameter online

for λ=0; $\lambda\le\Lambda$; λ++

{ $T_{k+1}^{(i)}=T_k^{(i)}+\lambda$;

$\hat{\boldsymbol{X}}_{k+1|k}^{(i)}=\boldsymbol{F}_{k+1}(T_{k+1}^{(i)})\hat{\boldsymbol{X}}_{k|k}^{(i)}$;

$$\text{if}\left\|\begin{gathered}\left(\frac{\sqrt{\hat{x}_{k|k}^2+\hat{y}_{k|k}^2}}{c}+\Delta T\right)-\left(\frac{\sqrt{\hat{x}_{k+1|k}^2+\hat{y}_{k+1|k}^2}}{c}\right.\\ \left.+\frac{\sqrt{(\hat{x}_{k+1|k}-\hat{x}_{k|k})^2+(\hat{y}_{k+1|k}-\hat{y}_{k|k})^2}}{\sqrt{\hat{v}_{x,k|k}^2+\hat{v}_{y,k|k}^2}}\right)\end{gathered}\right\|\le\gamma,$$

break;

}

end

Note that the search step λ may also take negative value in practice; the above code is just for simplicity. Λ is the parameter's perturbation bound, and can be calculated approximately as follows:

$$\Lambda_{\max}=\frac{\Delta T}{c-c_{\mathrm{t}}}\cdot c,\ \Lambda_{\min}=\frac{\Delta T}{c+c_{\mathrm{t}}}\cdot c. \tag{20}$$

If λ takes negative value, we use the upper bound and vice versa. Thus we get the estimated model parameter $T_{k+1}^{(i)}$.

(3) One-step prediction

$$\hat{\boldsymbol{X}}_{k+1|k}^{(i)}=\sum_{j=1}^{2a+1}\omega_k^{(j)}\boldsymbol{F}_{k+1}(T_{k+1}^{(i)})\hat{\boldsymbol{X}}_{k|k}^{(i)(j)}, \tag{21}$$

$$\boldsymbol{P}_{k+1|k}^{(i)}=\boldsymbol{Q}+\sum_{j=1}^{2a+1}\omega_k^{(j)}\left[\boldsymbol{F}_{k+1}(T_{k+1}^{(i)})\,\hat{\boldsymbol{X}}_{k|k}^{(i)(j)}-\hat{\boldsymbol{X}}_{k+1|k}^{(i)}\right]\cdot\left[\boldsymbol{F}_{k+1}(T_{k+1}^{(i)})\,\hat{\boldsymbol{X}}_{k|k}^{(i)(j)}-\hat{\boldsymbol{X}}_{k+1|k}^{(i)}\right]^{\mathrm{T}}, \tag{22}$$

$$\hat{\boldsymbol{z}}_{k+1|k}^{(i)}=\sum_{j=1}^{2a+1}\omega_k^{(j)}\boldsymbol{h}(\hat{\boldsymbol{X}}_{k+1|k}^{(j)}), \tag{23}$$

$$\boldsymbol{S}_{k+1}^{(i)}=\boldsymbol{R}+\sum_{j=1}^{2a+1}\omega_k^{(j)}\left[\boldsymbol{h}(\hat{\boldsymbol{X}}_{k+1|k}^{(i)(j)})-\hat{\boldsymbol{z}}_{k+1|k}^{(i)}\right]\cdot\left[\boldsymbol{h}(\hat{\boldsymbol{X}}_{k+1|k}^{(i)(j)})-\hat{\boldsymbol{z}}_{k+1|k}^{(i)}\right]^{\mathrm{T}}. \tag{24}$$

(4) Calculate the likelihood and the gain

$$\boldsymbol{K}_{k+1}^{(i)}=\sum_{j=1}^{2a+1}\omega_k^{(j)}\left[\hat{\boldsymbol{X}}_{k+1|k}^{(i)(j)}-\hat{\boldsymbol{X}}_{k+1|k}^{(i)}\right]\left[\hat{\boldsymbol{X}}_{k+1|k}^{(i)(j)}-\hat{\boldsymbol{X}}_{k+1|k}^{(i)}\right]^{\mathrm{T}}\cdot(\boldsymbol{S}_{k+1}^{(i)})^{-1}, \tag{25}$$

$$p(\boldsymbol{z}_{k'}\,|\,i)=\frac{1}{2\pi\sqrt{|\boldsymbol{S}_{k+1}^{(i)}|}}\exp\left\{\begin{gathered}-\frac{1}{2}[\boldsymbol{z}_{k'}-\boldsymbol{h}(\hat{\boldsymbol{X}}_{k+1|k}^{(i)})]\\ \cdot(\boldsymbol{S}_{k+1}^{(i)})^{-1}[\boldsymbol{z}_{k'}-\boldsymbol{h}(\hat{\boldsymbol{X}}_{k+1|k}^{(i)})]^{\mathrm{T}}\end{gathered}\right\}. \tag{26}$$

(5) Update the filter weights and manage the trackers intelligently

$$w_{k+1}^{(i)}=p(i\,|\,\boldsymbol{Z}^{k'})=\frac{p(\boldsymbol{z}_{k'}\,|\,i)w_k^{(i)}}{\sum_{i=1}^{N_{F,k}}p(\boldsymbol{z}_{k'}\,|\,i)w_k^{(i)}}. \tag{27}$$

If $\min\{w_k^{(i)}\}_{i=1}^{N_{F,k}}\le P_{\mathrm{LB}}$

{Delete the minimum weight tracker and renormalize the other weights, $N_{F,k+1}=N_{F,k}-1$}

Else if $\arg\max_i\{w_{k+1}^{(i)}\}_{i=1}^{N_{F,k}}=N_{F,k}$ or 1

{Add a new tracker by the corresponding side with a given weight and renormalize the other weights, $N_{F,k+1}=N_{F,k}+1$}

(6) Update

$$\hat{\boldsymbol{X}}_{k+1|k+1}^{(i)}=\hat{\boldsymbol{X}}_{k+1|k}^{(i)}+\boldsymbol{K}_{k+1}^{(i)}(\boldsymbol{z}_{k'}-\hat{\boldsymbol{z}}_{k+1|k}^{(i)}), \tag{28}$$

$$\boldsymbol{P}_{k+1|k+1}^{(i)}=\boldsymbol{P}_{k+1|k}^{(i)}-\boldsymbol{K}_{k+1}^{(i)}\boldsymbol{S}_{k+1}^{(i)}(\boldsymbol{K}_{k+1}^{(i)})^{\mathrm{T}}, \tag{29}$$

$$p(\boldsymbol{X}_{k+1}^{(i)}\,|\,\boldsymbol{Z}^{k'})=N(\boldsymbol{X}_{k+1}^{(i)};\hat{\boldsymbol{X}}_{k+1|k+1}^{(i)},\boldsymbol{P}_{k+1|k+1}^{(i)}). \tag{30}$$

(7) Output and return to Step 2(1)

$$\hat{\boldsymbol{X}}_{k+1|k+1}=\sum_{i=1}^{N_{F,k+1}}w_{k+1}^{(i)}\hat{\boldsymbol{X}}_{k+1|k+1}^{(i)}, \tag{31}$$

$$\boldsymbol{P}_{k+1|k+1}=\sum_{i=1}^{N_{F,k+1}}w_{k+1}^{(i)}(\boldsymbol{P}_{k+1|k}^{(i)}+[\hat{\boldsymbol{X}}_{k+1|k+1}^{(i)}-\hat{\boldsymbol{X}}_{k+1|k+1}]\cdot[\hat{\boldsymbol{X}}_{k+1|k+1}^{(i)}-\hat{\boldsymbol{X}}_{k+1|k+1}]^{\mathrm{T}}). \tag{32}$$

Performance analysis

For the nonlinear property of the problem, we replace the EKF with the UKF to improve the estimation precision at the cost of the elapsed time and computing resource. Each sample particle of the UKF can be regarded as a single Kalman filter, whose computational complexity approximately equals an extended Kalman filter. Except that, the proposed algorithm estimates the model parameter online using the linear search method to handle the signal time delay problem, the total elapsed time of the search method is related to the search step λ, the search original point and the bound γ. Denote the computation complexity of the EKF as Ω, the RPEKF is about $NF\times\Omega$, the IRPUKF is about $(2a+1)\times N_{F,k}\times\Omega+\Psi$. Ψ caters for the linear research consuming time.

From the above analysis, we can conclude that the IRPUKF algorithm improves the nonlinear estimation precision at the cost of the computation complexity. Compared with the batch type approaches, the proposed method costs less time.

Simulation

In the simulation, the IRPUKF approach is compared with the RPEKF for two cases: signal with time delay and signal without time delay. The given parameters are as follows: ΔT=1 s, c_t=200 m/s, c=340 m/s, $N_{F,0}$=5, a=4, $[R_{\min},R_{\max}]$=[0.5 km,0.5 km]. In the evaluation M=100 Monte Carlo simulations are performed. The track scenario is presented in Fig.2, Fig.3a and Fig.3b show the root mean square error curves for the range estimation of the two methods, respectively without and with signal time delay. Table 1 is the elapsed time of the methods.

Table 1 Elapsed time for 100 Monte Carlo simulations

	Elapsed time (s)	
	Without time delay	With time delay
RPEKF	4.278	4.286
IRPUKF	37.021	60.590

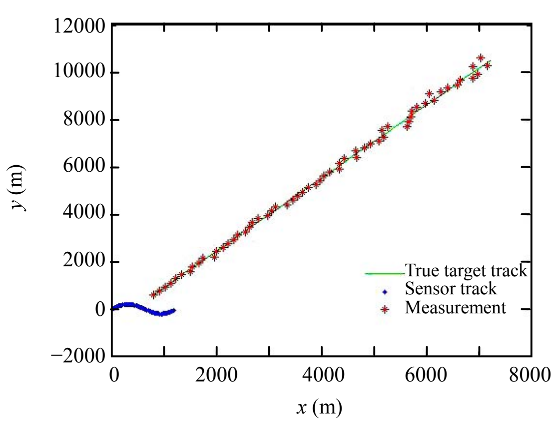

Fig.2 The tracking scenario illustration

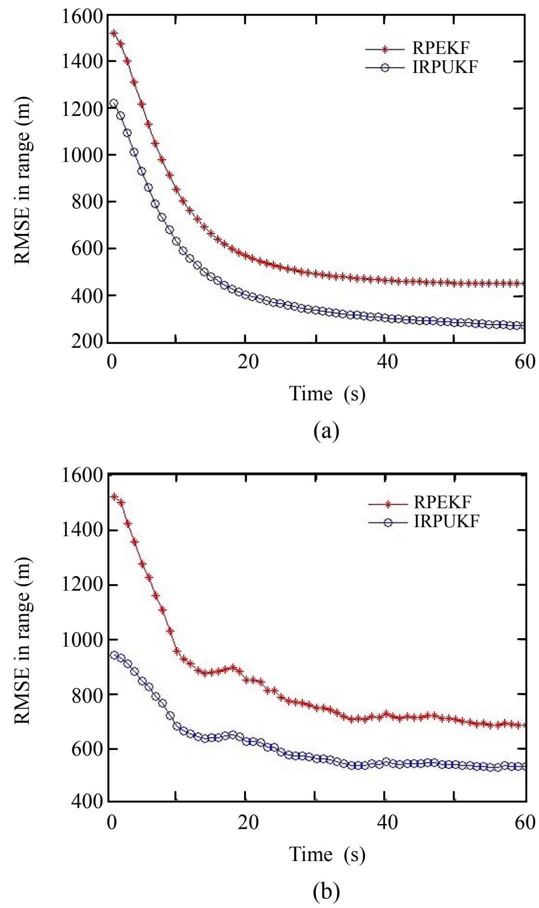

Fig.3 The RMSE in range comparison without signal time delay (a) and with signal time delay (b)

CONCLUSION

Based on the RPKEF approach and the VSMM method, we propose a new intelligent range parameterized unscented Kalman filter (IRPUKF) algorithm to track the target with bearings-only measurement. We also present a recursive line research method to estimate the model parameter T_k, which does not equal the scan period of the sensor in view of the signal time delay. In the simulation, the IRPUKF and RPEKF are compared respectively with and without time delay. The results verify the IRPUKF algorithm can improve the estimation precision and can deal with the bearings-only tracking with signal time delay. The elapsed time in one running is less than the sensor scan period and hence is acceptable.

References

Dommermuth, F., Schiller, J., 1984. Estimating the trajectory of an accelerationless aircraft by means of a stationary acoustic sensor. *Journal of Acoustical Society of America*, **76**(4):1114-1122. [doi:10.1121/1.391403]

Foy, W.H., 1976. Position-location solutions by Taylor-series estimation. *IEEE Trans. Aerospace and Electronic Systems*, **12**:187-194.

Gai, M.J., Yi, X., He, Y., Shi, B., 2005. An Approach to Tracking a 3D-target with 2D-radar. 2005 IEEE International Radar Conference, Arlington, Virginia, USA, p.763-768. [doi:10.1109/RADAR.2005.1435928]

Julier, S., Uhlmann, J., Durrant-White, H.F., 2000. A new method for nonlinear transformation of means and covariances in filters and estimators. *IEEE Trans. Automatic Control*, **45**(3):477-482. [doi:10.1109/9.847726]

Kronhamn, T.R., 1998. Bearings-only target motion analysis based on a multihypothesis Kalman filter and adaptive ownship motion control. *Radar, Sonar Navigation, IEE Proceedings*, **145**(4):247-252. [doi:10.1049/ip-rsn:1998 2130]

Li, X.R., Jilkov, V.P., Ru, J.F., 2005. Multiple-model estimation with variable structure—part VI: expected-mode augmentation. *IEEE Trans. Aerospace and Electronic Systems*, **41**(3):853-867. [doi:10.1109/TAES.2005.1541435]

Lo, K.W., Ferguson, B.G., 2000. Broadband passive acoustic technique for target motion parameter estimation. *IEEE Trans. Aerospace and Electronic Systems*, **36**(1):163-175. [doi:10.1109/7.826319]

Passerieux, J.M., van Cappel, D., 1998. Optimal observer maneuver for bearings-only tracking. *IEEE Trans. Aerospace and Electronic Systems*, **34**(3):777-788. [doi:10.1109/7.705885]

Peach, N., 1995. Bearings-only tracking using a set of range-parameterized extended Kalman filters. *Control Theory and Applications, IEE Proceedings*, **142**(1):73-80. [doi:10.1049/ip-cta:19951614]

Ristic, B., Arulampalam, S., Gordon, N., 2004. Beyond the Kalman Filter. Artech House, Boston, London, p.103-108.